Recent developments in planetary formation and migration theory

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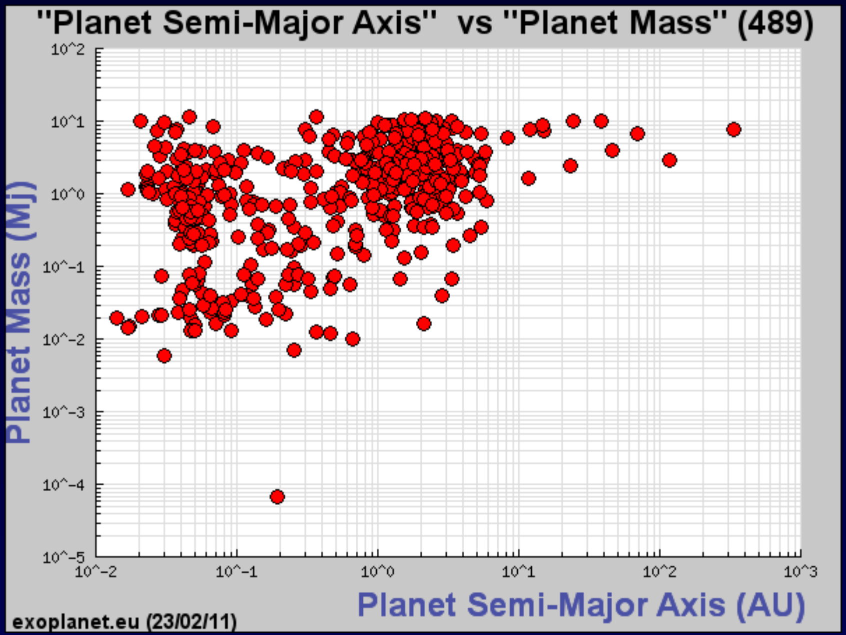
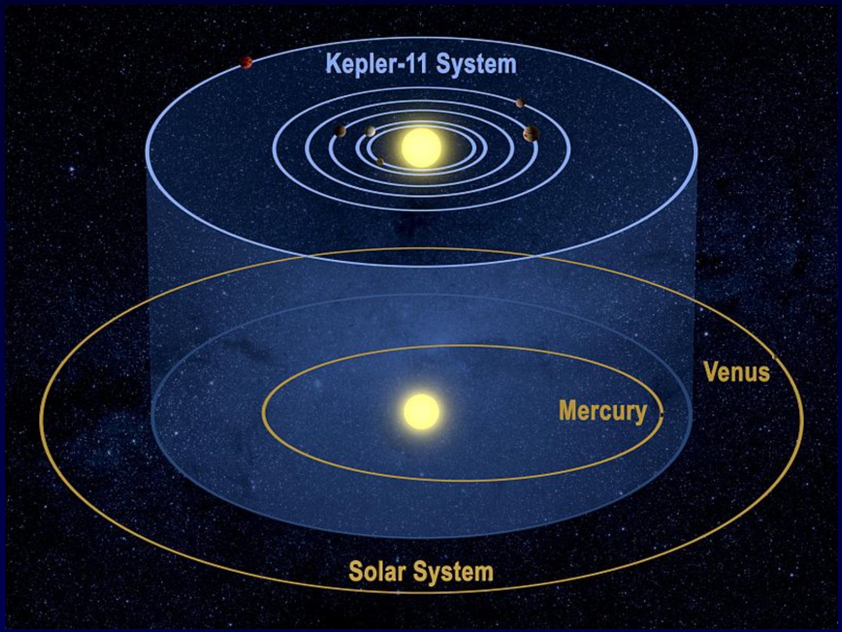
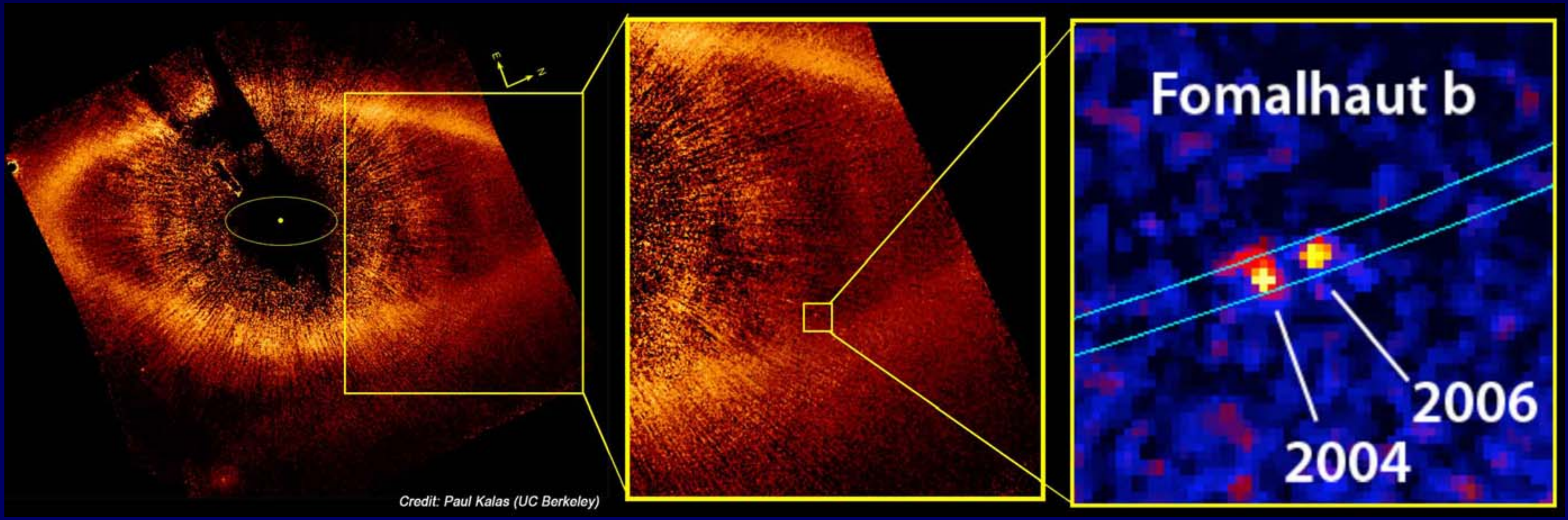
Coordinator of WP: Exoplanetary formation and orbital evolution

Queen Mary
University of London


"Planet Semi-Major Axis" vs "Planet Mass" (489)
Planet Mass (Mj)
Planet Semi-Major Axis (AU)
exoplanet.eu (23/02/11)


Kepler-11 System
Venus
Mercury
Solar System


Fomalhaut b
2006
2004
Credit: Paul Kalas (UC Berkeley)

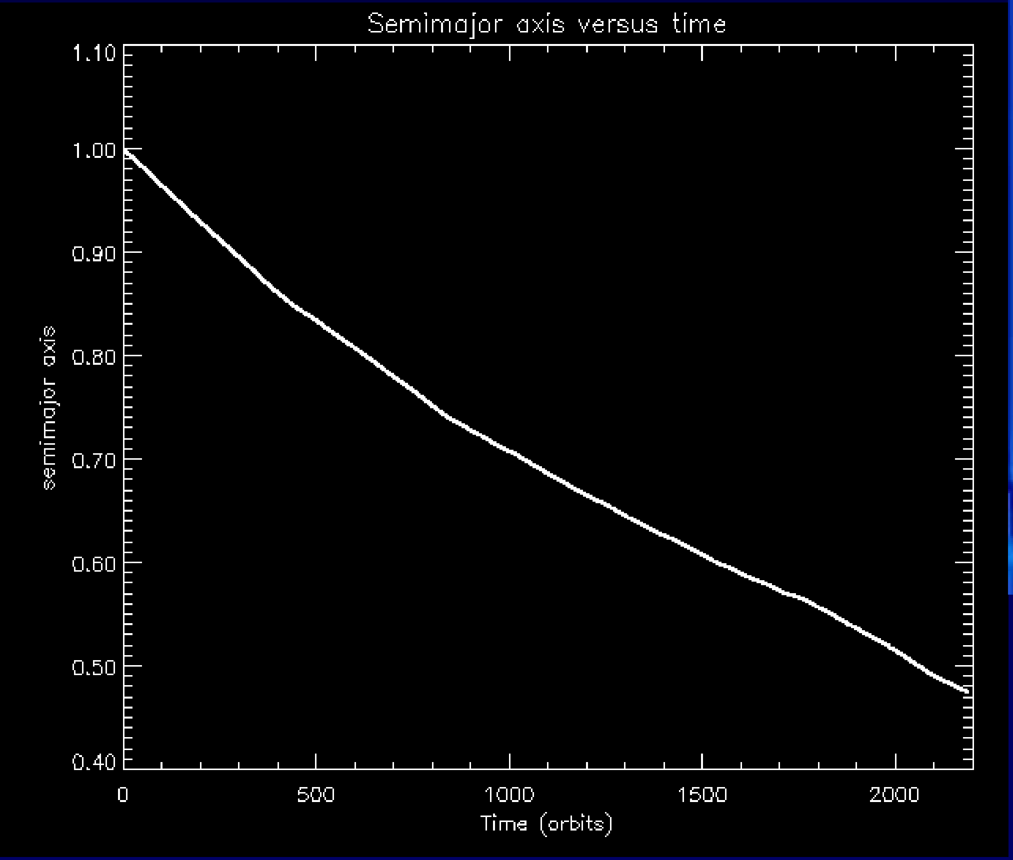
Planetary migration

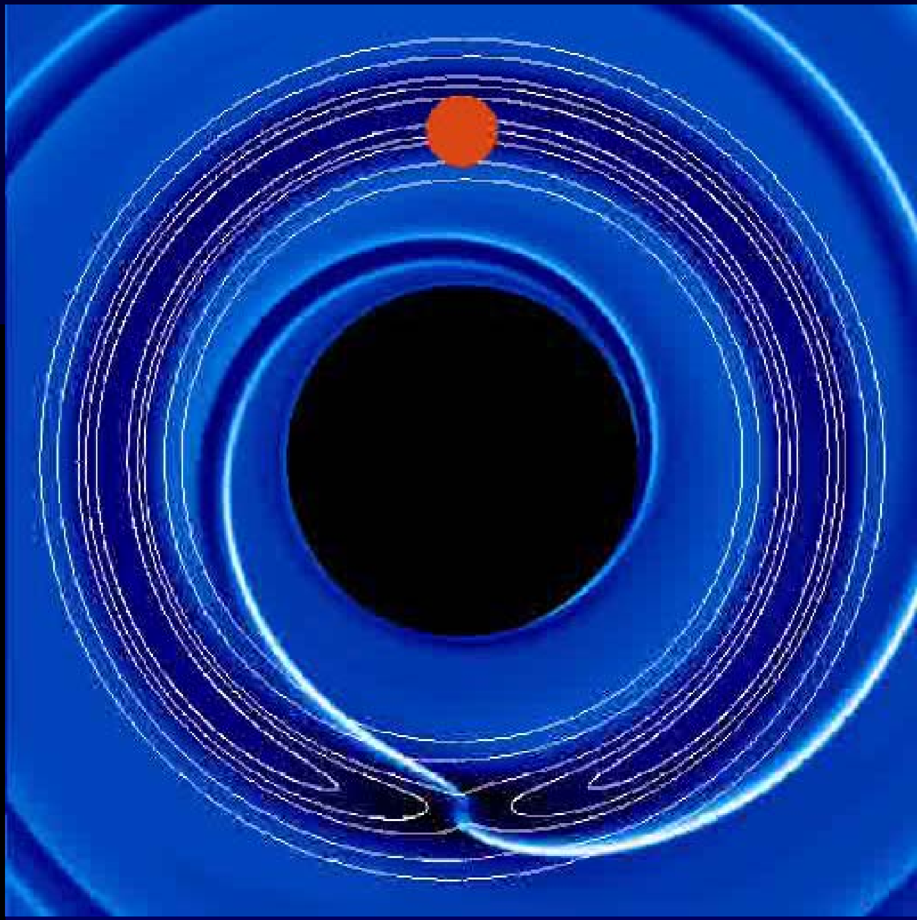
- Migration of planets can occur because of various processes

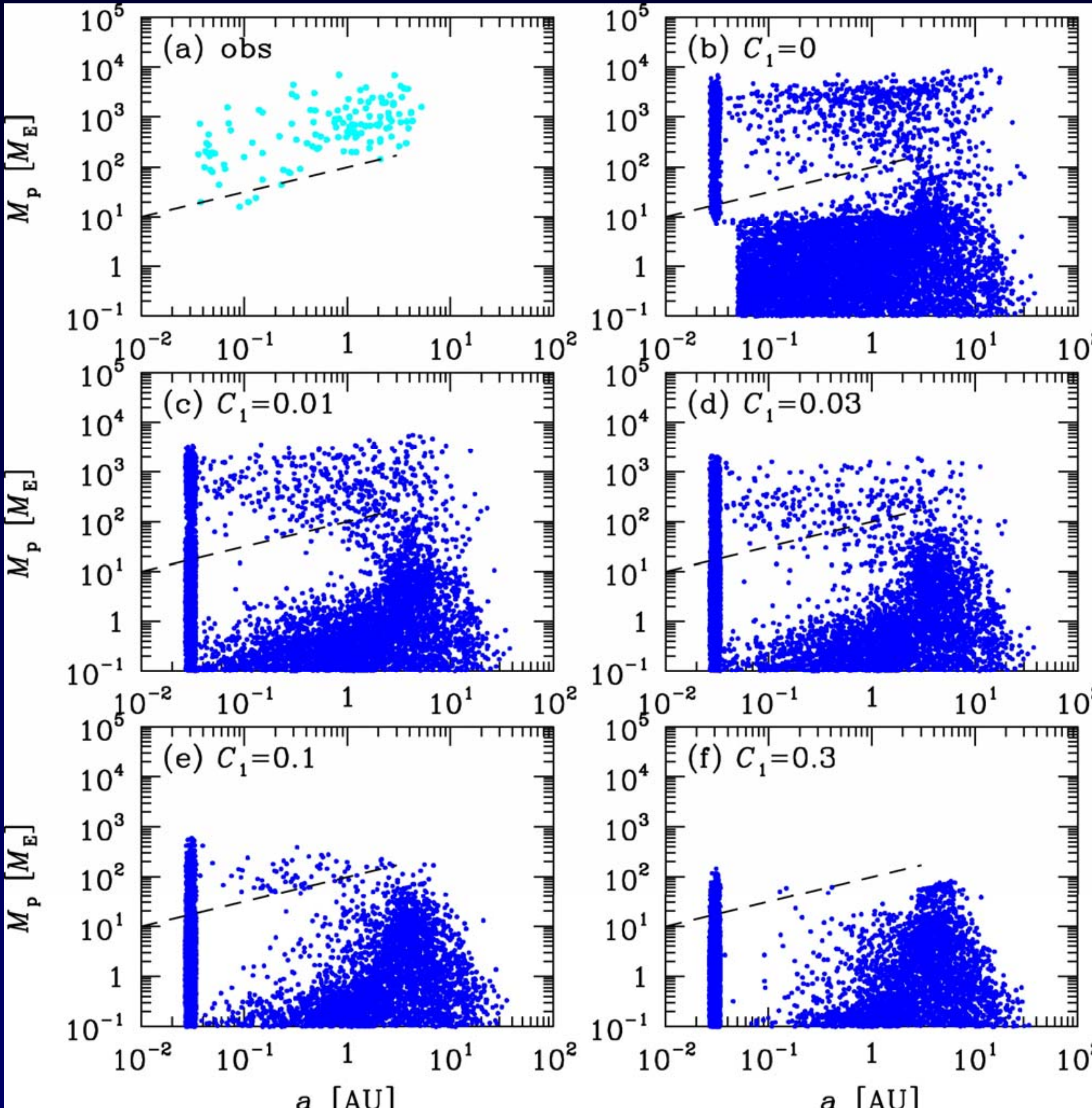
  - Gravitational interaction with protoplanetary disc
  - Planet-planet scattering
  - Kozai effect + tidal interaction with central star
  - Scattering of planetesimals

Low mass planets - type I migration

- Planet generates spiral waves in disc at Lindblad resonances
- Gravitational interaction between planet and spiral wakes causes exchange of angular momentum





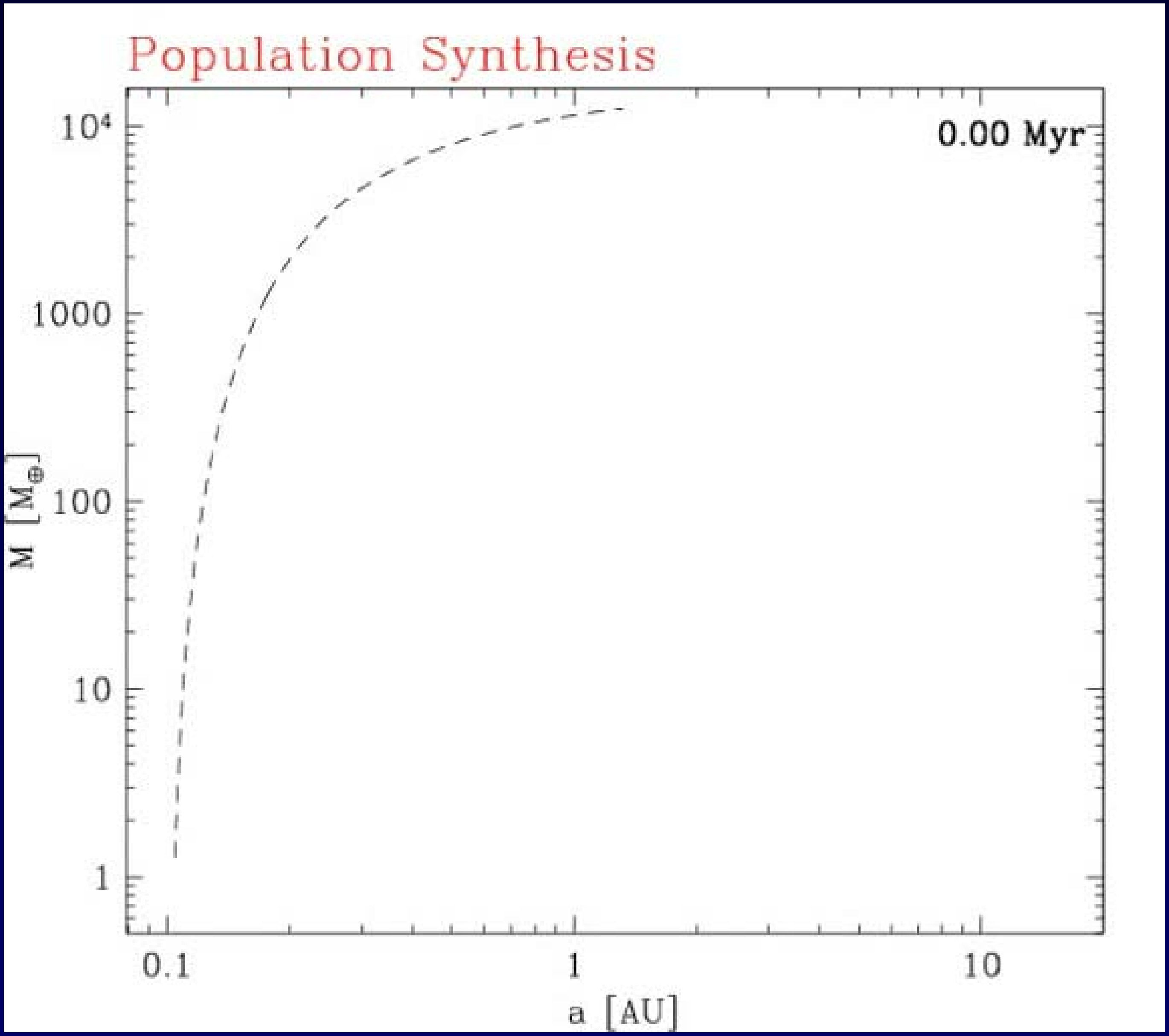


Ida & Lin (2008)

Population synthesis
+ type I migration

Type I migration leads to “desert” of low and intermediate mass planets within ~ 2 AU, even when attenuated by x 10
Rapid gas accretion also reduces number of intermediate mass planets

This “planet desert” is inconsistent with observations
Howard et al (2010)



Mordasini et al (2009)

Even with type I migration switched off the planet desert is apparent

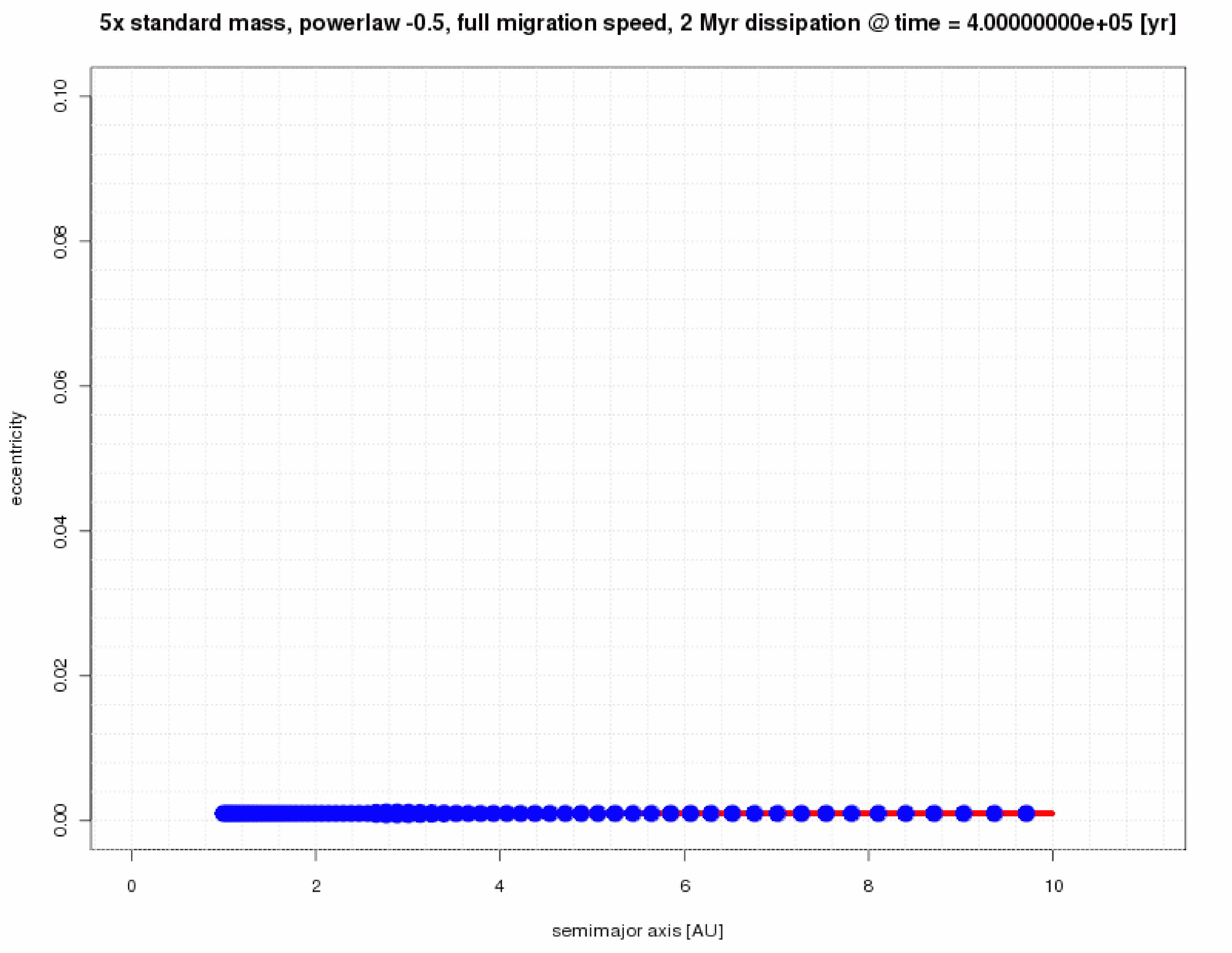
Forming hot Neptunes and super-earths via accretion and migration

Question: Is it possible to form hot Neptunes and super-earths that are consistent with the observations by combining standard type I migration with oligarchic accretion ?

N-body simulations plus type I migration (McNeil & Nelson 2009, 2010)

Approximately 15000 planetesimals + approx 100 planetary embryos

Dissipating gas disc (time scale ~ 1-2 Myr)


5x standard mass, powerlaw -0.5, full migration speed, 2 Myr dissipation @ time = 4.00000000e+05 [yr]
eccentricity
0.00
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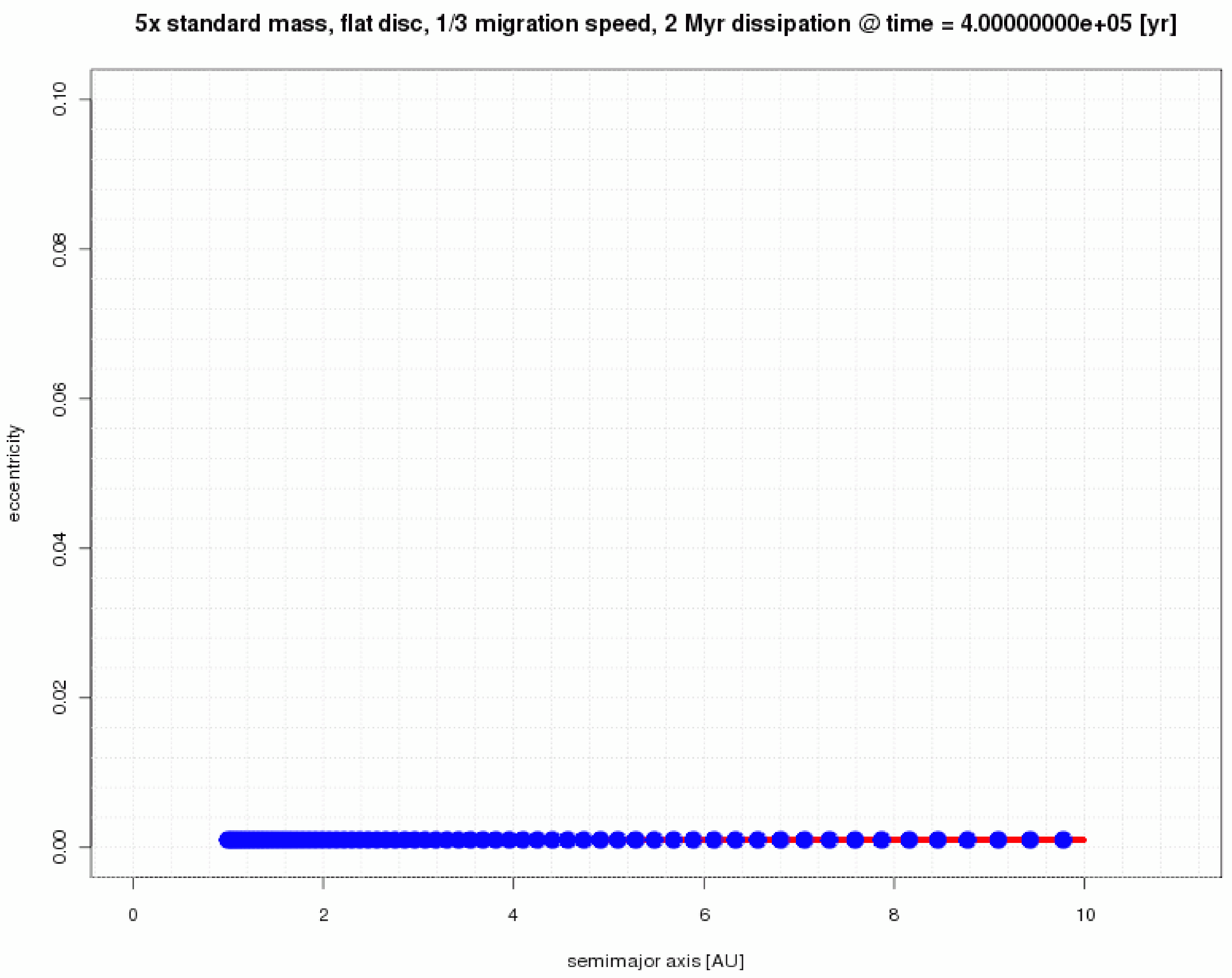
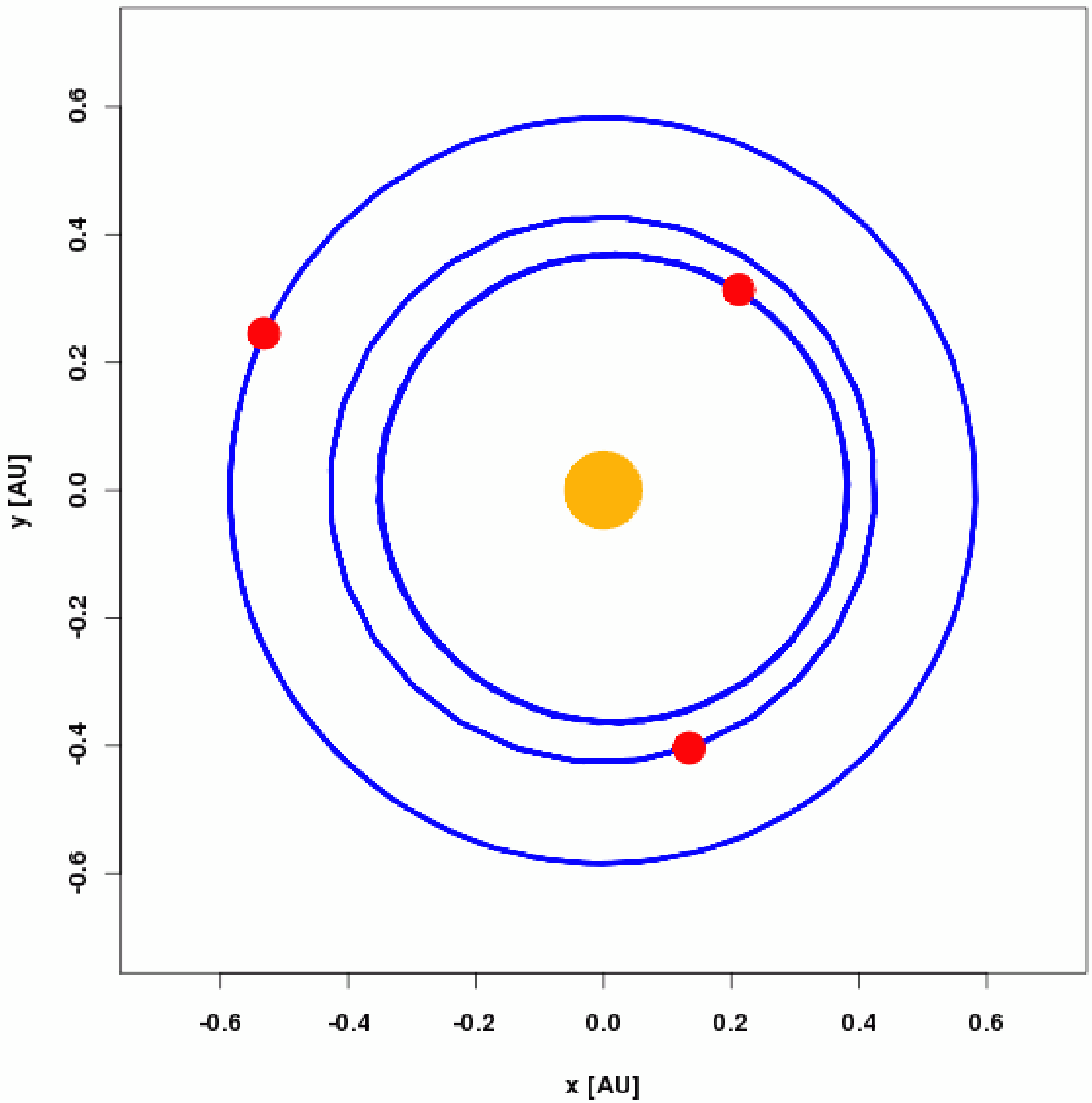
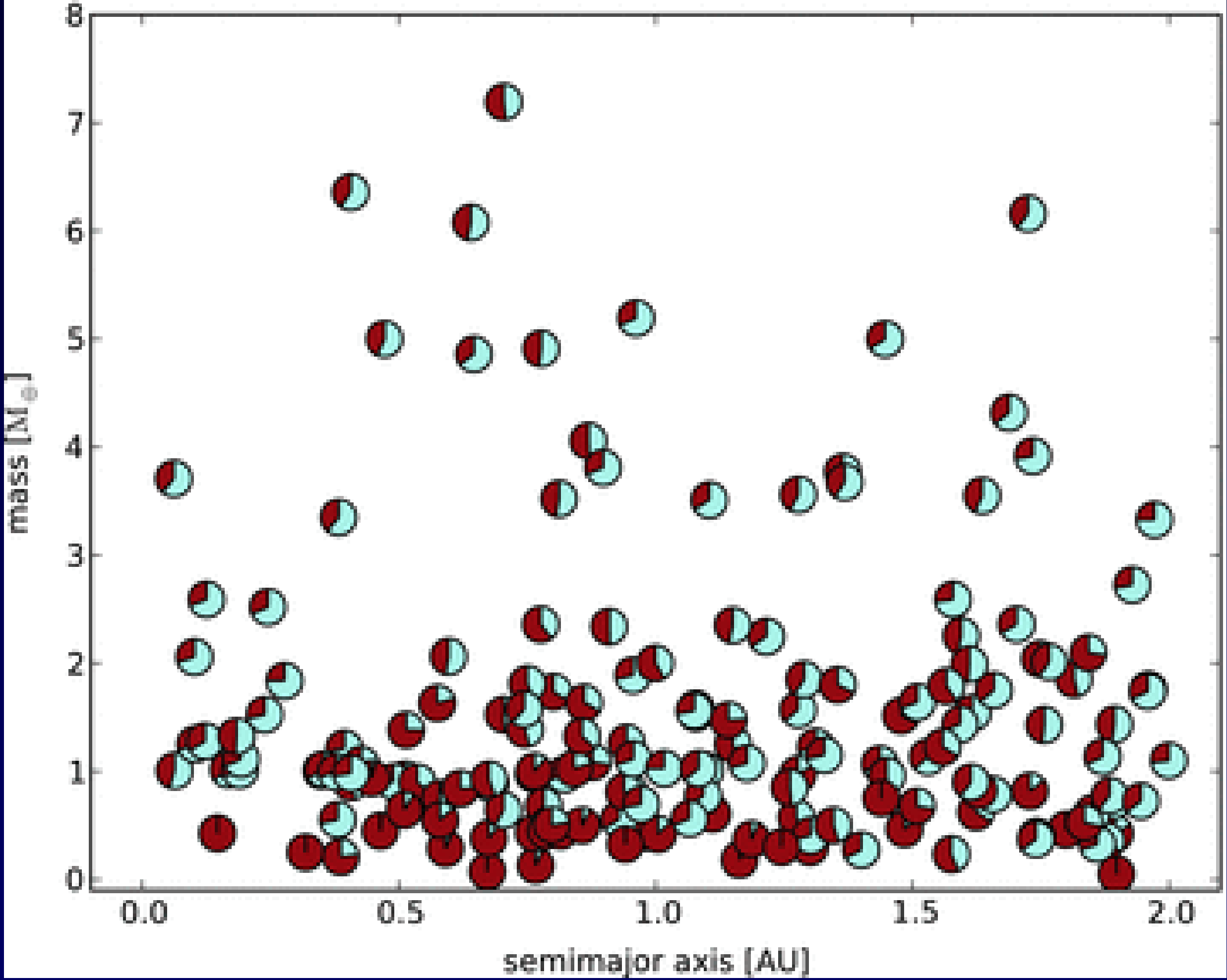
semimajor axis [AU]


5x standard mass, flat disc, 1/3 migration speed, 2 Myr dissipation @ time = 4.00000000e+05 [yr]
eccentricity
0.00
0.02
0.04
0.06
0.08
0.10
0
2
4
6
8
10
semimajor axis [AU]

5x standard mass, flat disc, 1/3 migration speed, 2 Myr dissipation
0.9 ME, 0.37 AU; 5.0 ME, 0.43 AU; 4.9 ME, 0.58 AU



Mass versus semimajor axis



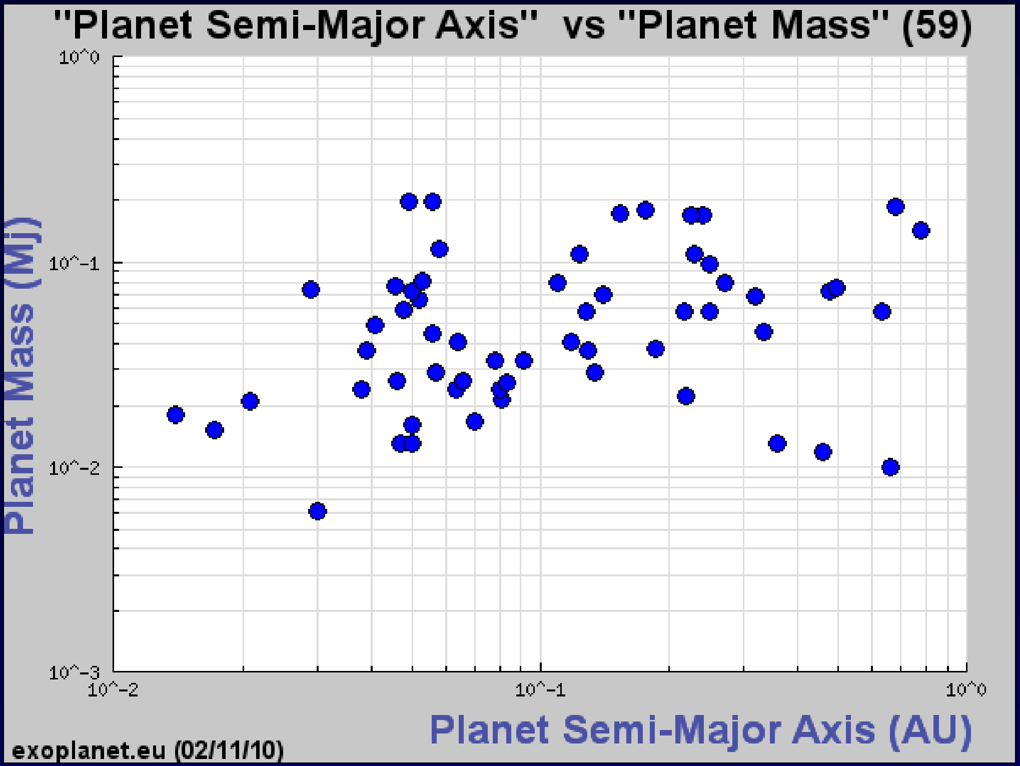
Super-earths are formed
But no systems containing sufficient mass at small radii
(such as the systems Gl581 or HD69830) were formed

Evidence for type I migration

- Short period neptunes and super-earths

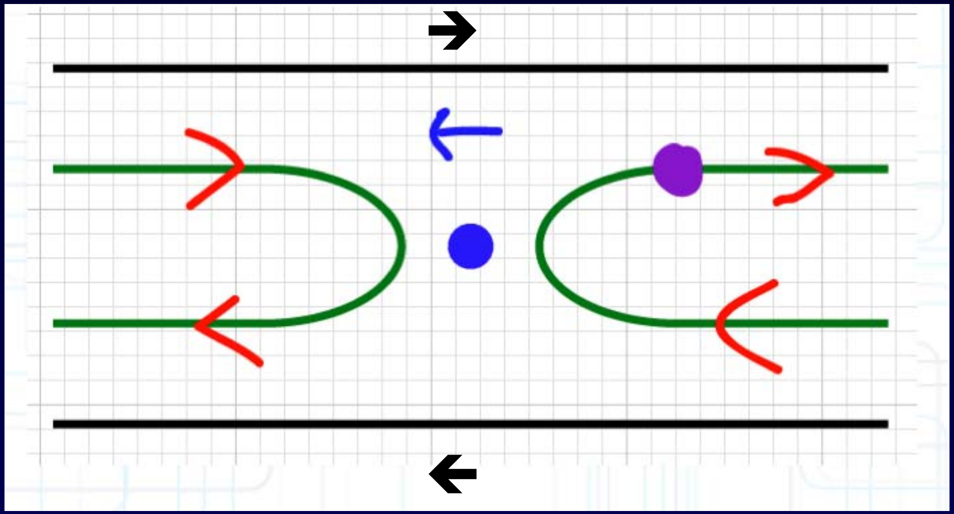
  > 50 planets with
  m sini < 40 M_{earth}
  (e.g. Gl 581- 4 planets)

- Disc model properties:
  - mass of solids too small within 1 AU
  - planets must form further out and migrate in



- Type I migration does occur !
  - but probably more slowly than predicted by basic theory

Corotation torques

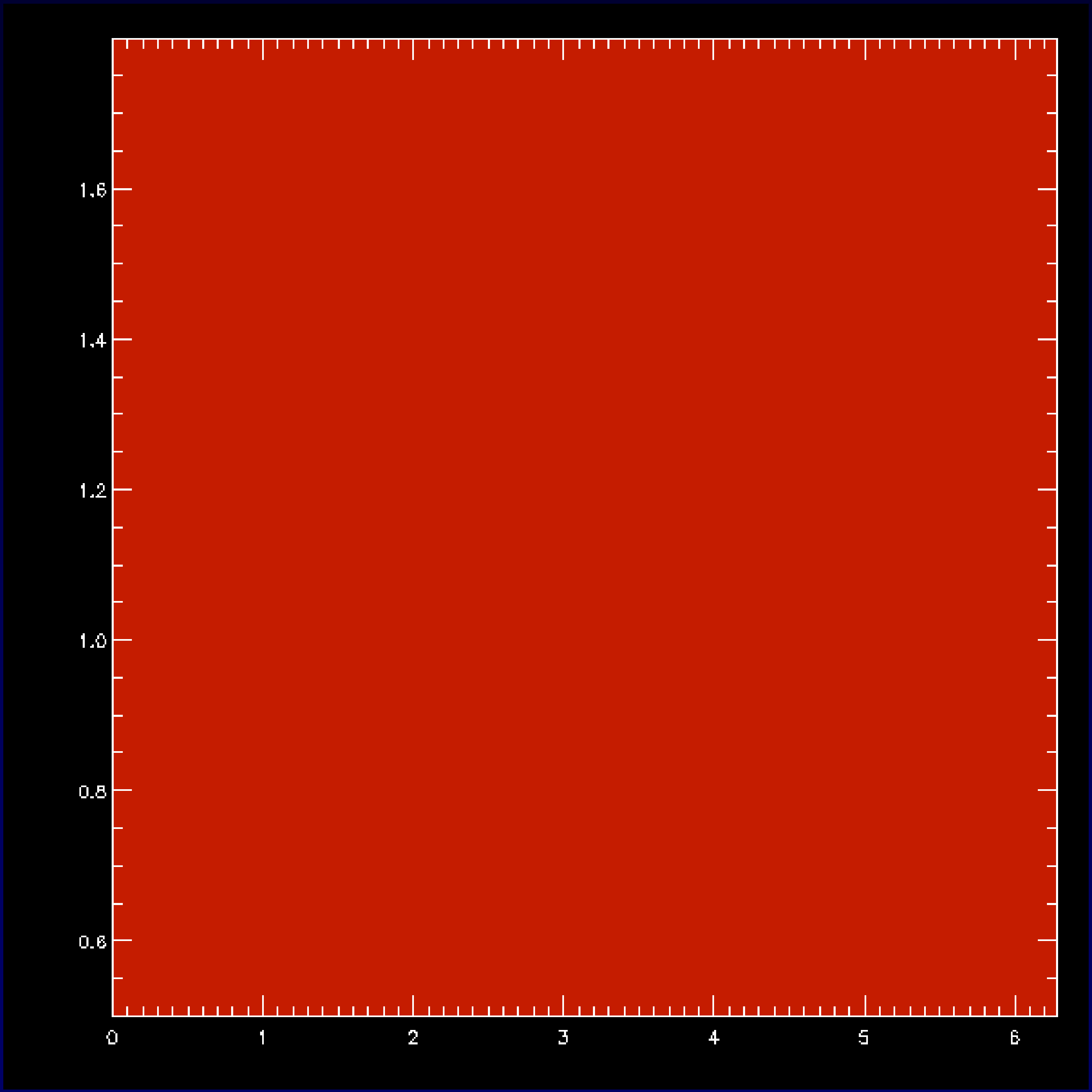


- Corotation torques arise when gas interacts with planet while performing horseshoe orbits

- Conservation of either specific vorticity (vorticity/density) and/or entropy during U-turn causes change in density structure near planet

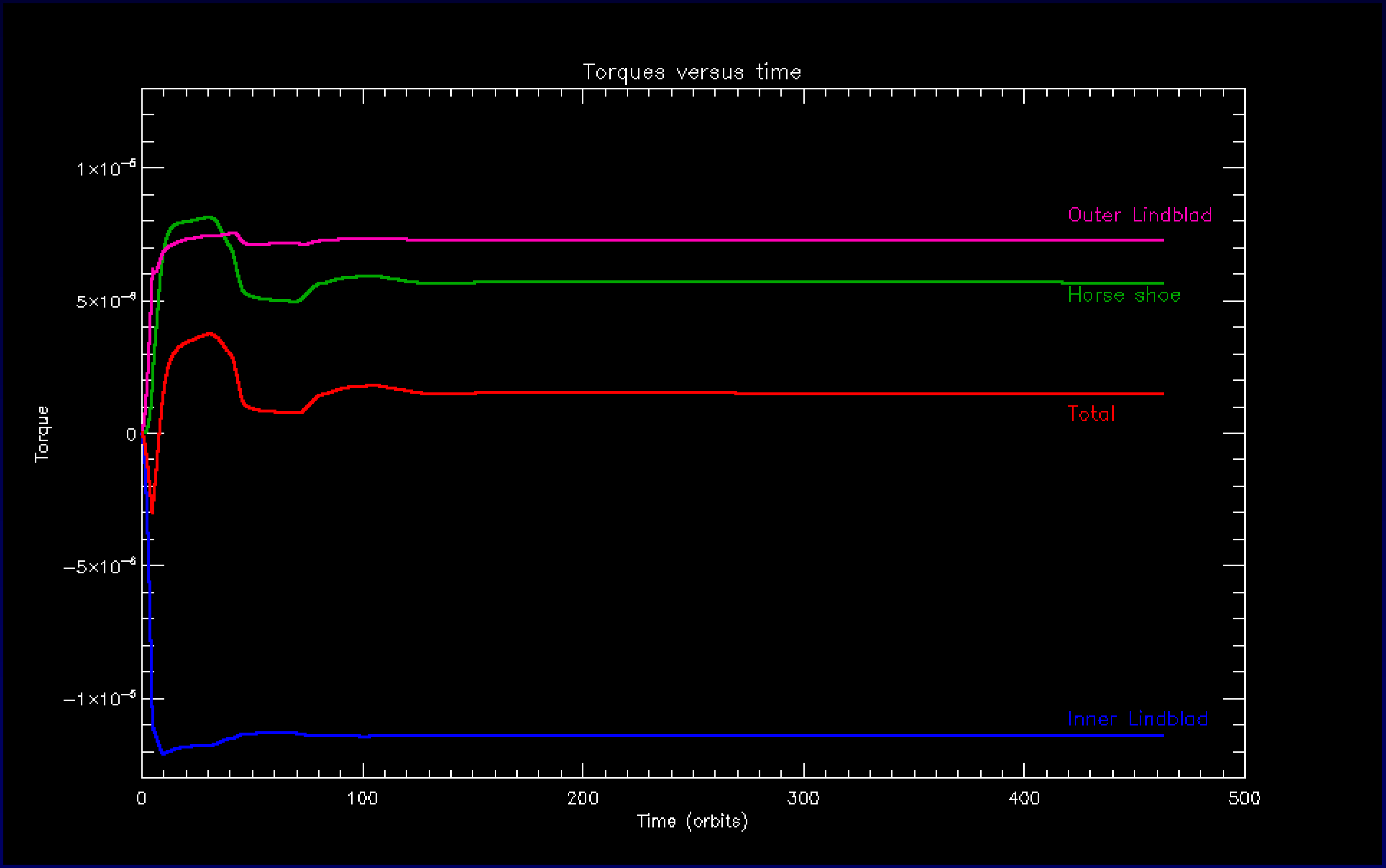
- In optically thick discs corotation torque can exceed Lindblad torques - stalling or even reversing type I migration (Paardekooper & Mellema 2007; Baruteau & Masset 2008; Pardekooper & Papaloizou 2008)

Effect of entropy gradient in disc

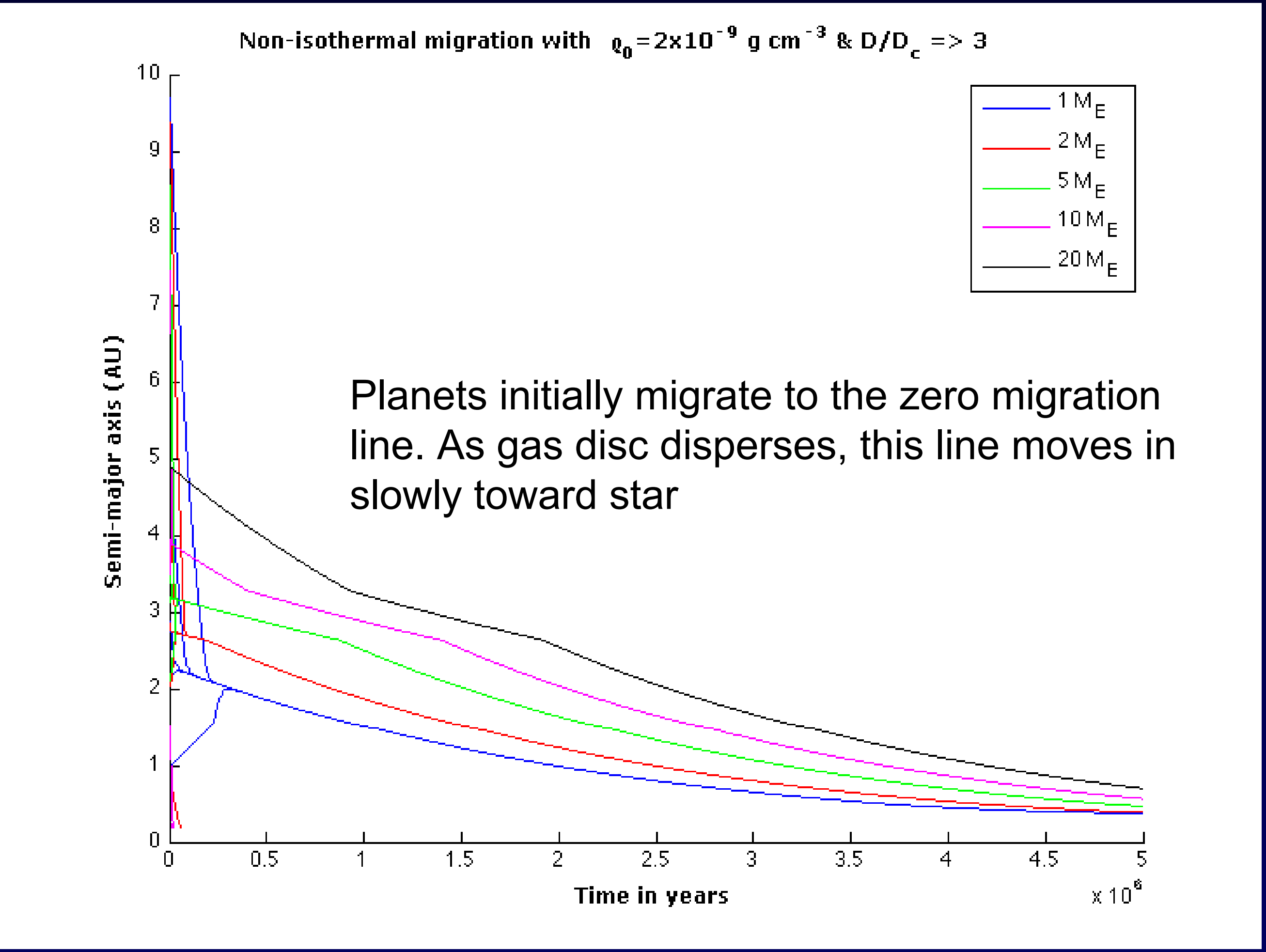


Sustaining the corotation torque requires viscosity & radiative diffusion to act on the time scale of ~ ½ a horseshoe orbital period

Evolution of the torque with radiative diffusion



To prevent the entropy-related corotation torque from saturating, require that local thermal and viscous diffusion times ~ horseshoe libration time

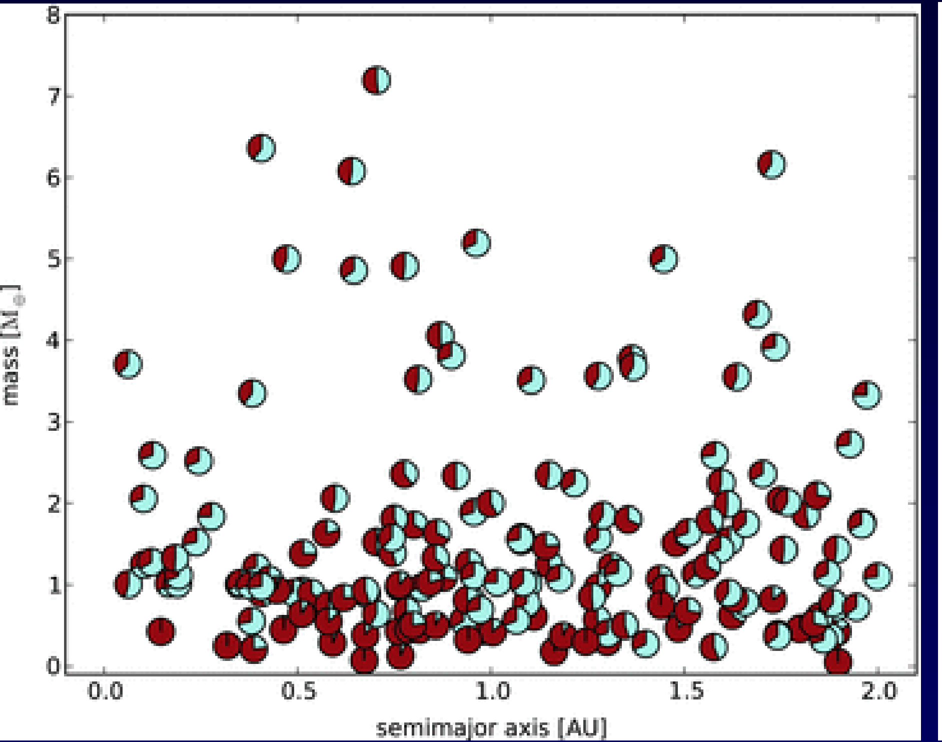


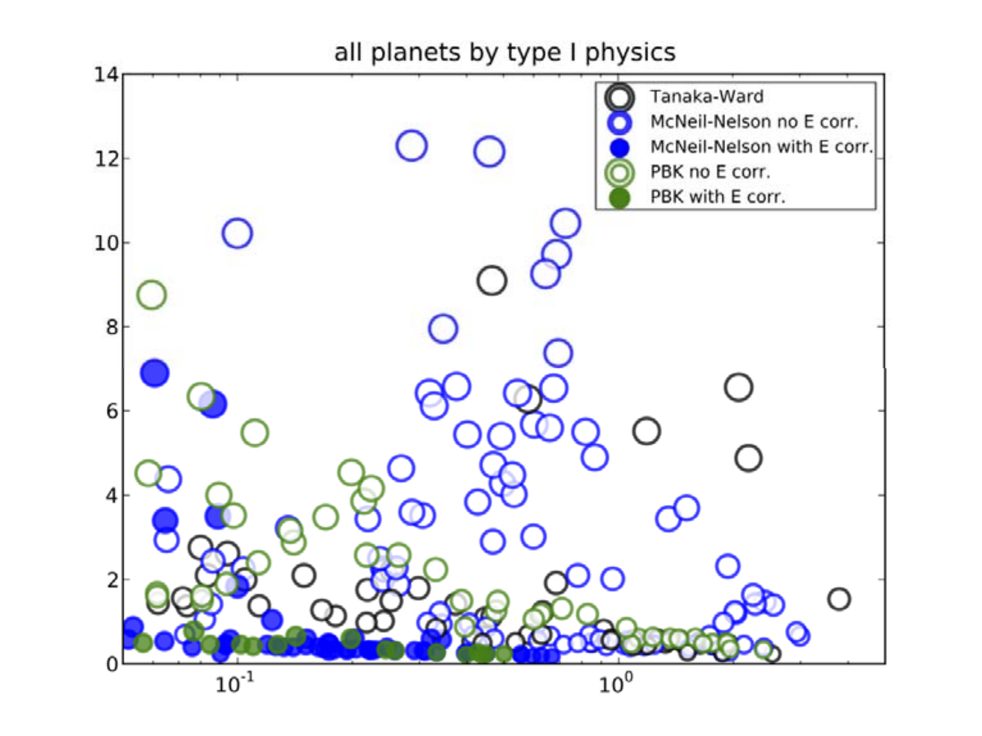
If thermal relaxation time << horseshoe libration time → migrate inward

If thermal relaxation time >> horseshoe libration time → migrate inward

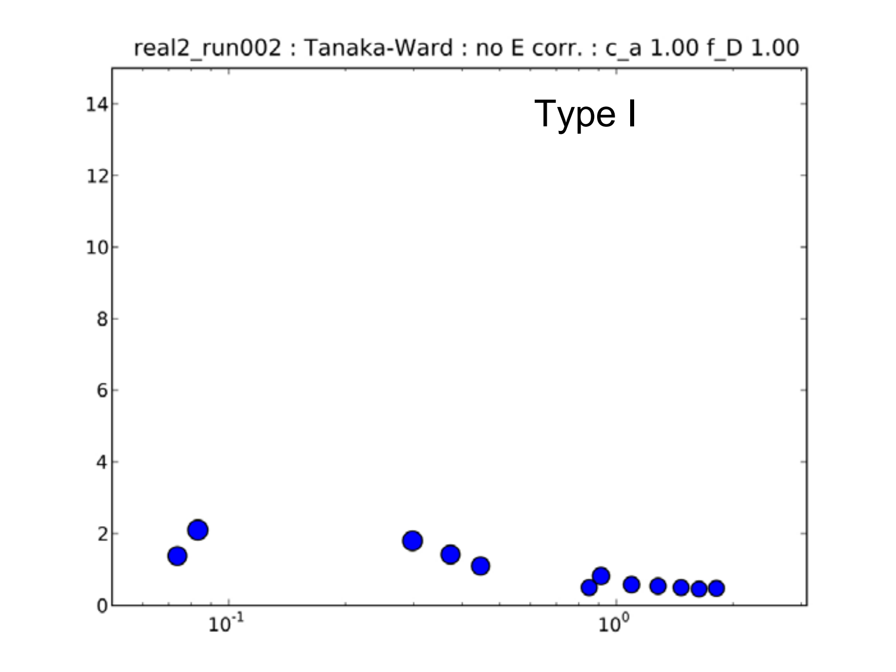
If thermal relaxation time ~ horseshoe libration time → migrate outward

N-body simulations of oligarchic growth of planets with migration and corotation torques (McNeil & Nelson – in prep.)





The inclusion of corotation torques leads to formation of more massive objects, and short period multiplanet systems - convergent migration assists growth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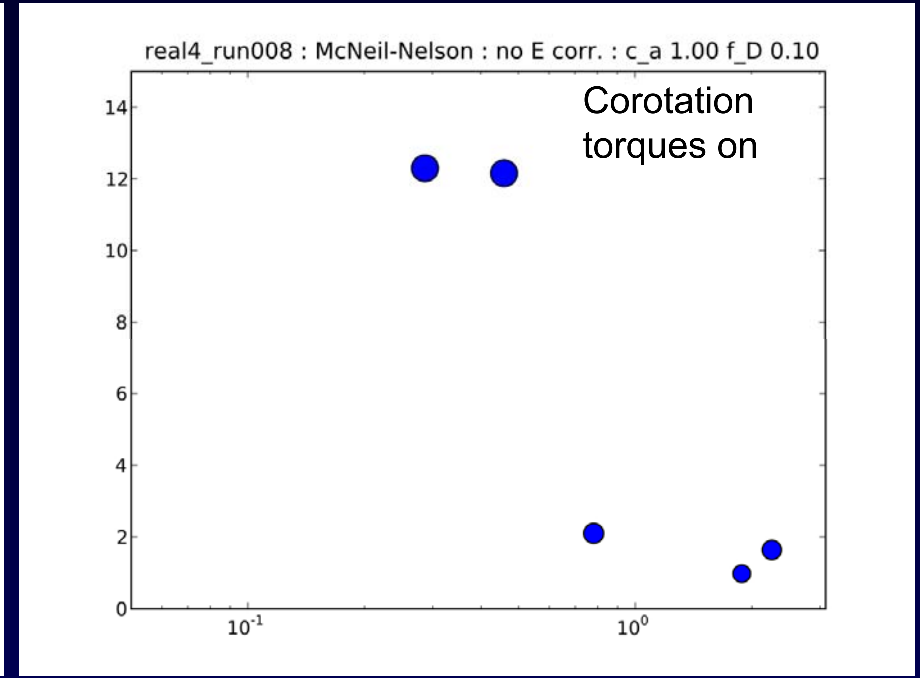
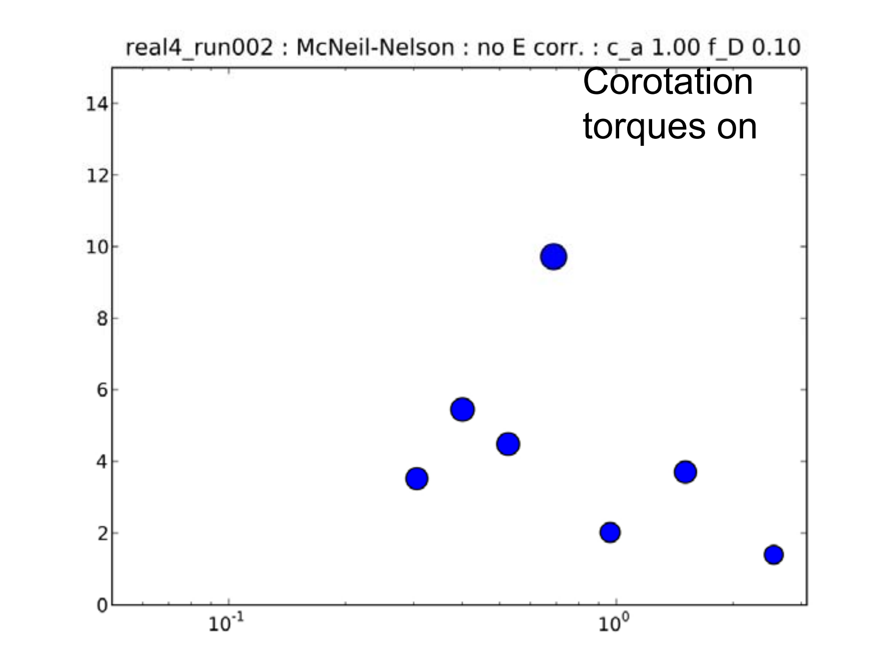
Type I
0
2
4
6
8
10
12
14
10^{-1}
10^{0}


real4_run008 : McNeil-Nelson : no E corr. : c_a 1.00 f_D 0.10
Corotation torques on
0
2
4
6
8
10
12
14
10^{-1}
10^{0}


real4_run002 : McNeil-Nelson : no E corr. : c_a 1.00 f_D 0.10
Corotation torques on
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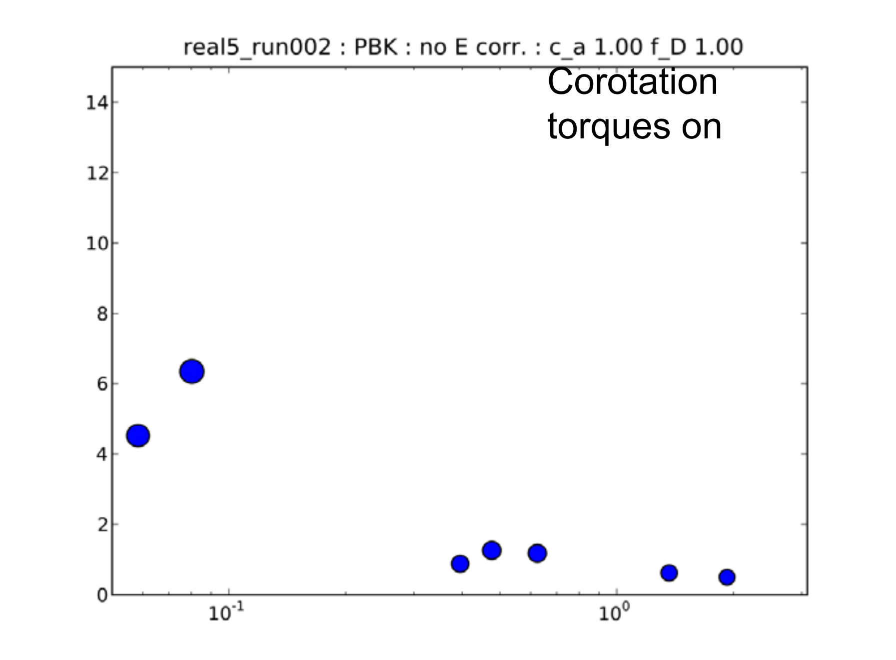
real5_run002 : PBK : no E corr. : c_a 1.00 f_D 1.00
Corotation torques on
0
2
4
6
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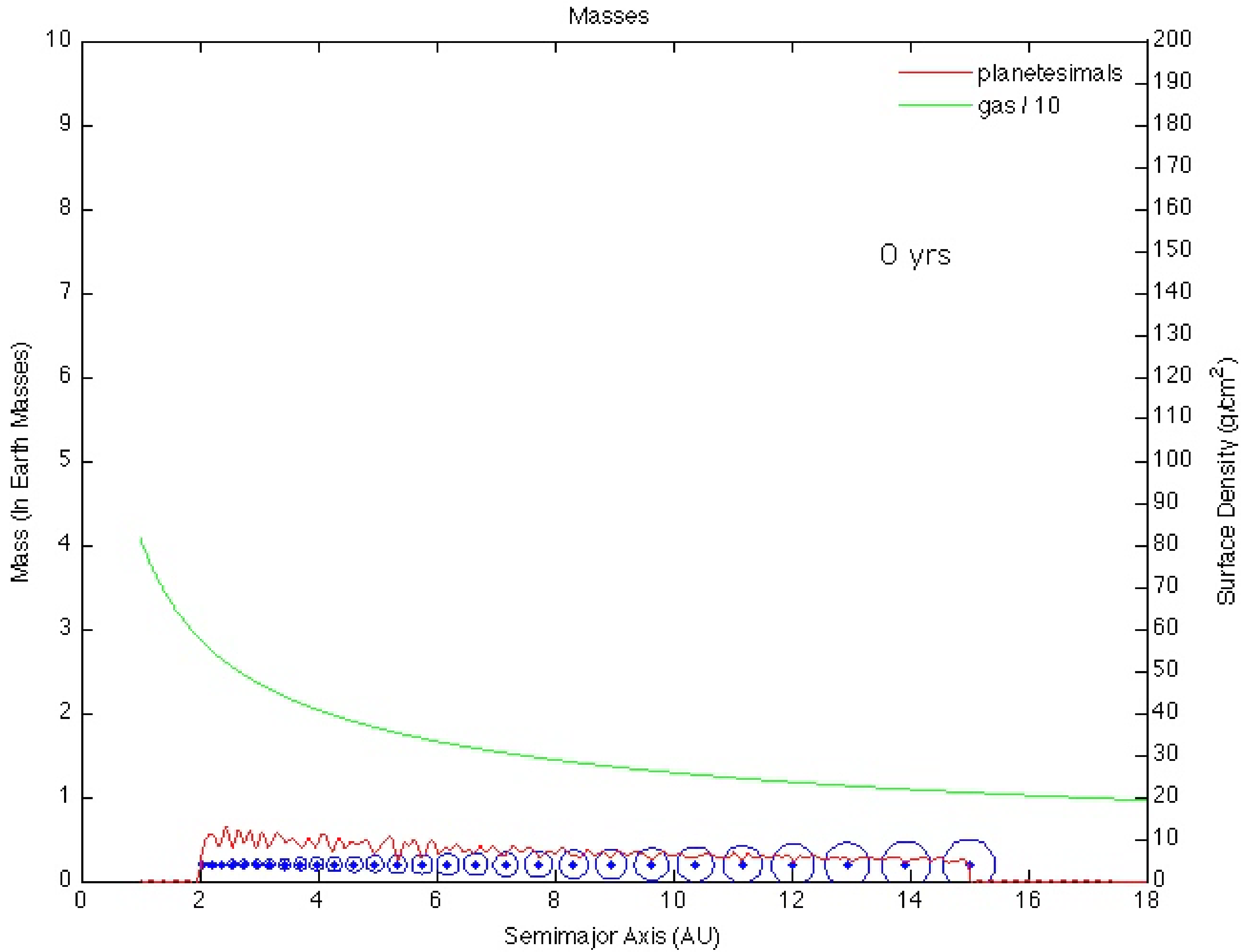
12
14
10^{-1}
10^{0}

N-body simulations with migration, collisional growth and gas accretion onto planetary cores (Hellary & Nelson – in prep)

- Planetary embryos + planetesimals
- Migration with corotation torques
- Gas settling onto planetary cores – enhanced planetesimal capture (Inaba et al 2003)
- Gas accretion for cores with mass > 3 Earth masses (Movshovitz et al 2010)
- Transition to gap formation and type II migration when planet mass > Saturn's mass
- Gas disc dispersal on time scale ~ 3 Myr



Masses
planetesimals
gas / 10
0 yrs
Mass (In Earth Masses)
Surface Density (g/cm^2)
Semimajor Axis (AU)
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10
12
14
16
18
0
10
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190
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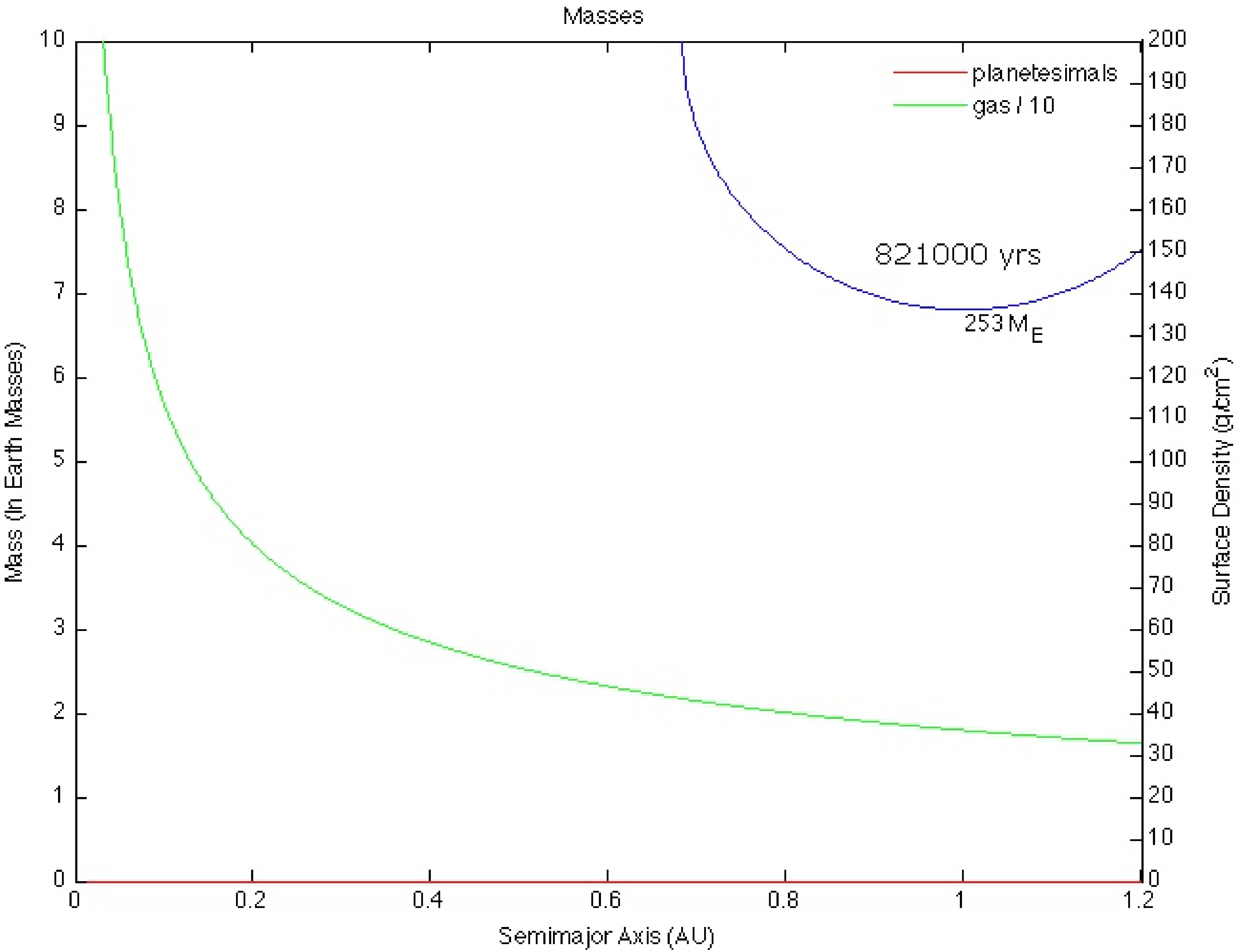
Masses
planetesimals
gas / 10
821000 yrs
253 M_E
Mass (In Earth Masses)
Surface Density (g/cm^2)
Semimajor Axis (AU)
0
1
2
3
4
5
6
7
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9
10
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0.2
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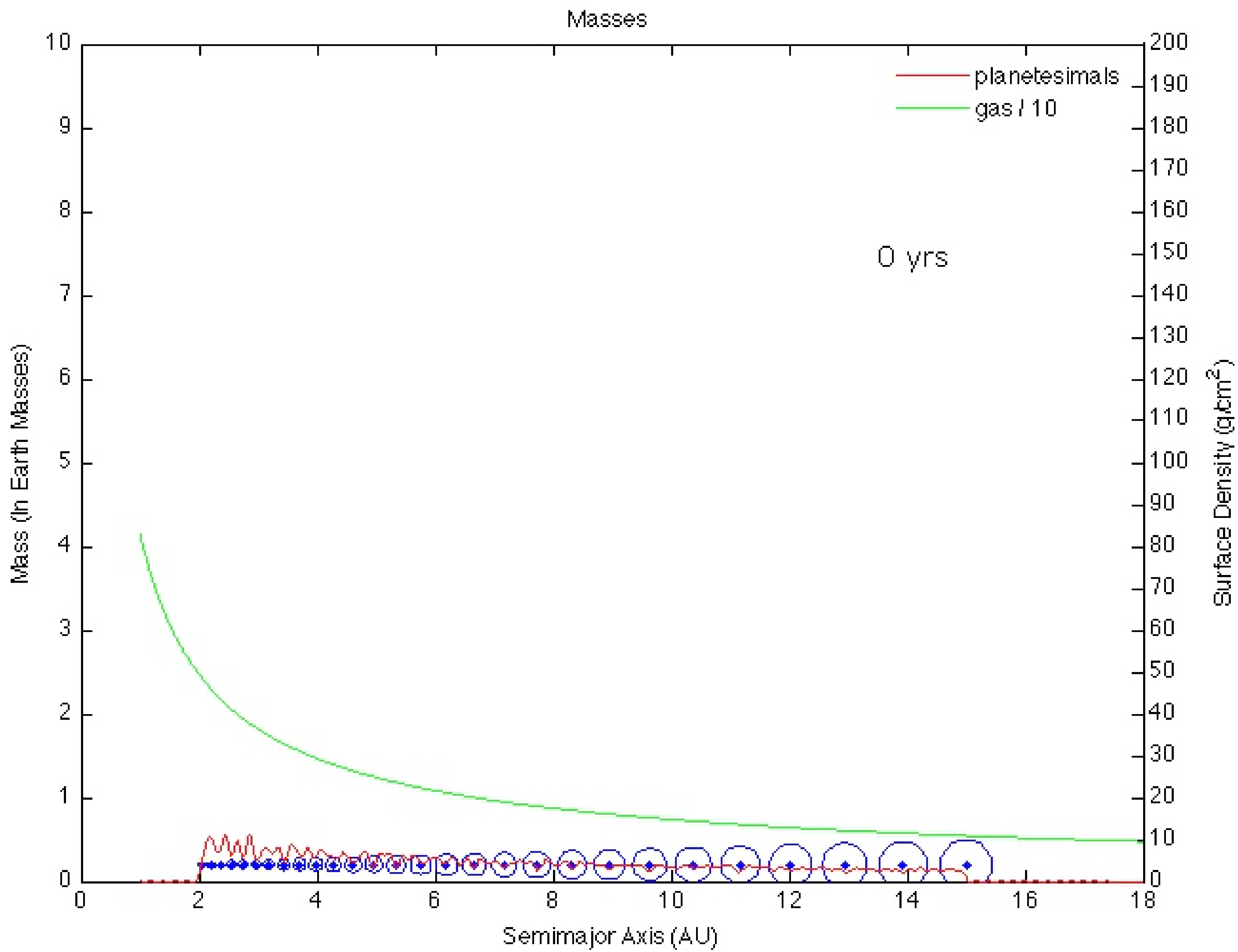
0.8
1
1.2



Masses
planetesimals
gas / 10
0 yrs
Mass (In Earth Masses)
Surface Density (g/cm2)
Semimajor Axis (AU)
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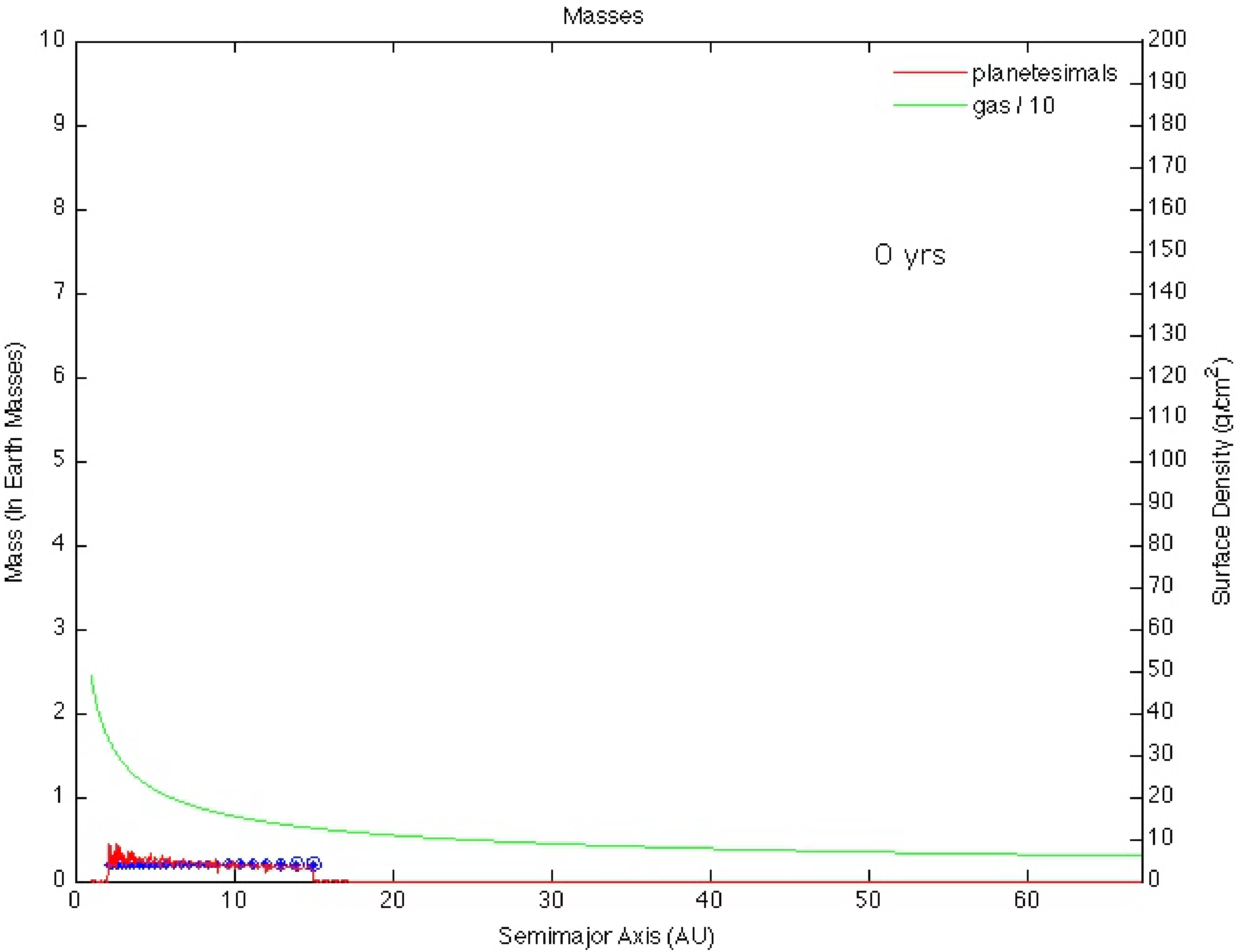

Masses
planetesimals
gas / 10
0 yrs
Mass (In Earth Masses)
Surface Density (g/cm^2)
Semimajor Axis (AU)
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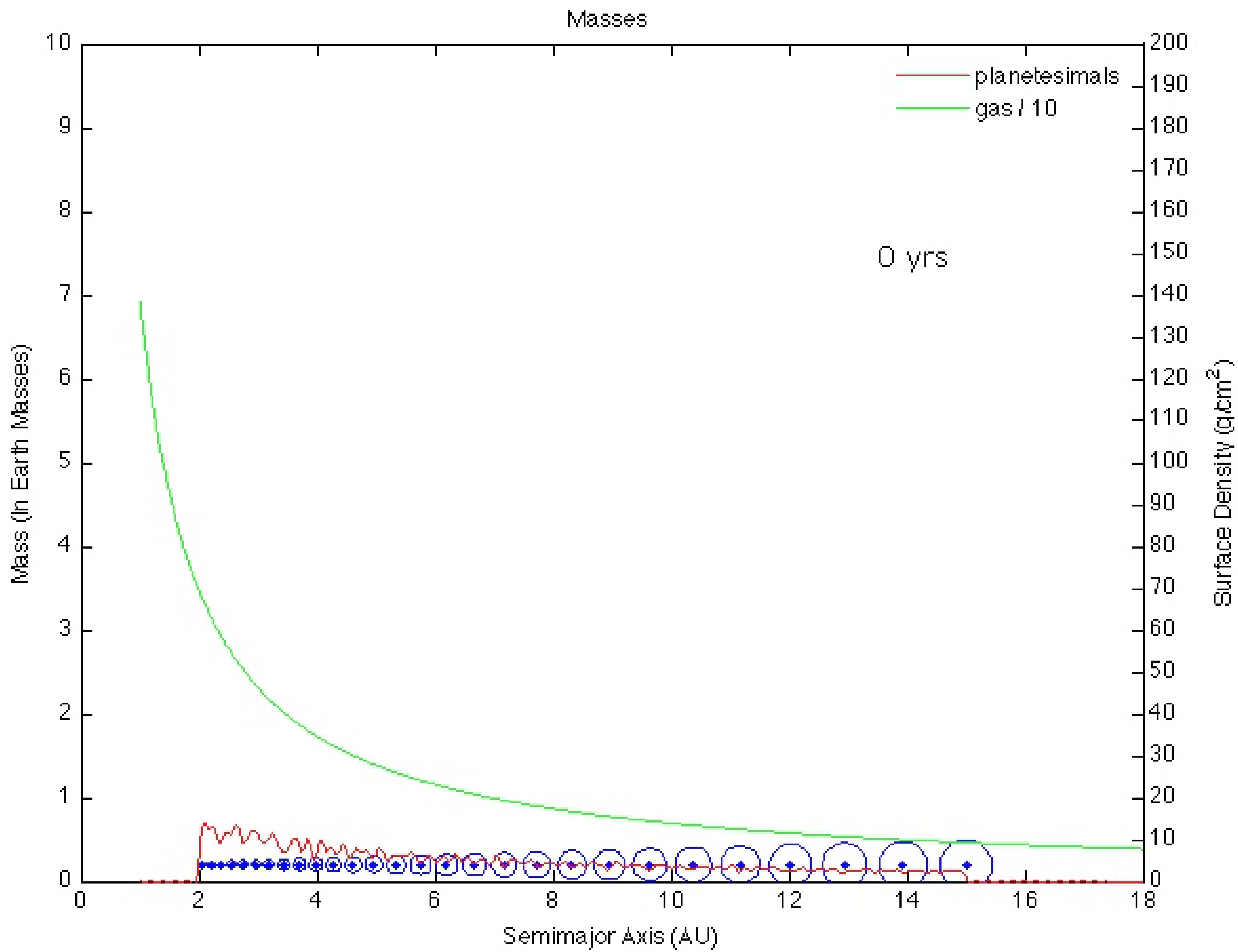

Masses
planetesimals
gas / 10
0 yrs
Mass (In Earth Masses)
Surface Density (g/cm^2)
Semimajor Axis (AU)
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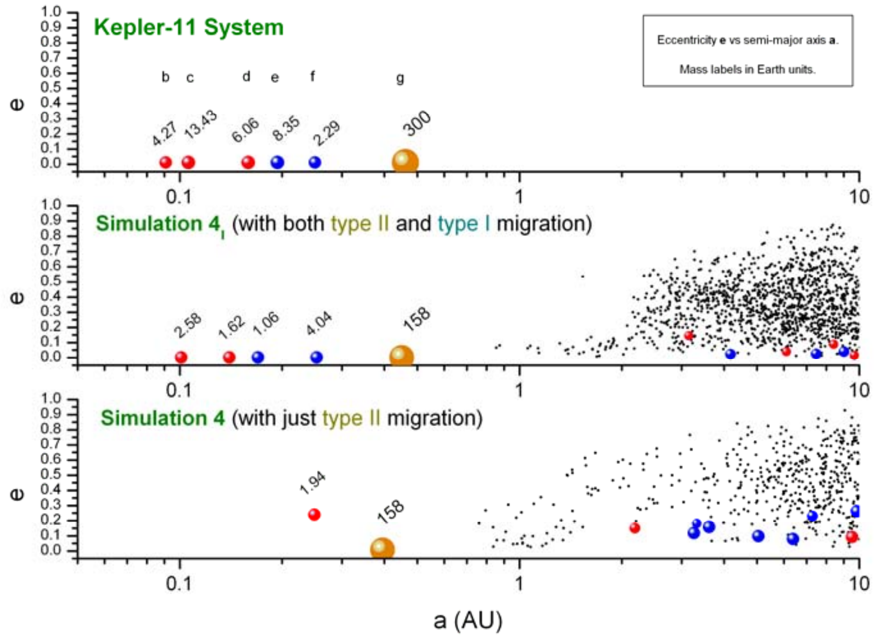
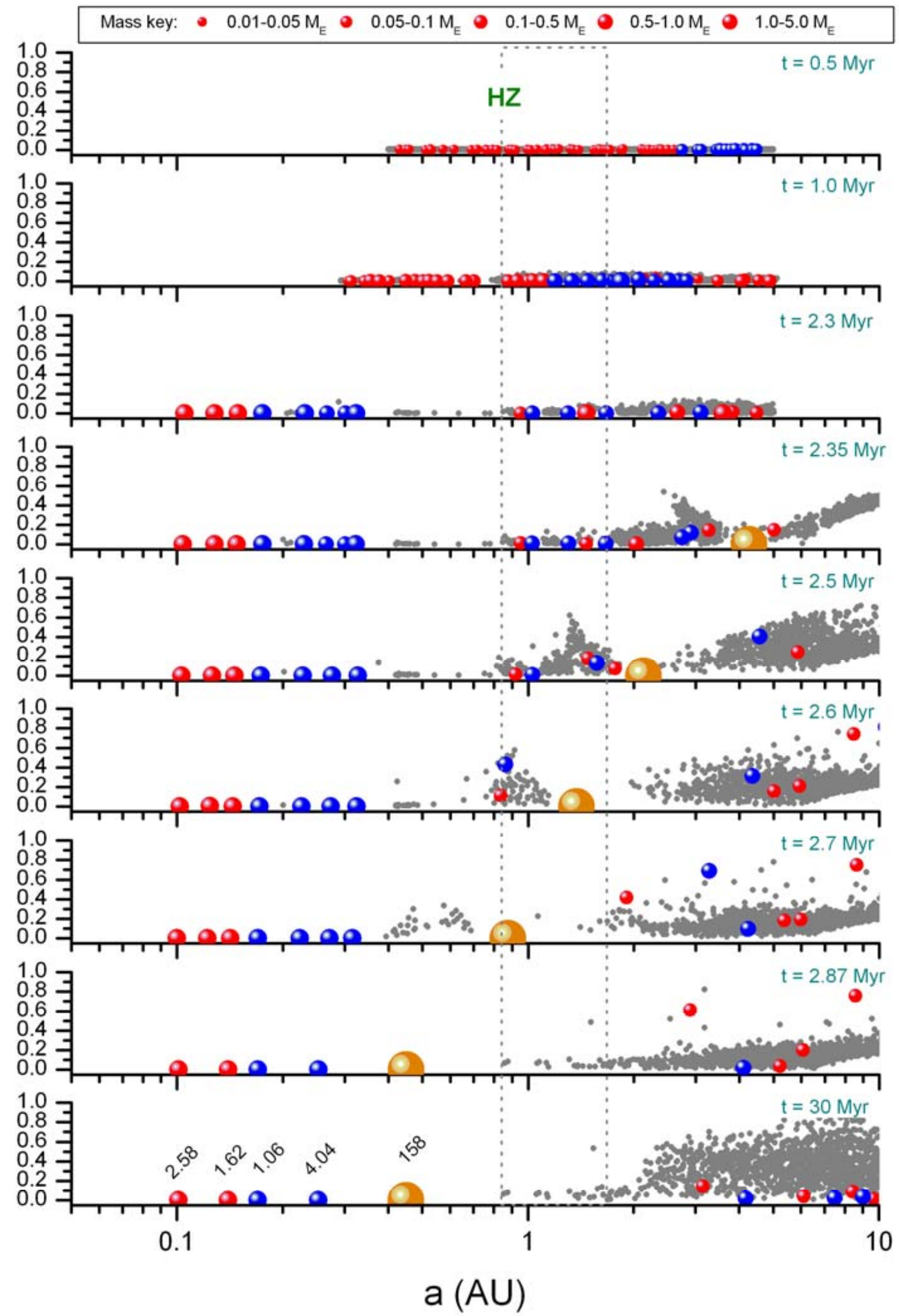
Terrestrial Planet Formation During Giant Planet Migration

- N-body simulations performed (Fogg & Nelson 2005, 2006, 2009)
- Initial conditions: inner solids disk undergoing different stages of `oligarchic growth' within a viscously evolving gas disc
- Giant planet is introduced which migrates through inner planet-forming disc

A possible formation scenario for Kepler 11 (Fogg & Nelson 2009)





Conclusions

• Significant progress has been made in understanding planetary migration – but much work remains to be done (turbulence, dead zones, etc…)

• Pop. synthesis models and N-body simulations with prescriptions for migration, gas growth etc. are beginning to produce systems which bracket the different exoplanetary system architectures that have been observed – but their realism needs to be improved

• Developments in theory and modelling will allow a detailed comparison between exoplanet data and planetary formation theory on a time scale of a 2 - 5 years – just in time for the PLATO mission

Planetesimal evolution

- Recent shearing box simulations indicate slow growth in velocity dispersion of planetesimals due to gravitational interaction with turbulent density fluctuations (Yang, Maclow & Menou 2009)
- How does velocity dispersion vary as function of box size (or size of sphere of influence in global runs) ?
- Once converged, what do simulations predict for the equilibrium velocity dispersion for planetesimals of different size ? How quickly do planetesimals diffuse radially via a random walk ?
- Do global simulations and local shearing box simulations agree on outcomes ?

A few remarks about planetary growth…

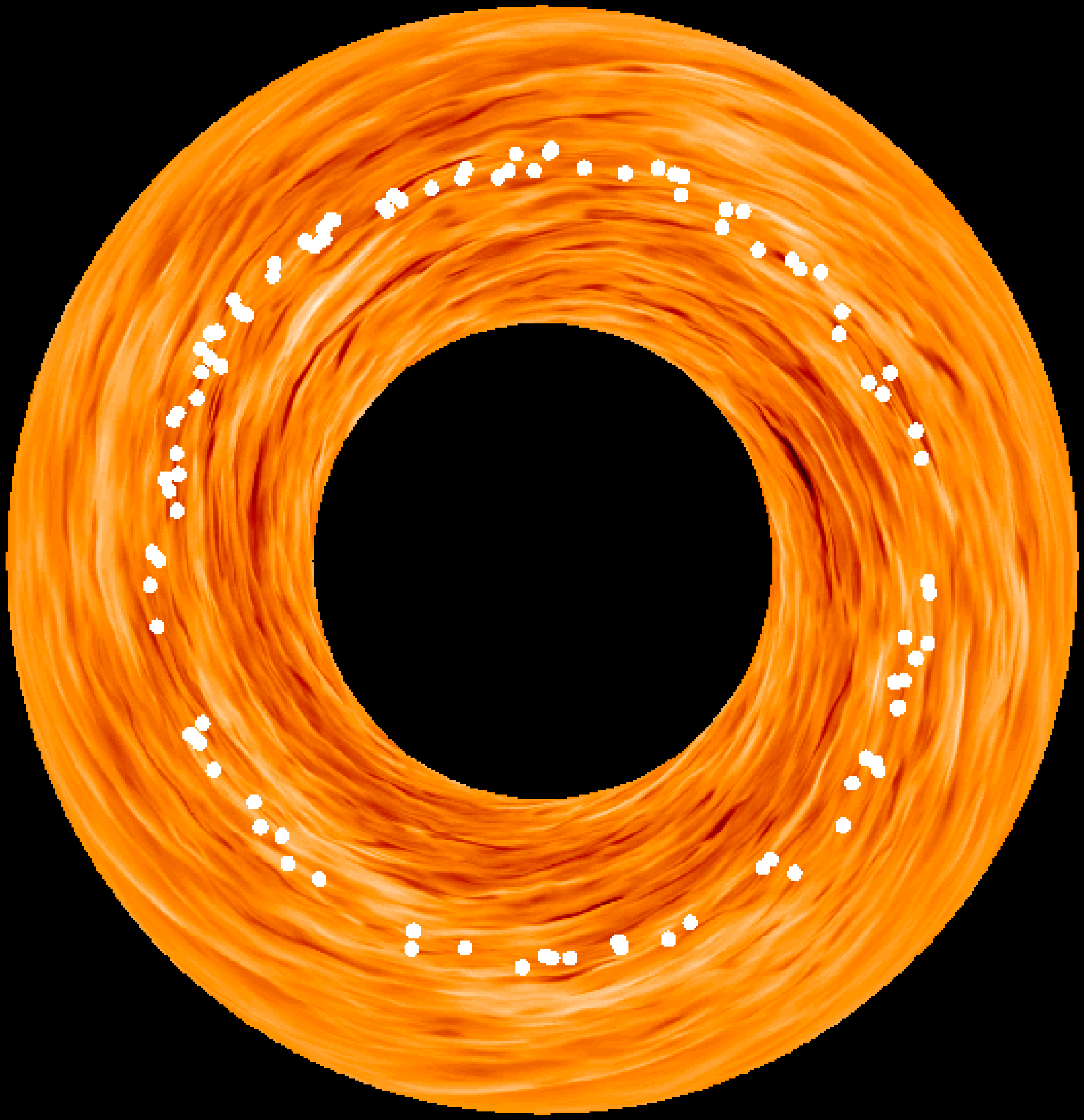
For runaway growth require planetesimal velocity dispersion to be significantly smaller than escape velocity from largest accreting objects:

$$\frac{dm_p}{dt} = \pi r_p^2 \langle v \rangle n m_{pl} \left(1 + \frac{2Gm_p}{r_p \langle v \rangle^2}\right)$$

For 10 km sized bodies with ρ=2 g/cm^3 escape velocity=10 m/s

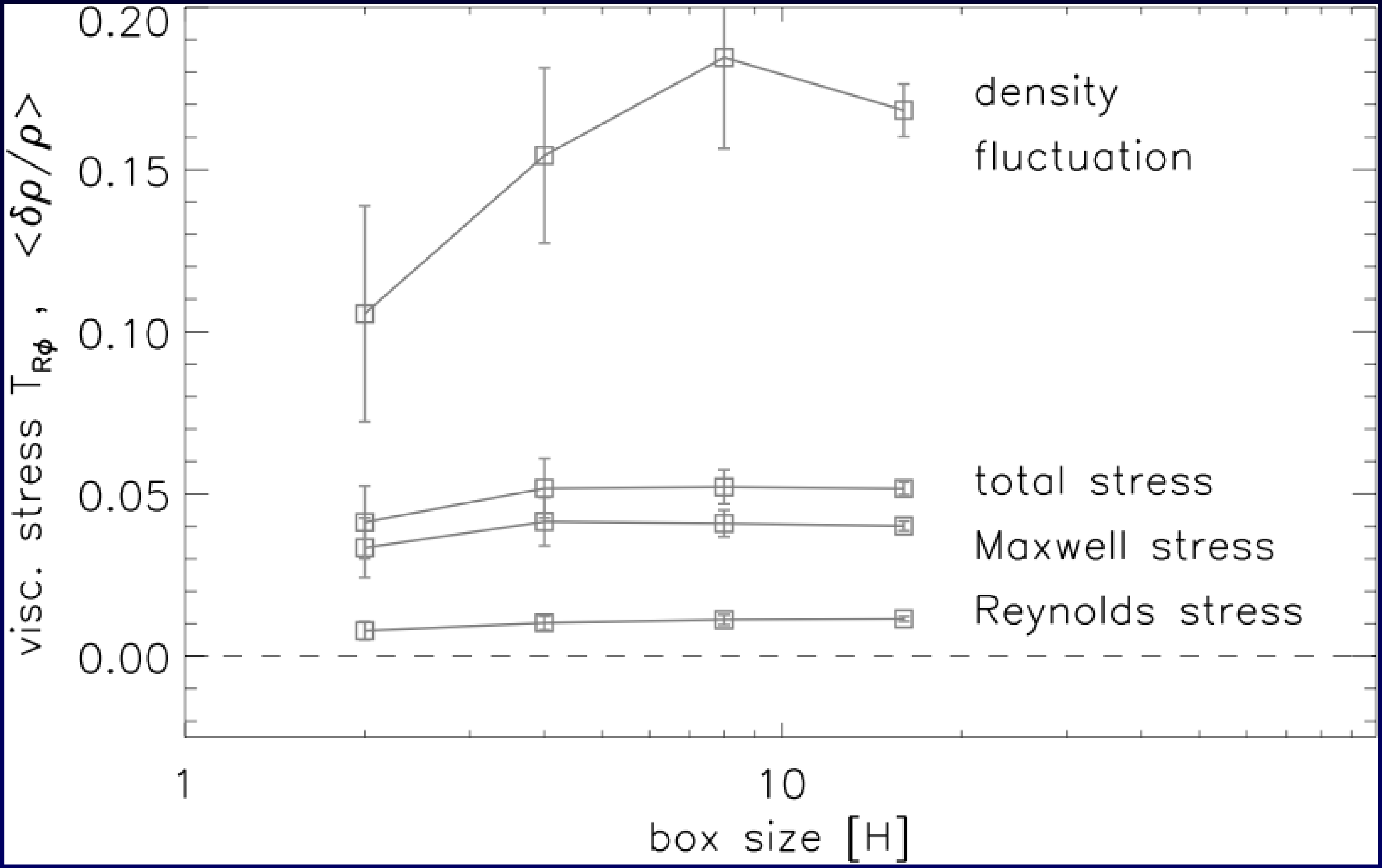
$$\begin{aligned} v_{esc} &= \sqrt{\frac{8\pi G\rho}{3} r_p} \\ &= 10 \left(\frac{r_p}{10\mathrm{km}}\right) \mathrm{m\,s}^{-1} \end{aligned}$$

For <v> ~ 10 m/s collisions may lead to catastrophic disruption for km sized bodies (Benz & Asphaug 1999; Stewart & Leinhardt 2009)

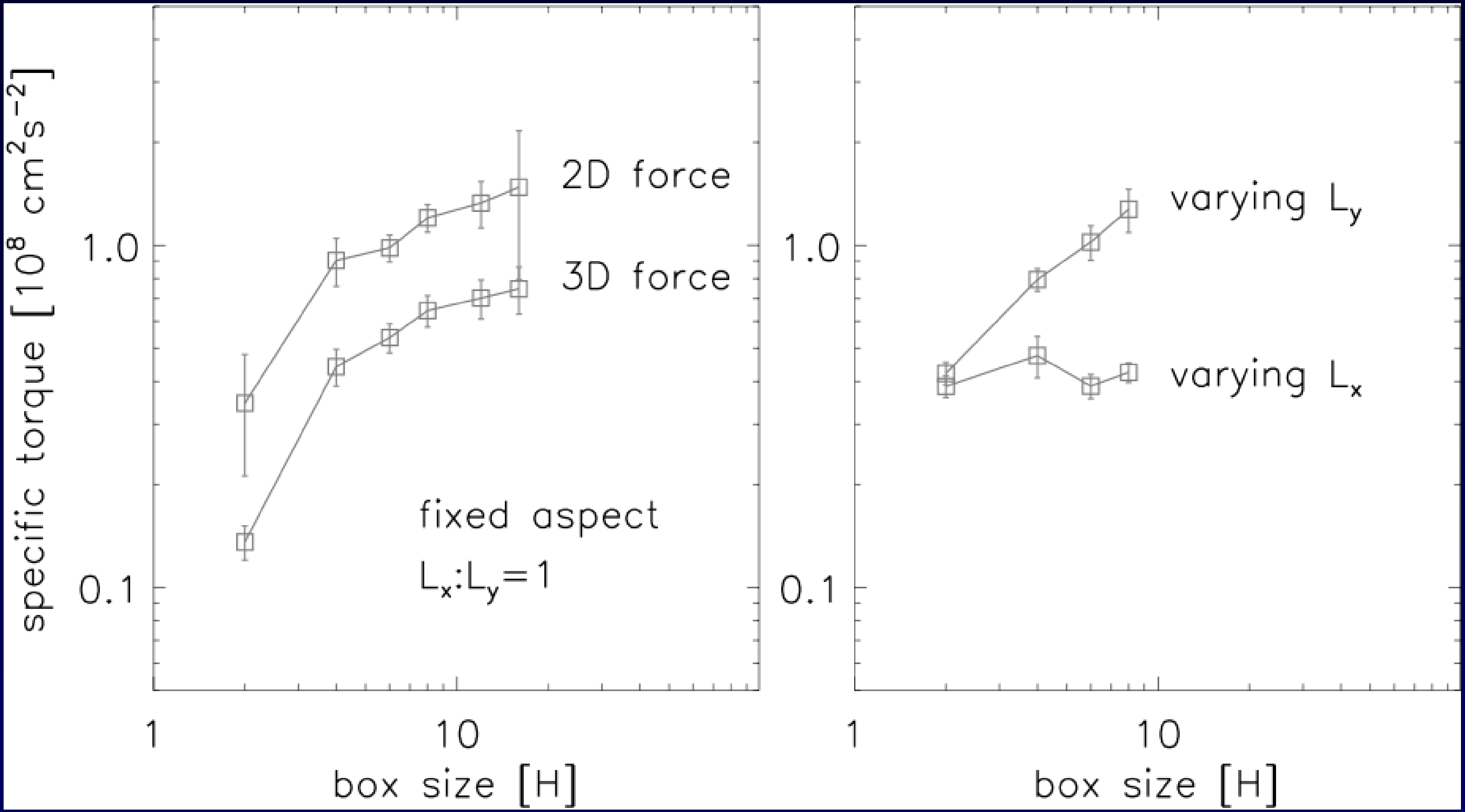


Turbulent disc simu
performed with 100
planetesimals which
evolve under influen
of gravitational field
disc and gas drag

Shearing boxsize requirements

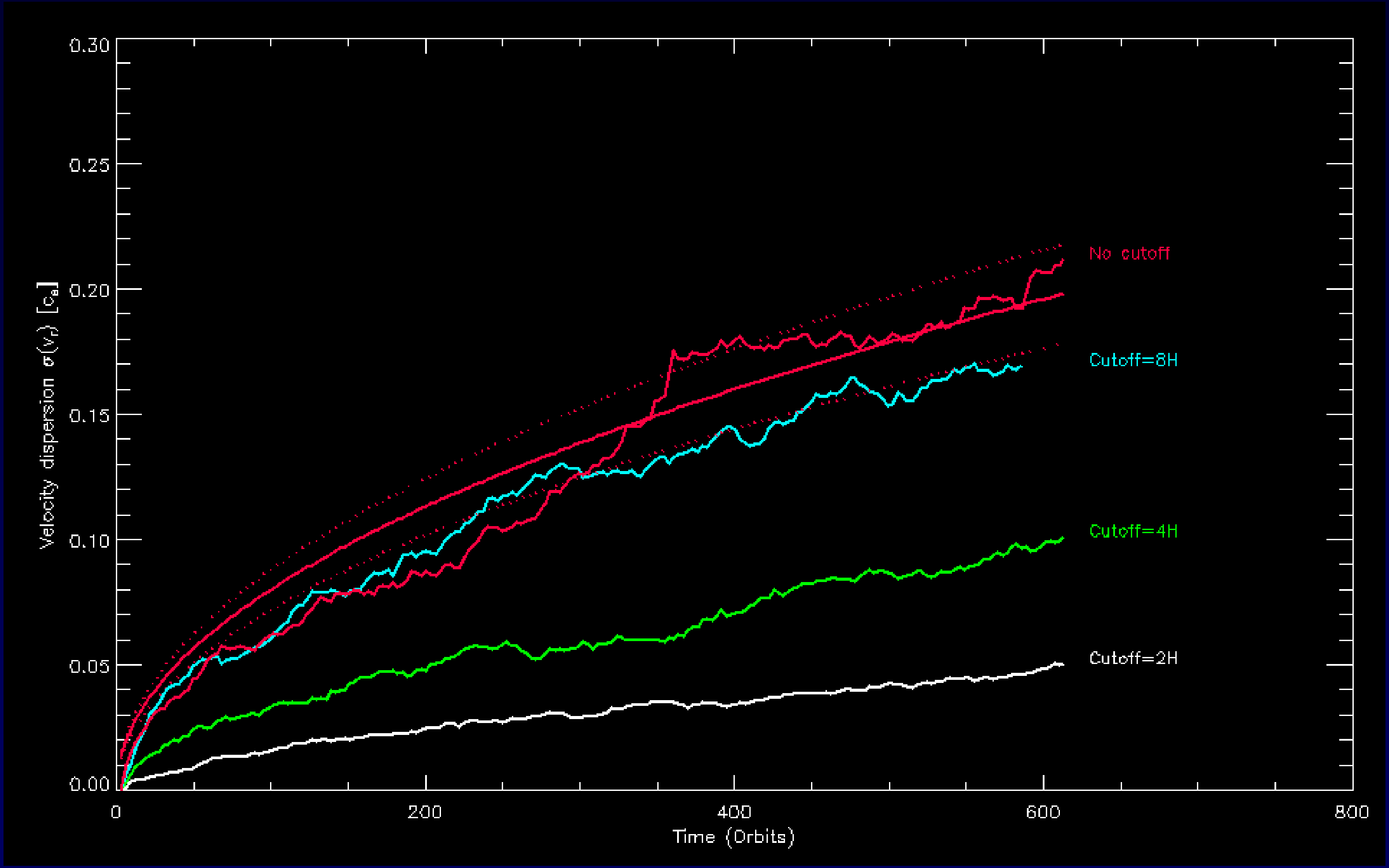


Boxsize ≥ 10H required to correctly model excitation, propagation and non-linear steepening of spiral density waves (Heinemann & Papaloizou 2009)



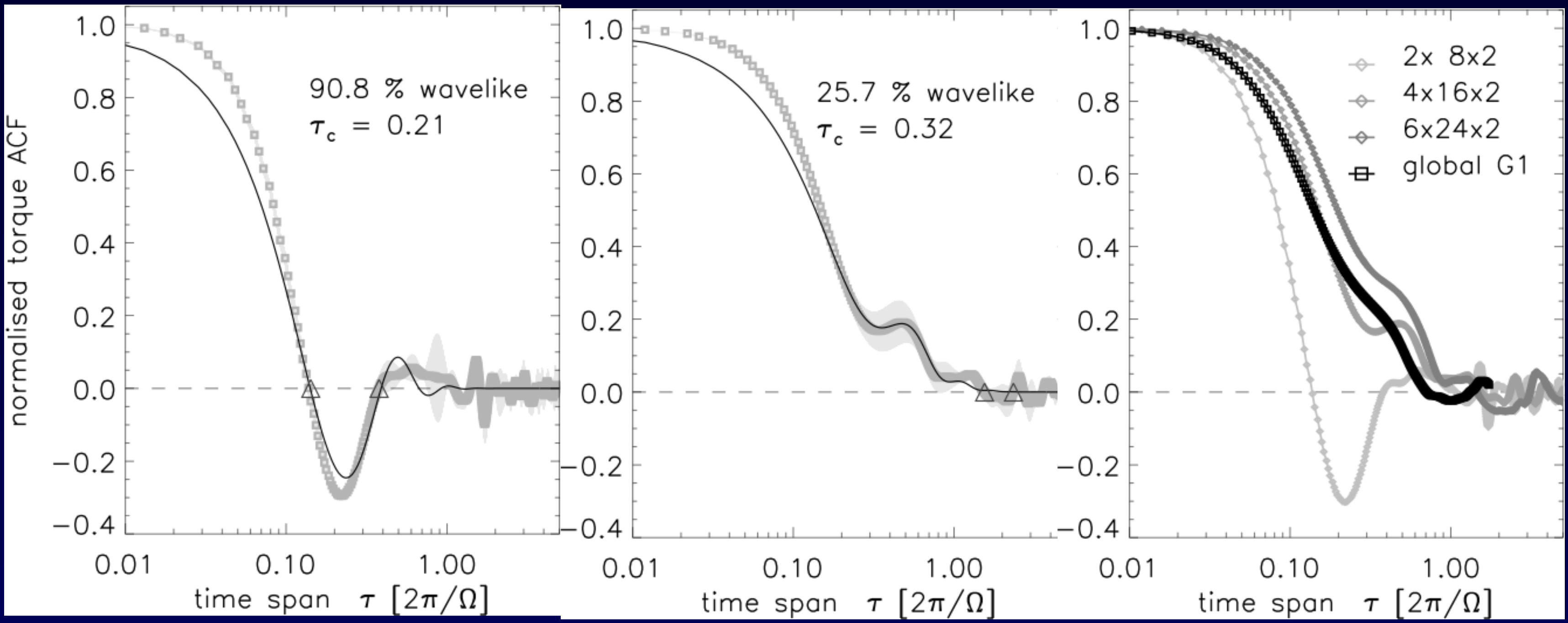
Most important to have an elongated box in the azimuthal direction

Gravitational cut-off in global models



Radial velocity dispersion for 25 planetesimals initially on circular orbits as function of disc gravity cut-off distance.
Convergence obtained when sphere of influence > 8 scale heights
- similar to convergence requirements for shearing boxes.

Autocorrelation function



The box size and aspect ratio also affect the torque autocorrelation function – and the measured correlation time.

Small boxes allow spiral density waves to undergo multiple interactions with embedded bodies before they damp – creating an oscillation in the autocorrelation function

Evolution with gas drag included

Consider bodies ≥ 10m → Stokes drag regime

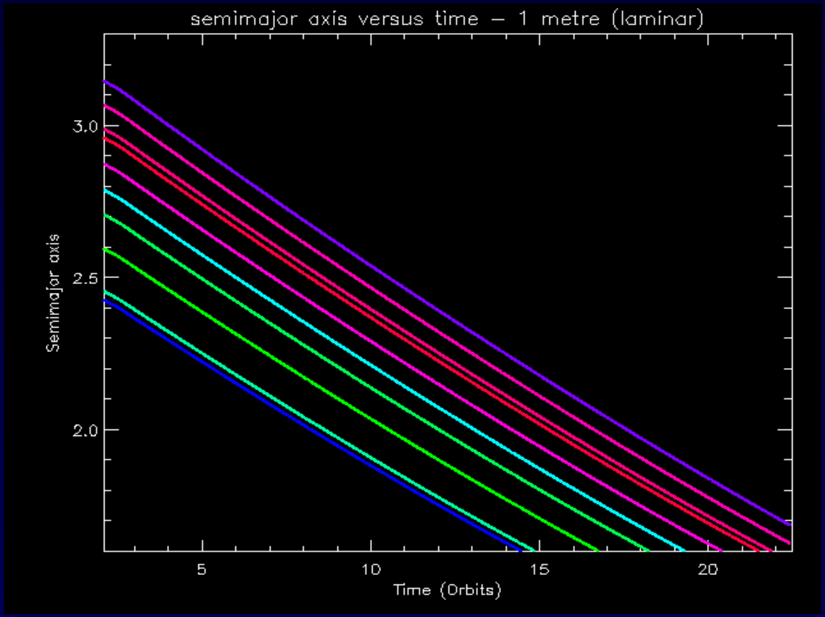
$$\mathbf{F} = C_D \pi a^2 \rho \frac{v^2}{2}$$

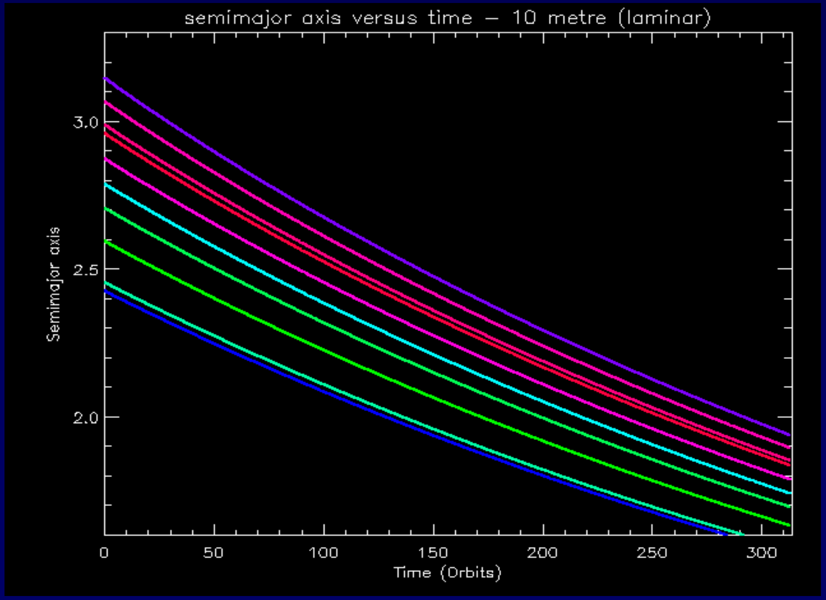
$$C_D = 24 \mathcal{R}_e \quad \text{for } \mathcal{R}_e < 1$$

$$C_D = 24 \mathcal{R}_e^{0.6} \quad \text{for } 1 < \mathcal{R}_e < 800$$

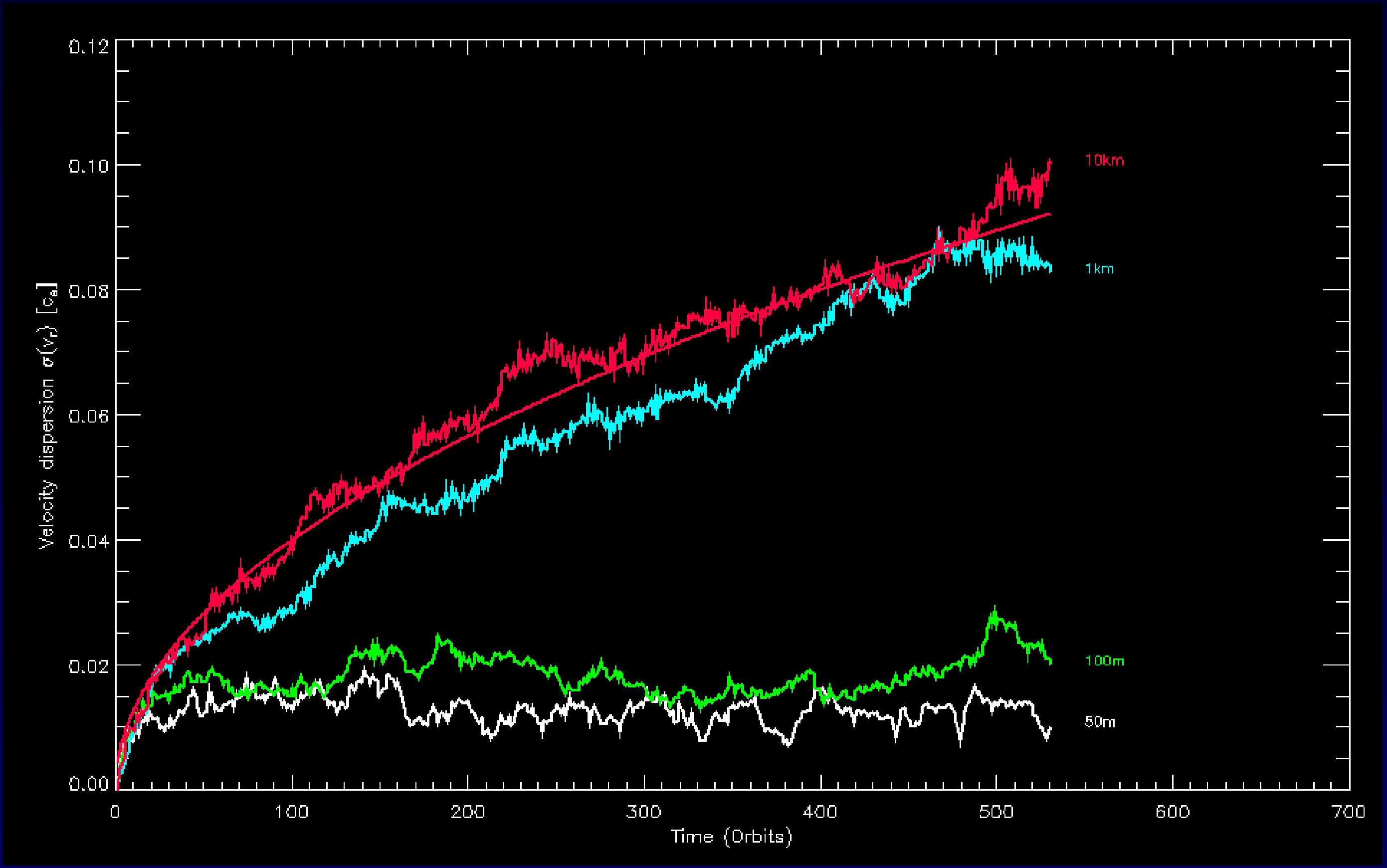
$$C_D = 0.44 \quad \text{for } \mathcal{R}_e > 800$$

Taken from Weidenschilling (1977)





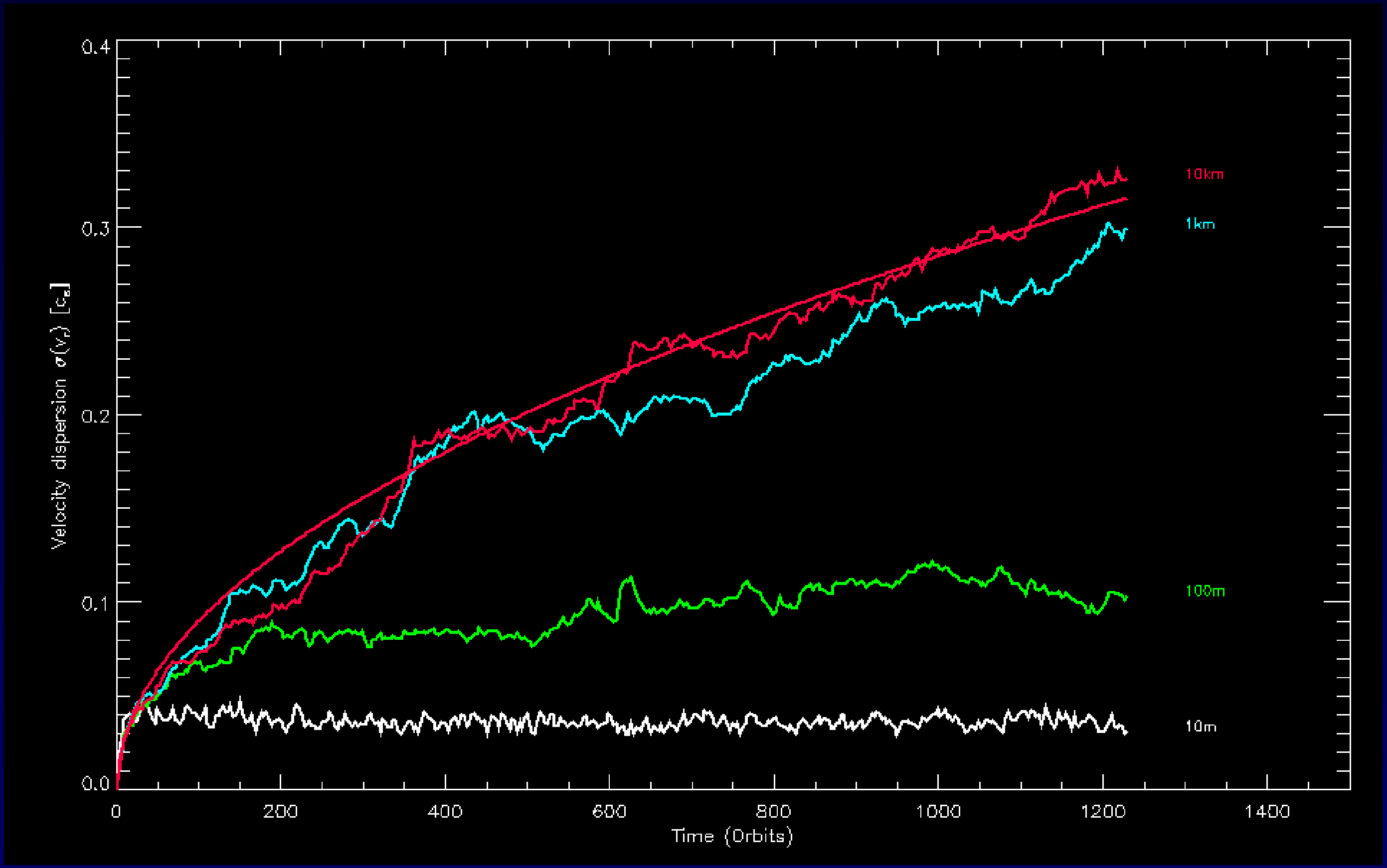
$\sigma(v_r)$ in local and global simulations



For local and global models with H/R=0.075, α=0.035, we obtain $\sigma(v_r) \sim 0.1\ c_s$ (where c_s = 1 km/s) after 500 orbits for 10 km sized planetesimals.

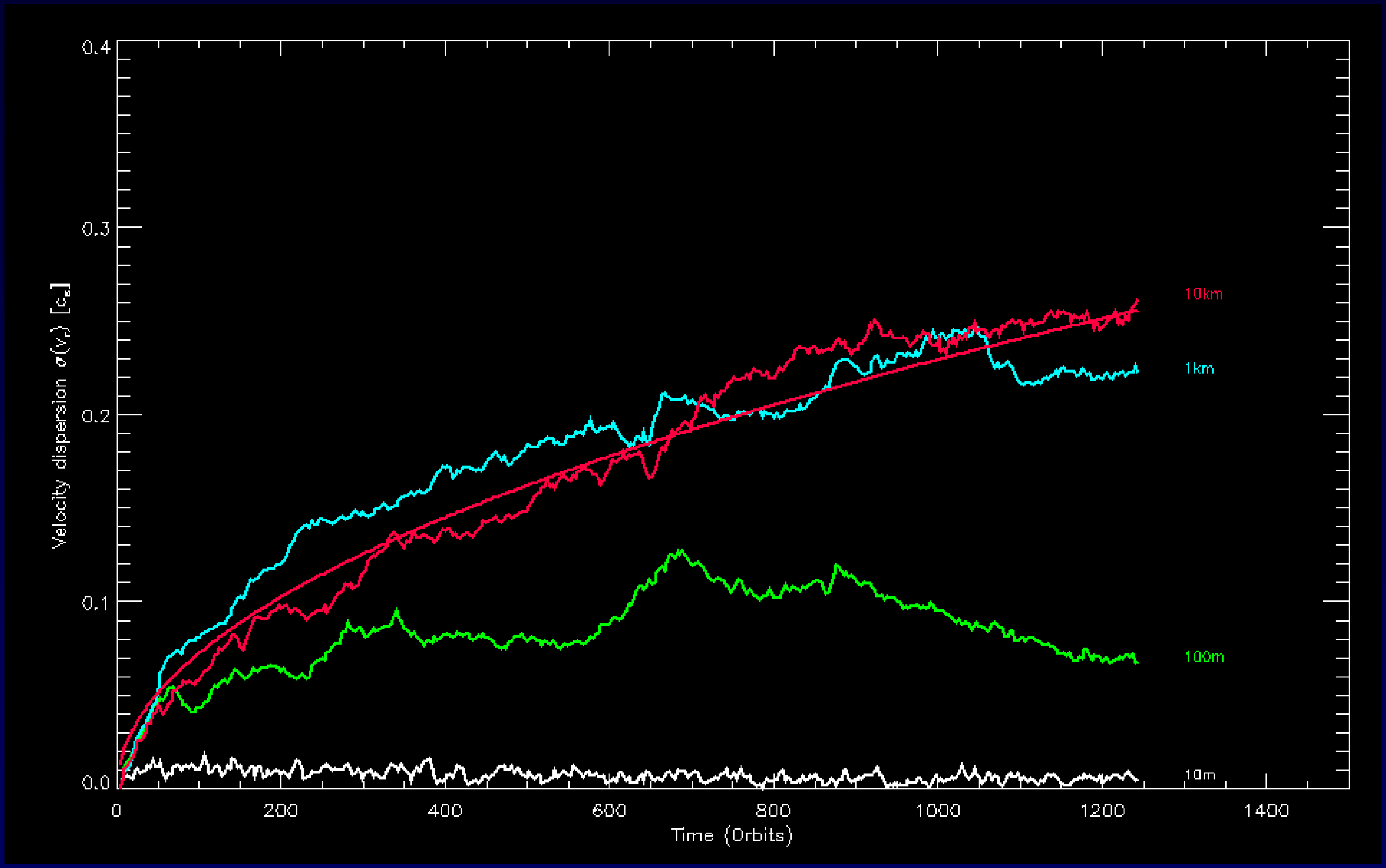
Smaller bodies with size ~ 10 - 50 m achieve equilibrium with $\sigma(v_r) \sim 20$ m/s

Velocity dispersion versus α



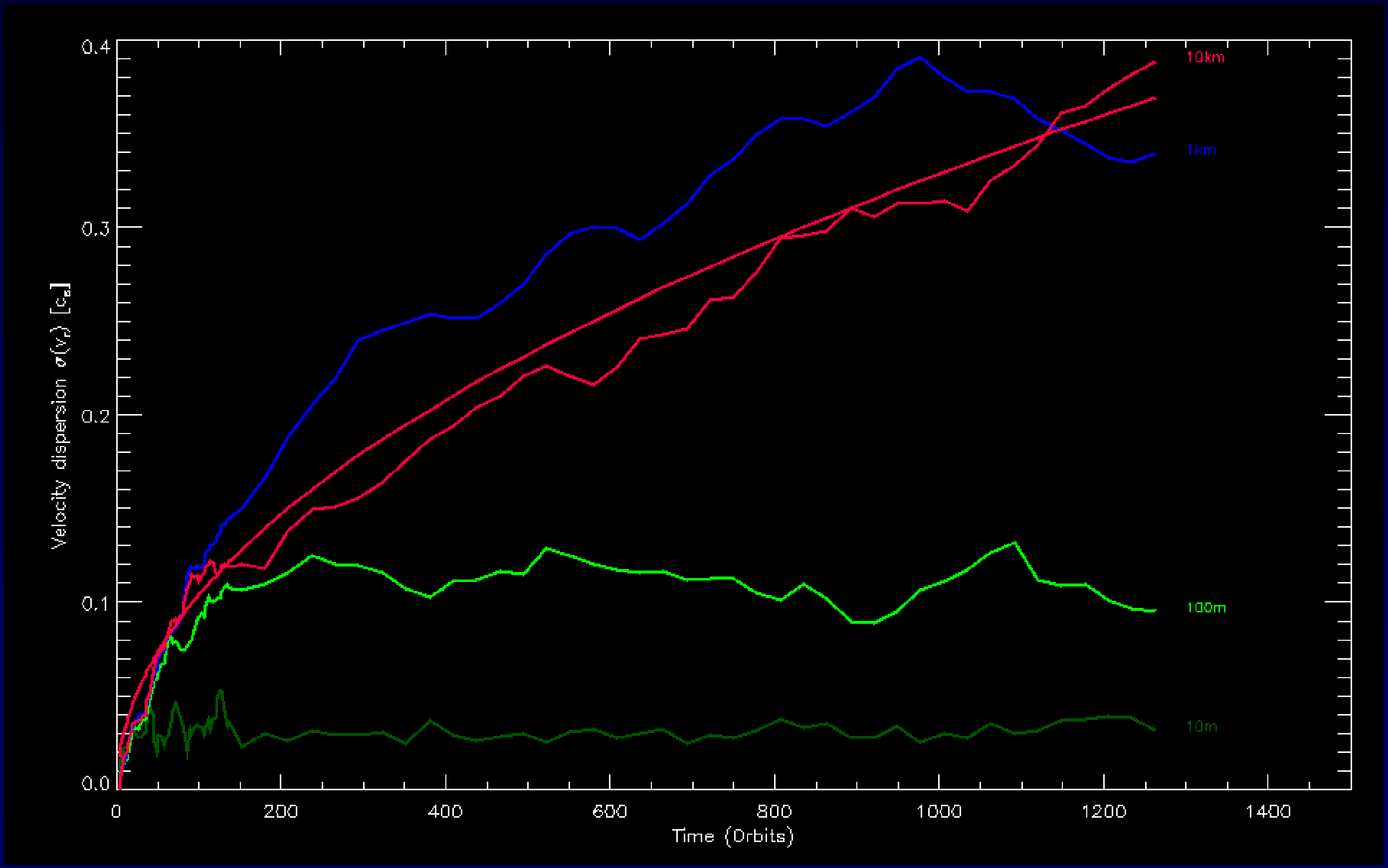
H/R=0.05, α = 0.035, c_s = 666 m/s

Velocity dispersion versus α



H/R=0.05, α = 0.017, c_s = 666 m/s

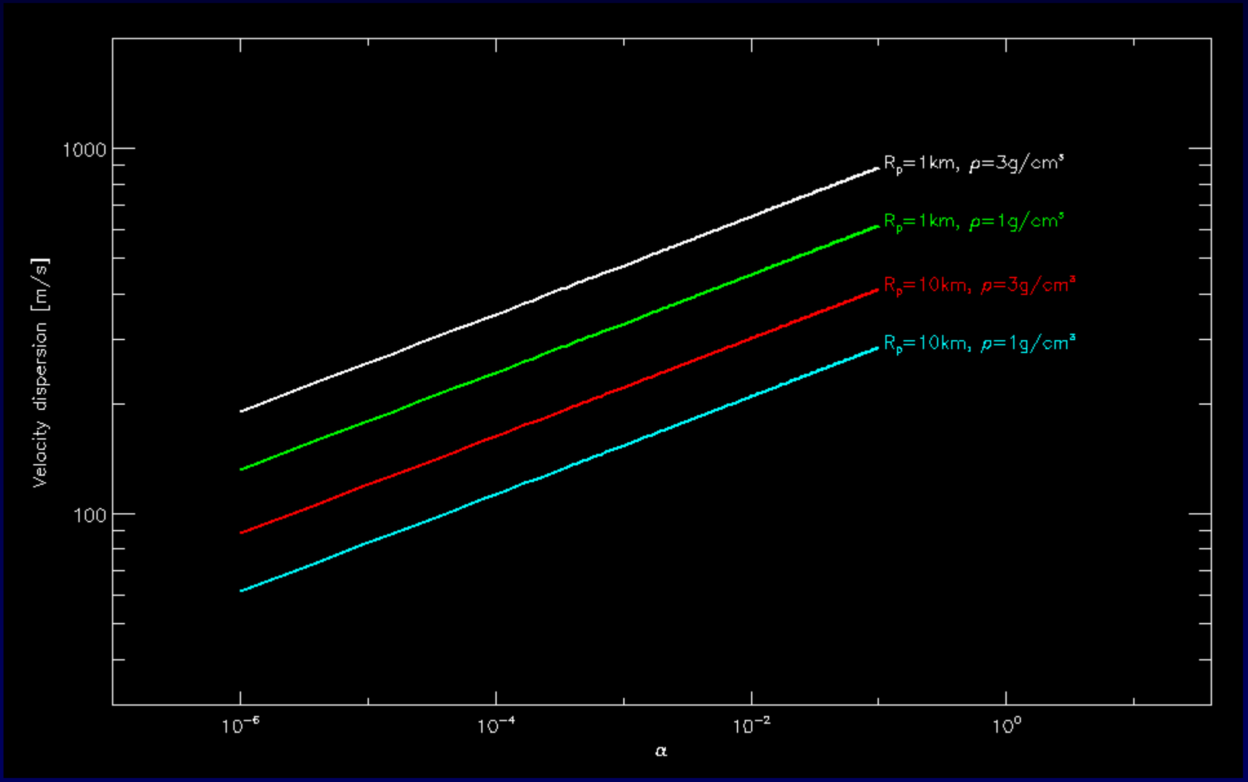
Velocity dispersion versus α



H/R=0.05, α = 0.101, c_s = 666 m/s

Assuming that $\sigma(v_r) \sim \alpha^b$ we find b ~ 1/4

Equilibrium $\sigma(v_r)$ versus α



$$\tau_{\text{grow}} = \frac{e}{de/dt} = \frac{2e^2 v_{\text{K}}^2}{C_\sigma(v_r)},$$

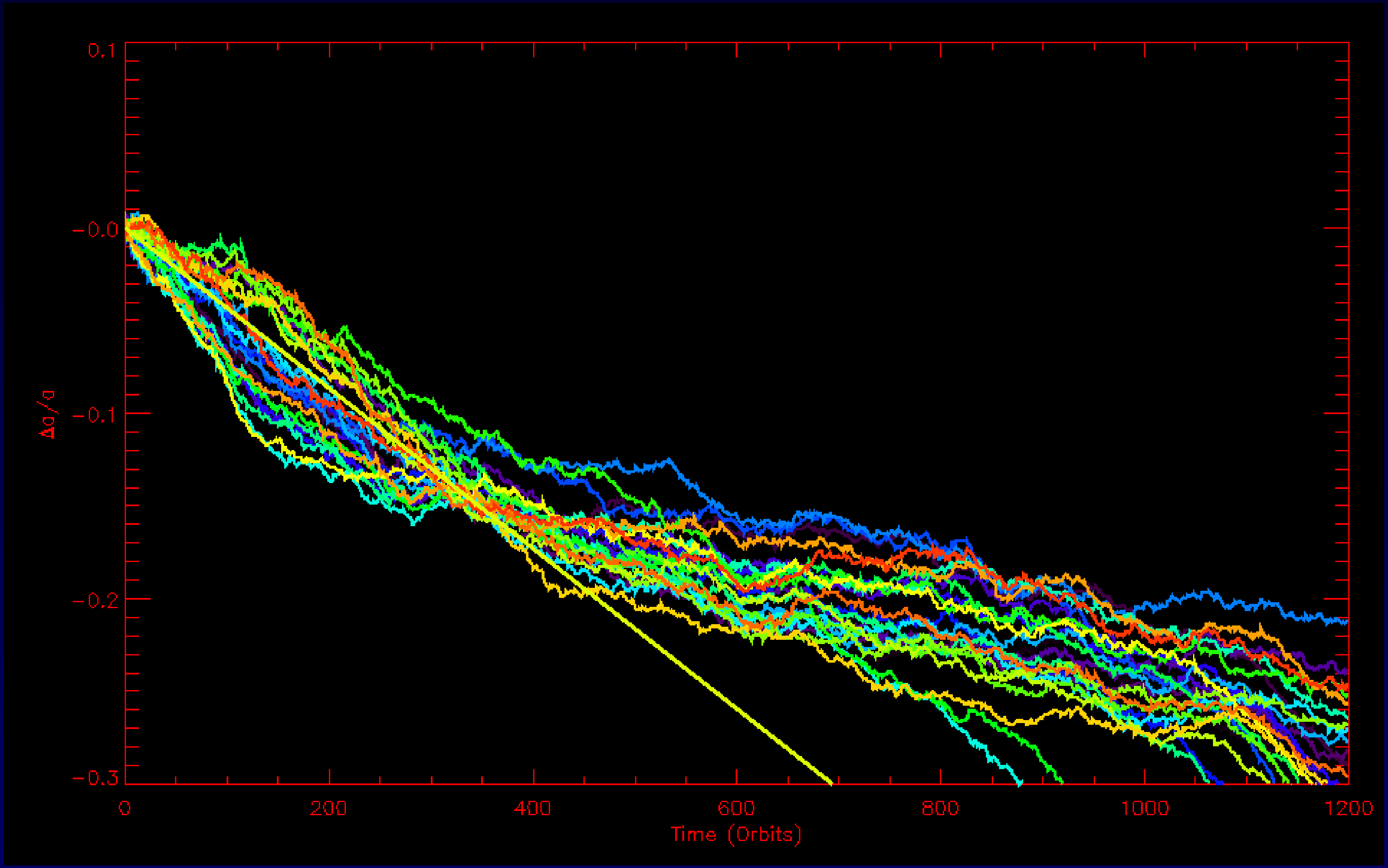
$$\tau_{\text{damp}} = \frac{2 m_{\text{p}} v_{\text{disp}}}{C_D \pi R_{\text{p}}^2 \varrho v_{\text{disp}}^2}$$

$$v_{\text{disp}} = \left(\frac{4 \varrho_{\text{p}} R_{\text{p}} K_{v_r}^2 \alpha^{0.4}}{3 C_D \varrho} \right)^{1/3}$$

Using fits: $\sigma(v_r) = C\sigma(v_r)\sqrt{t}$ and $C\sigma(v_r) \sim \alpha^{1/4}$ we can estimate equilibrium $\sigma(v_r)$ as a function of α and planetesimal size (Ida, Guillot & Morbidelli 2008)

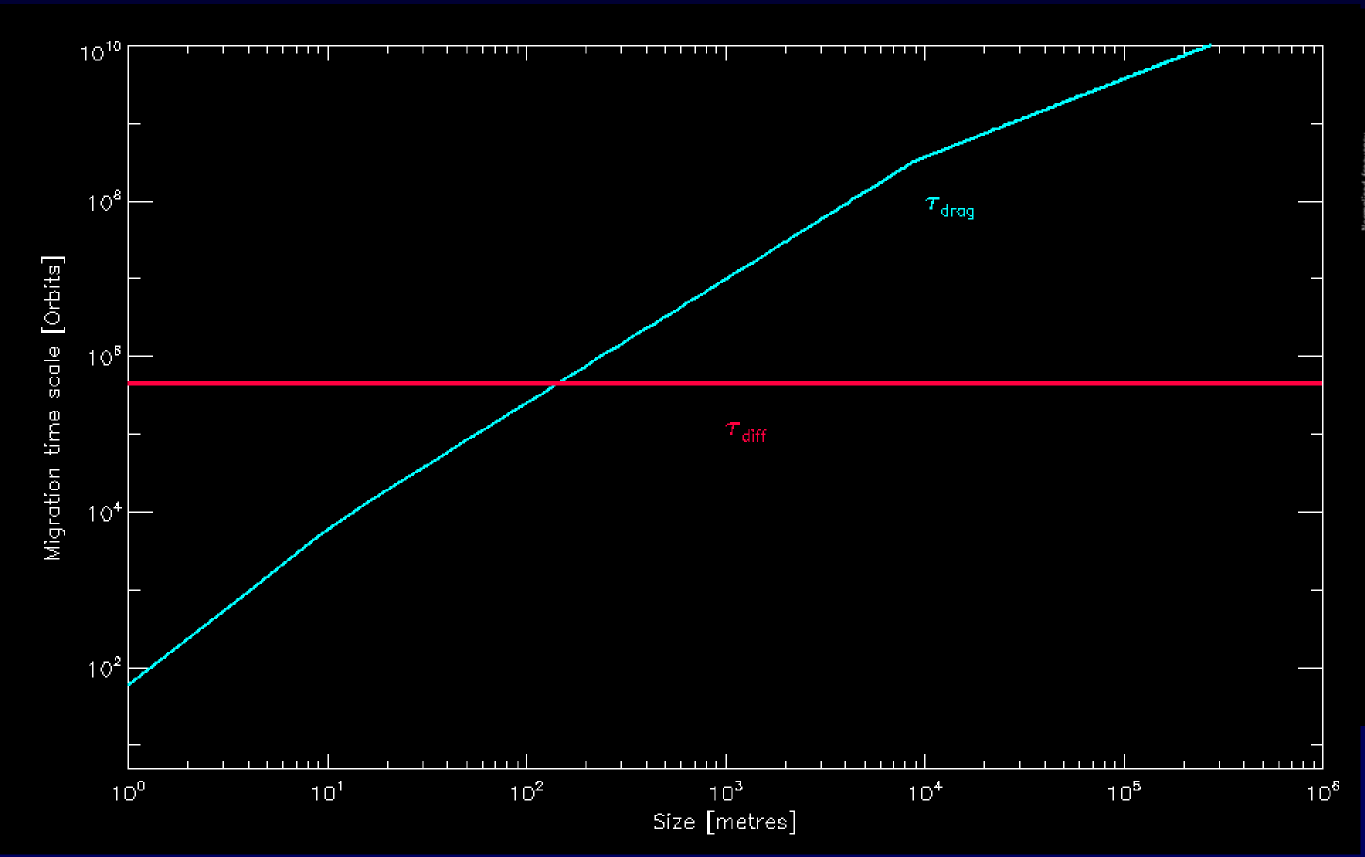
Catastrophic disruption threshold for collisions between 10km sized bodies is ~ 10 – 20 m/s (Benz & Asphaug 1999; Stewart & Leinhardt 2009)

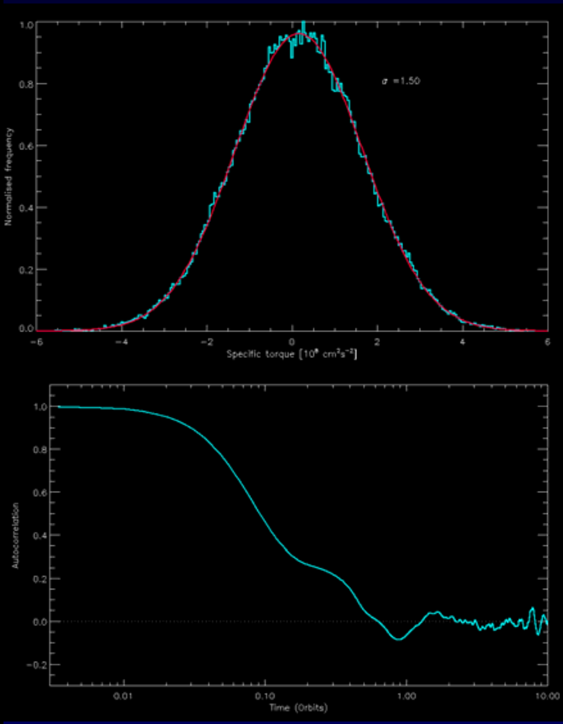
Radial migration/diffusion



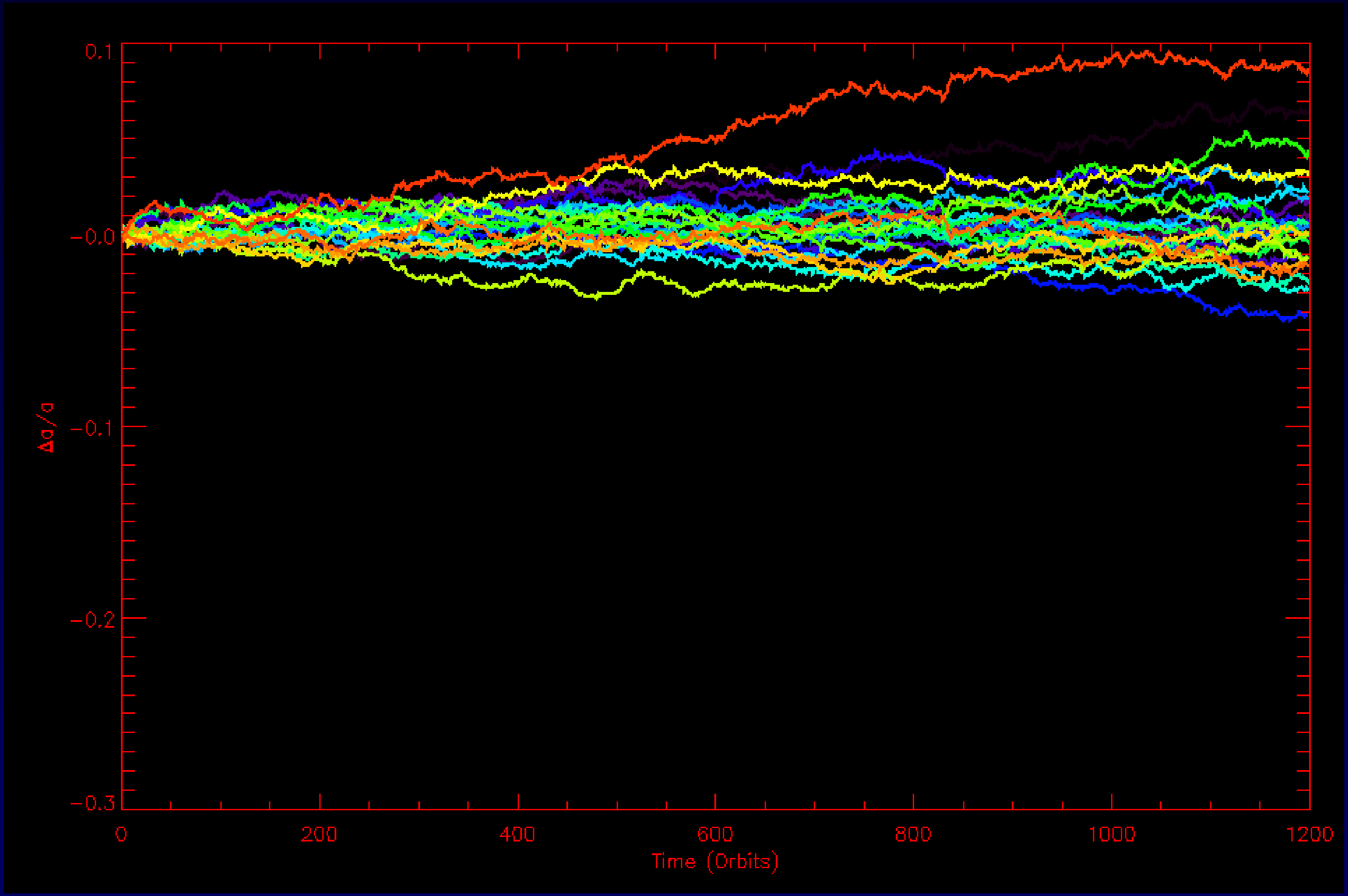
10m sized bodies migrate inward rapidly due to gas drag. Deviation from laminar case caused by surface density profile being modified by radial variation in turbulent stresses. Stochastic forces have little effect on migration rate.

Gas drag migration versus stochastic migration



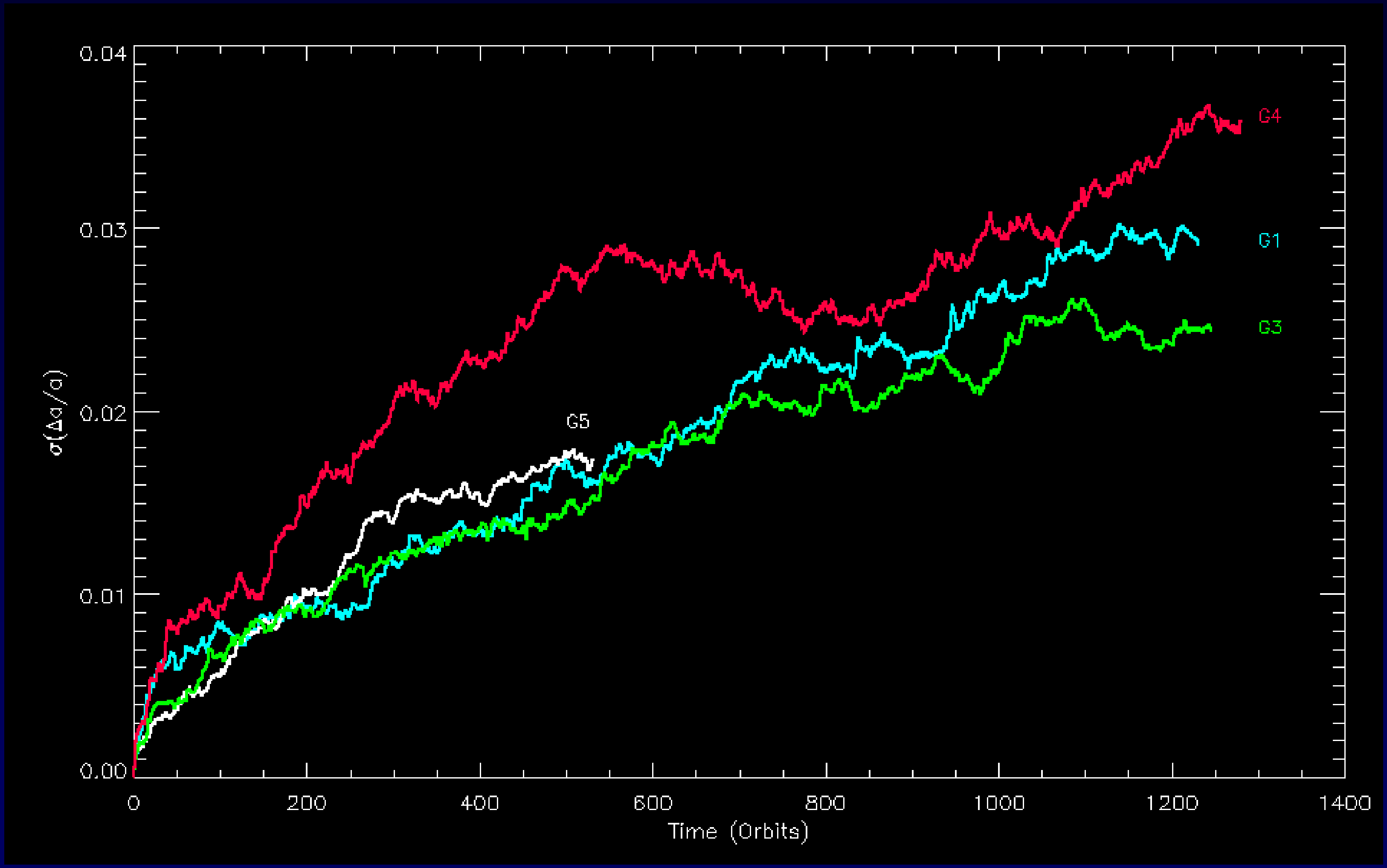


Predict that after 2 Myr of evolution, gas drag induced migration will dominate stochastic migration for bodies of size < 100 m



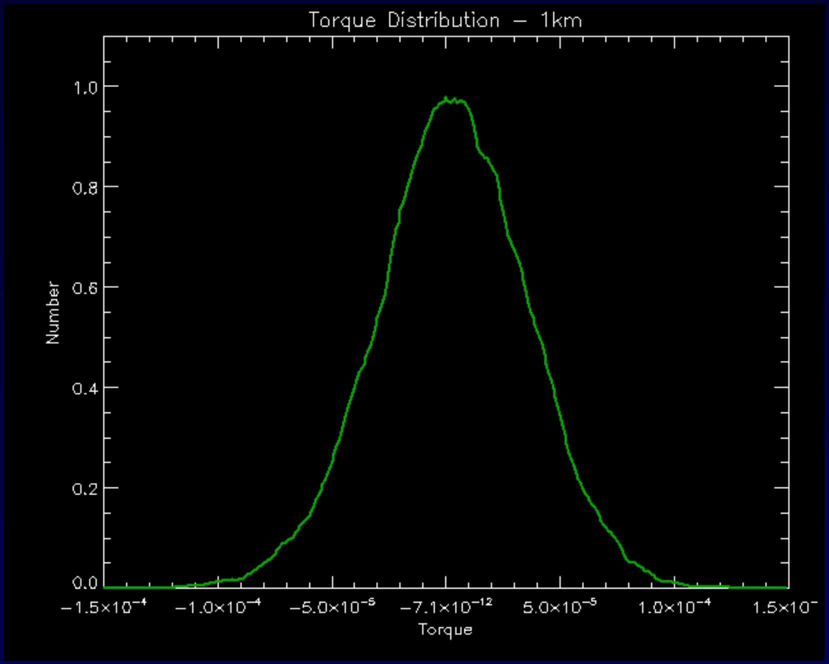
100m, 1km & 10km bodies undergo radial diffusion/ stochastic migration

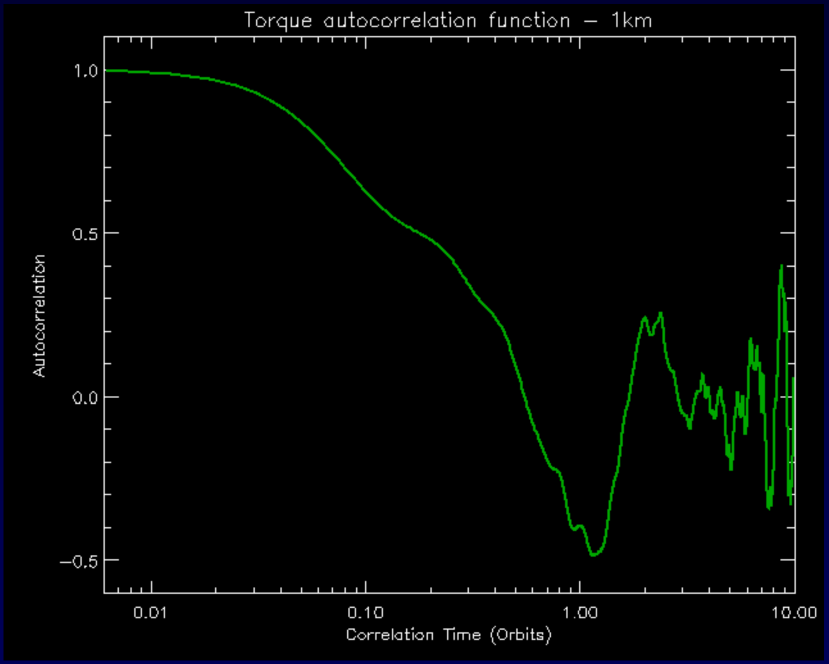
Radial diffusion versus α



Adopting fit: $\sigma(\Delta a/a) = C_a \sqrt{t}$ we find $C_a \sim \alpha^{1/4}$

Long-term orbital evolution of planetesimals





r.m.s. torque fluctuations

$\sigma_T \sim 3.5 \times 10^{-5}$

Angular momentum diffusion coefficient

$D_J \sim (\sigma)^2 \tau_{corr}$

(e.g. Johnson, Goodman, Menou 2006)

Stochastic migration time

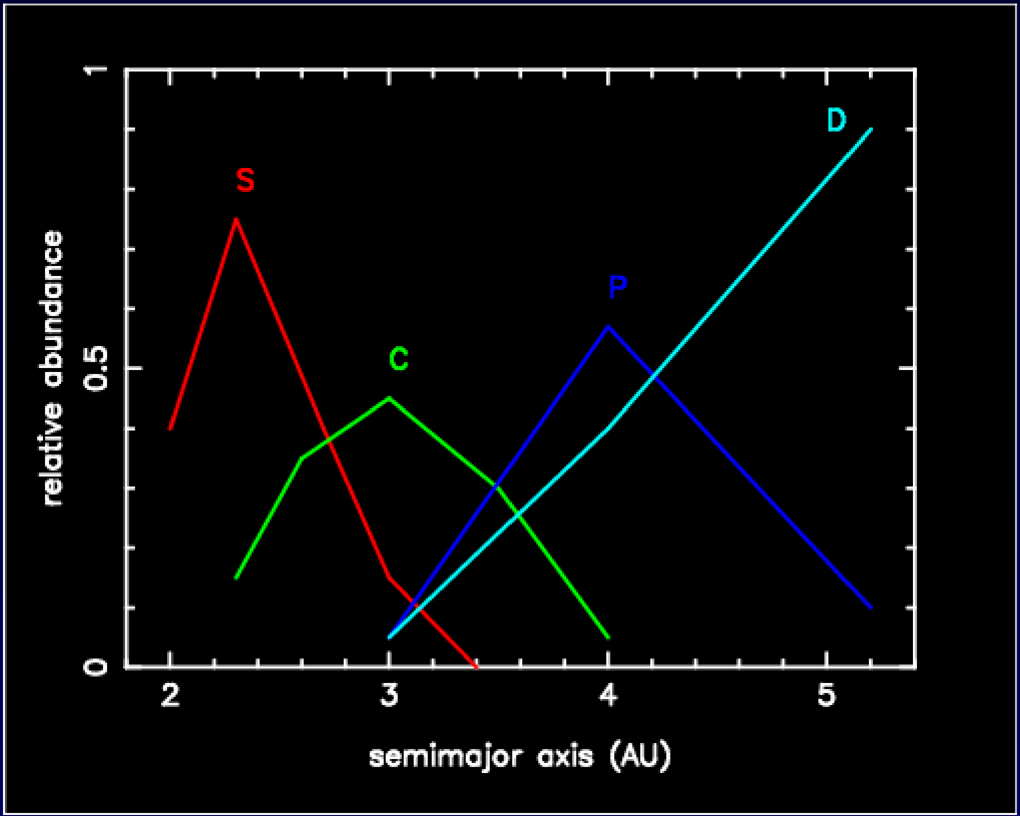
$t_{mig} \sim (\Delta J)^2 / D_J$

Fluctuating torque correlation time

$\tau_{corr} \sim 0.3$ orbits (~ 3 years at 5 AU)

Time for planetesimals to stochastically migrate from 5 AU to 2.5 AU ~ 5 Myr in MMSN

Asteroids in Solar System



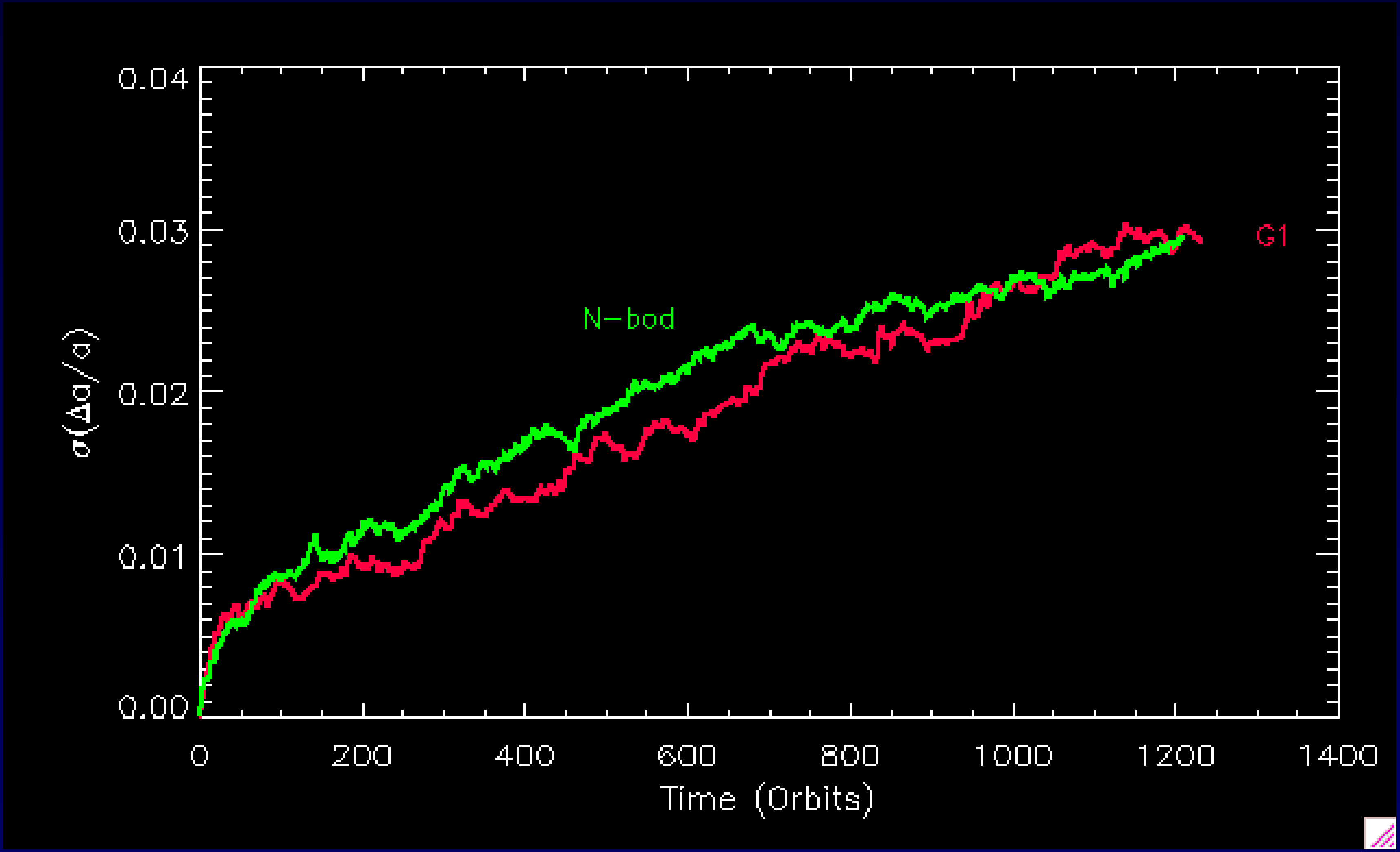
Gradie et al (1982)

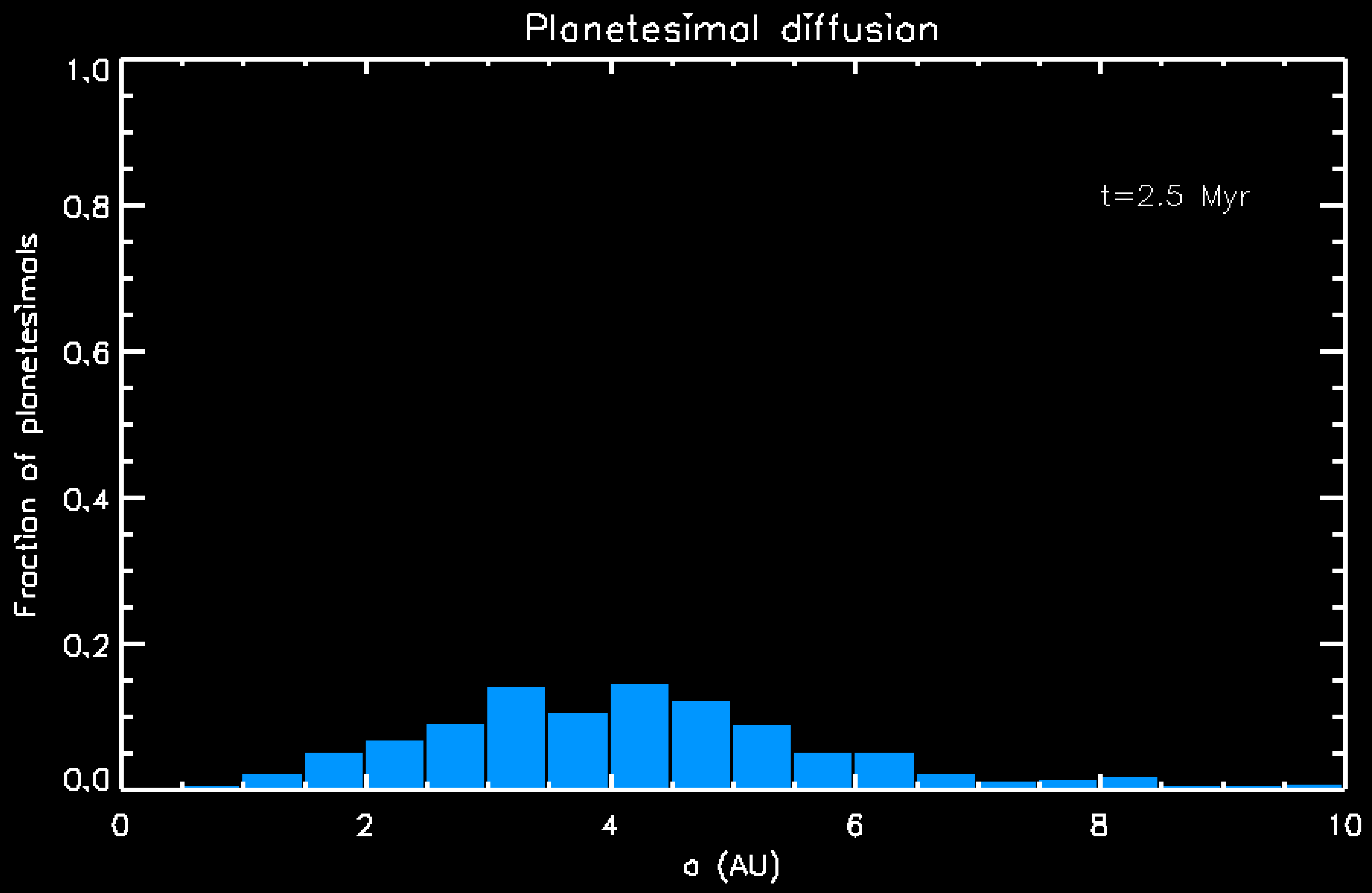
Observations of asteroids in the asteroid belt show that different taxonomic classes are reasonably well ordered as a function of heliocentric distance

➔ radial mixing in the Solar nebula was relatively modest

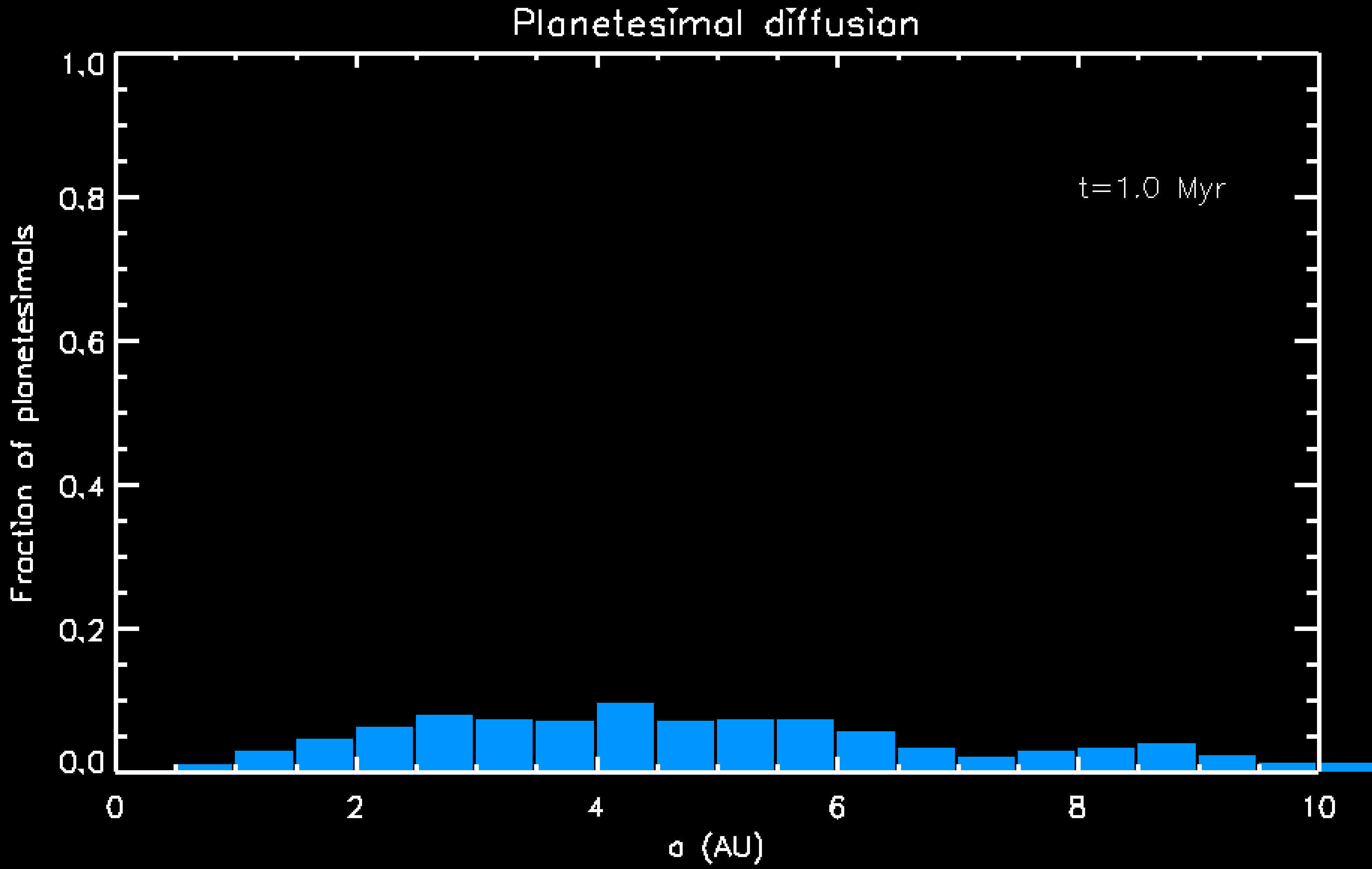
Significant radial mixing of icy asteroids would substantially enhance water content of the Earth (O'Brien et al 2007)

N-body simulations with stochastic migration

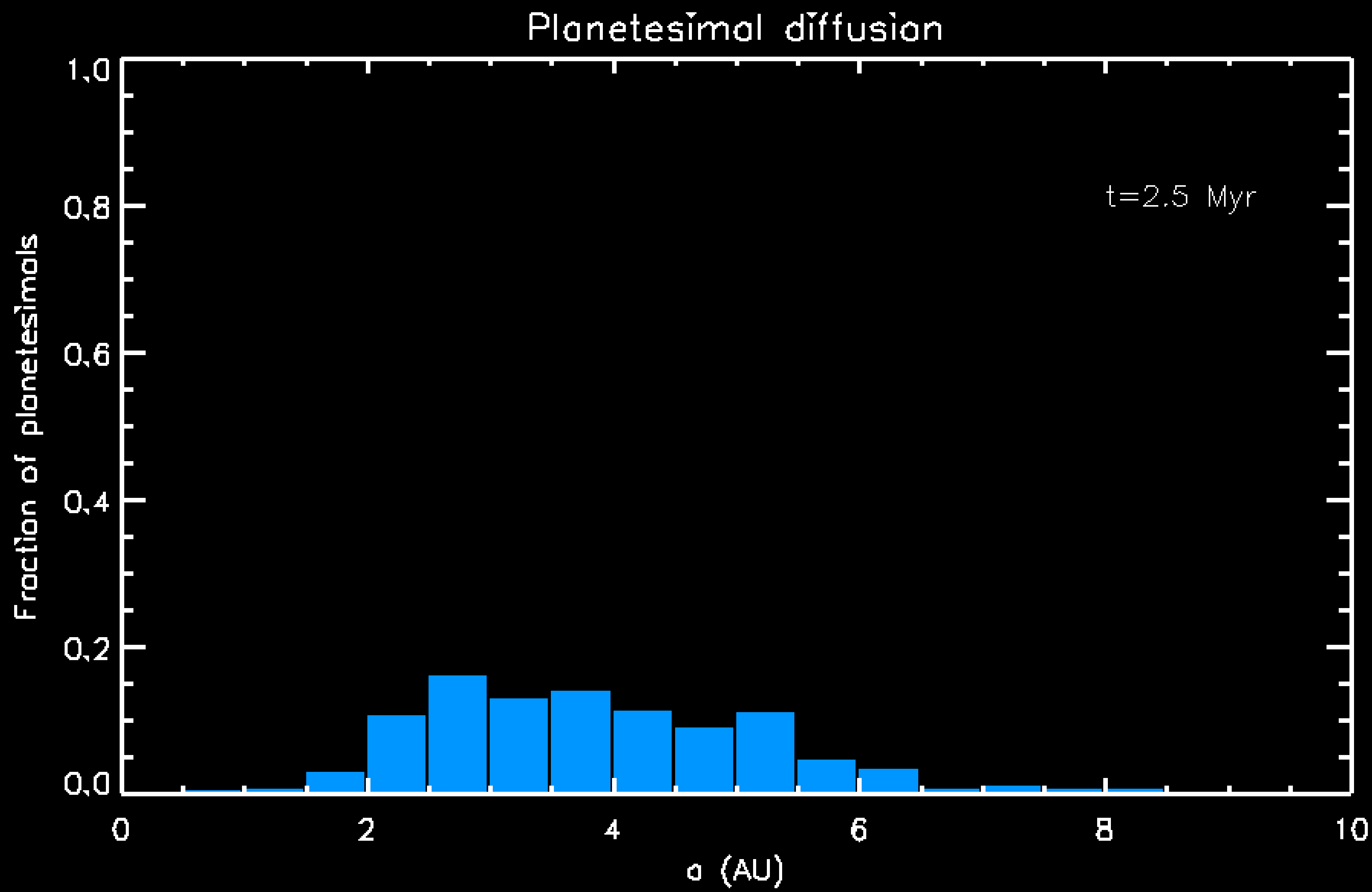



Planetesimal diffusion
t=2.5 Myr
Fraction of planetesimals
a (AU)
0,0
0,2
0,4
0,6
0,8
1,0
0
2
4
6
8
10

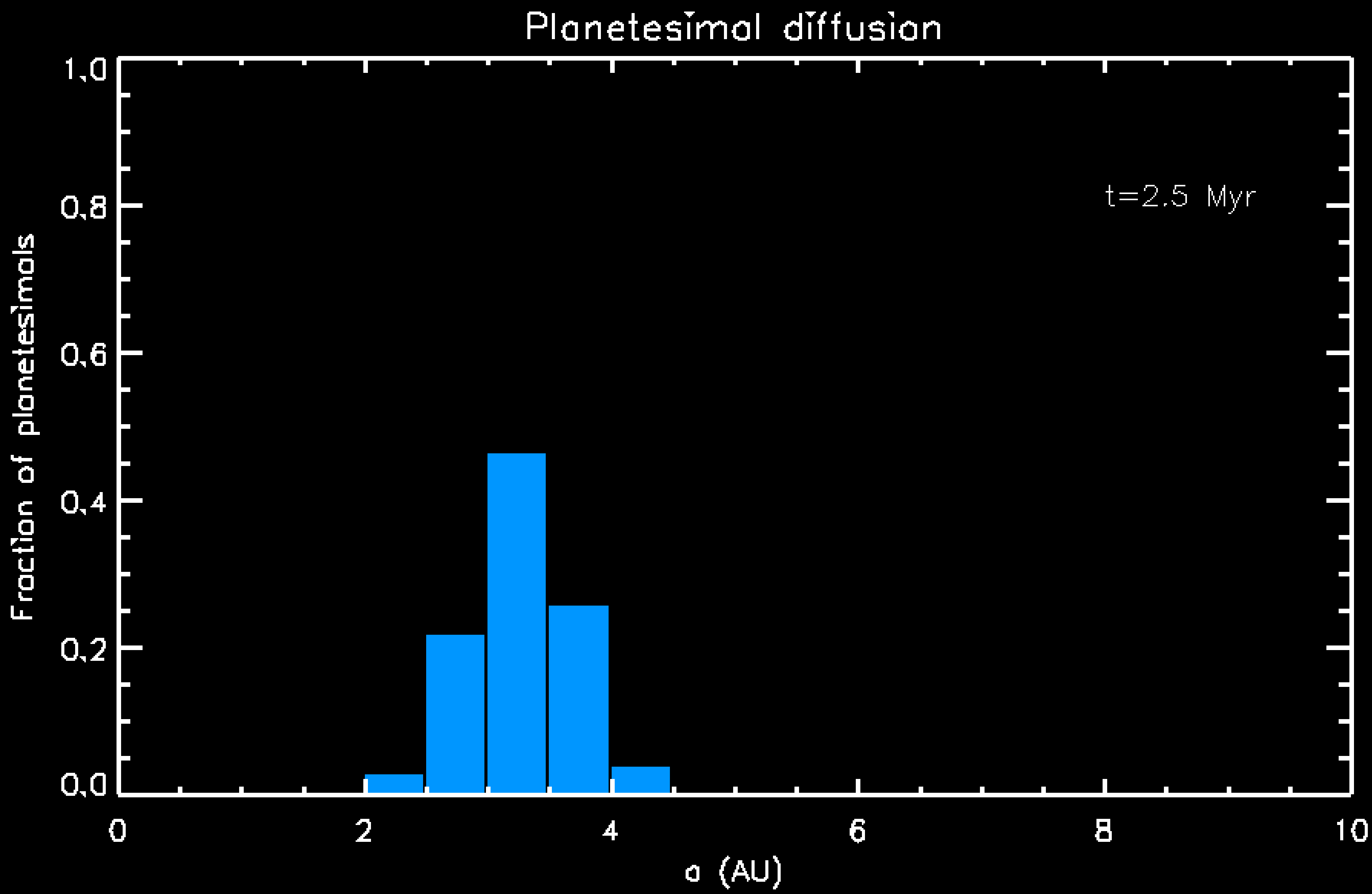
α= 0.03, MMSN


Planetesimal diffusion
t=1.0 Myr
Fraction of planetesimals
a (AU)
0.0
0.2
0.4
0.6
0.8
1.0
0
2
4
6
8
10

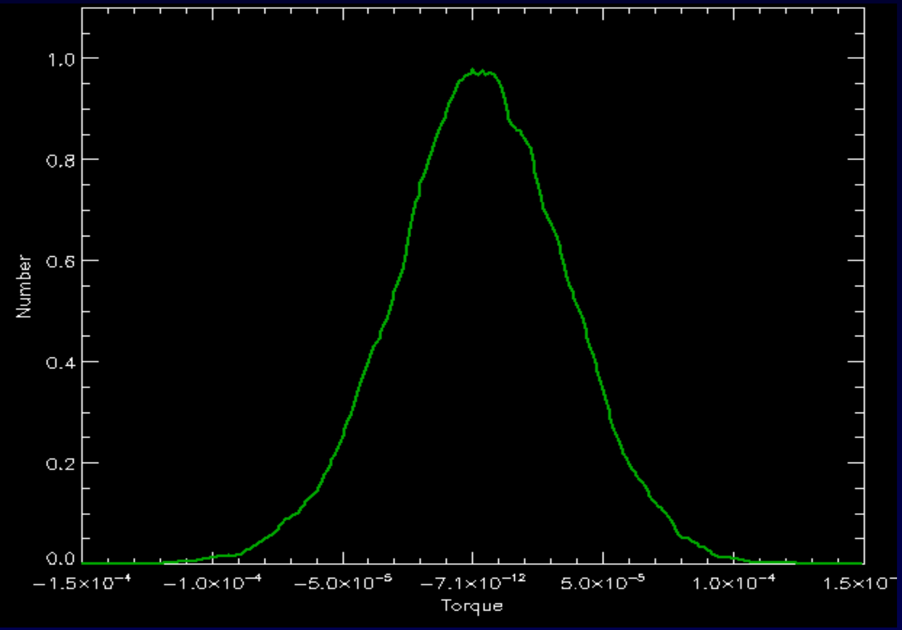
α=0.03, 3 x MMSN

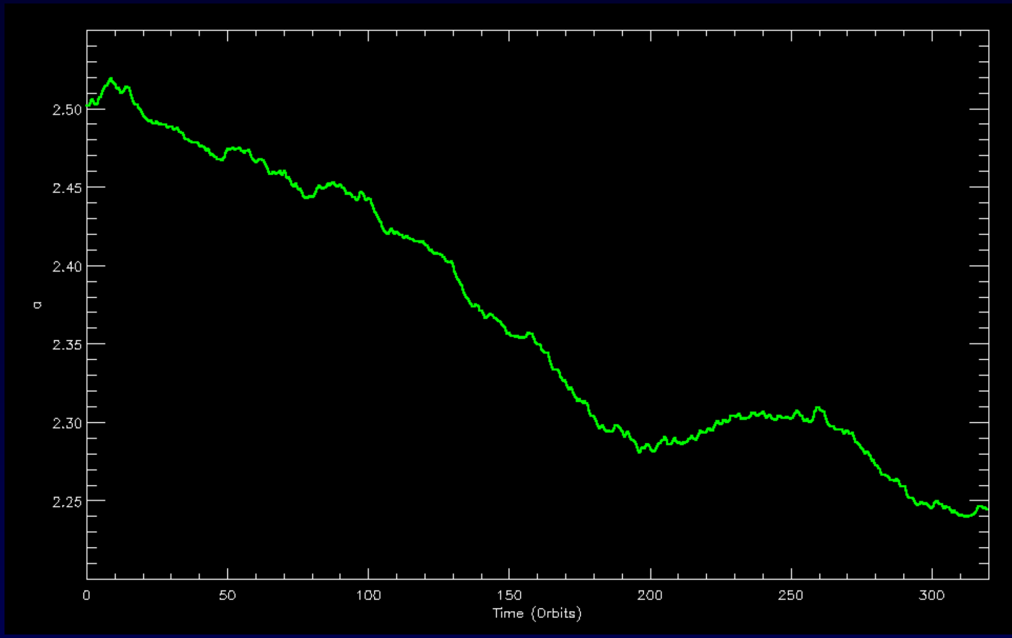


$\alpha=10^{-3}$, 3 x MMSN



$\alpha=10^{-5}$, 3 x MMSN





$$\overline{T} = \langle T \rangle + \frac{\sigma}{\sqrt{t}}$$

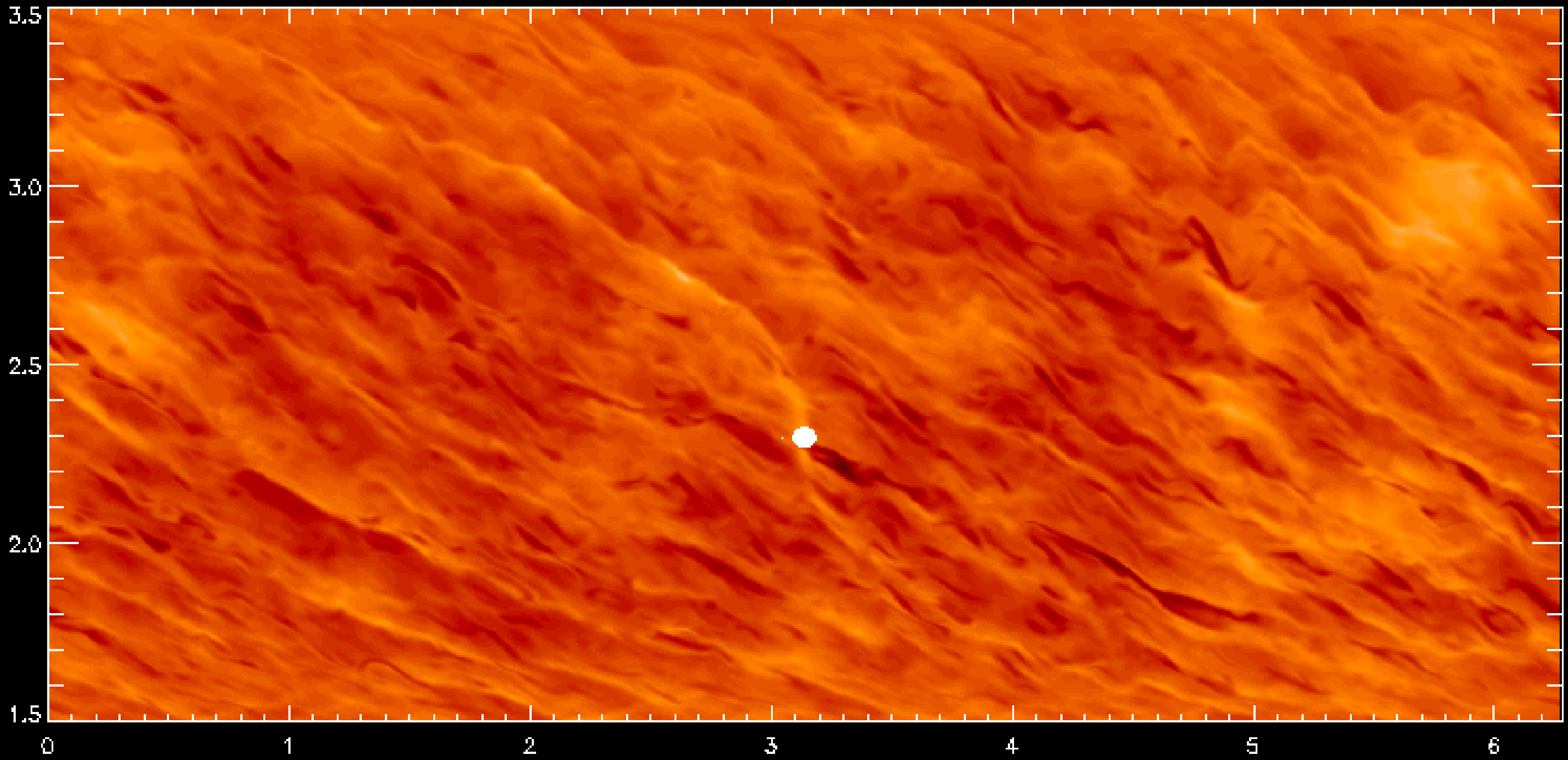
Can treat stochastic migration as a signal to noise problem (assume linear superposition of type I + stochastic torques)

Calculate time scale over which type I torque dominates random walk

For mp=10 Earth masses we estimate that $t_{type\ 1} \sim 500\ t_{corr}$

For mp=1 Earth mass $t_{type\ 1} \sim 5 \times 10^4\ t_{corr}$

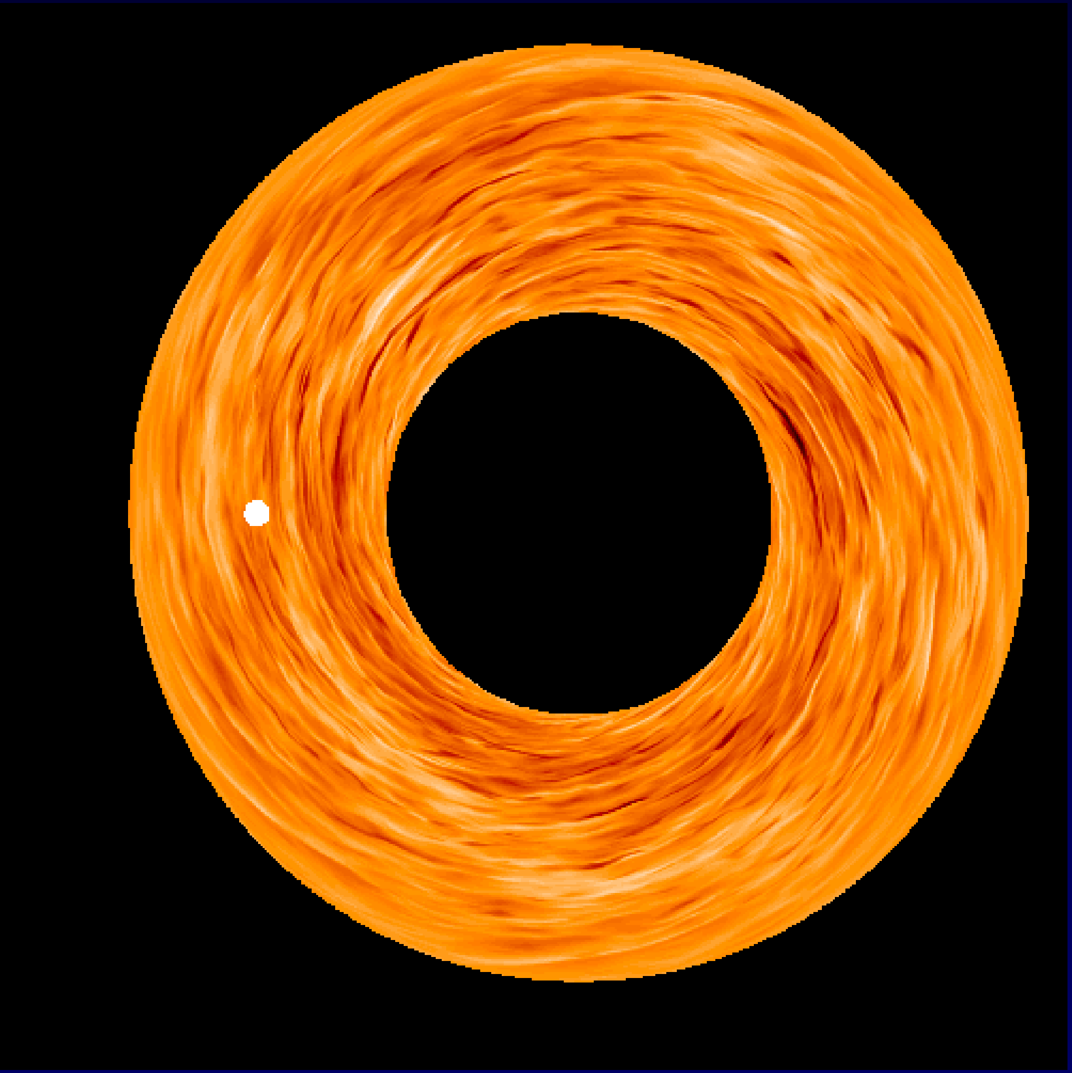
Expect stochastic migration to dominate over type I migration only for planets with masses ~ Mars masses for disc life times of 5 Myr

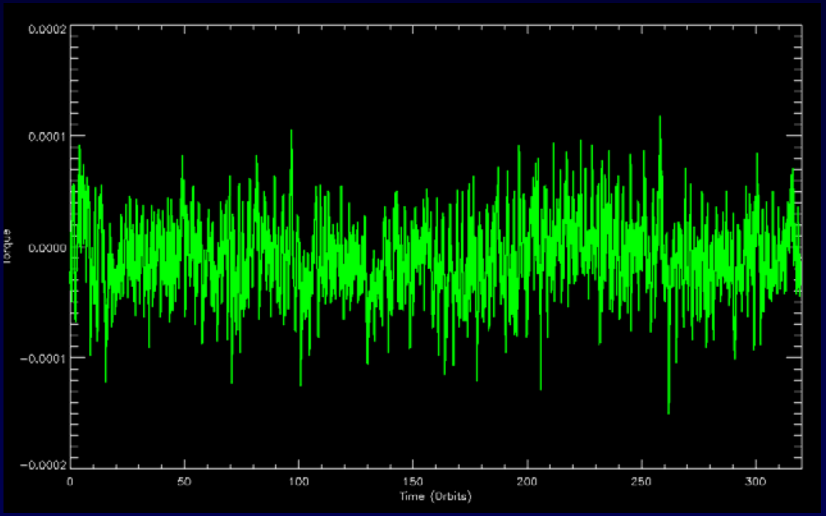


Does migration/torque correlate with local surface density profile ➔ corotation torques
Results are inconclusive so far…
but expect to observe corotation torques in turbulent discs (Baruteau & Lin 2010)

Persistent vortex-like features appear in flow – long term impact on migration ?

Low mass planets in turbulent discs





10 M_{earth} in MMSN disc model experiences strong stochastic torques

Conclusions + future work

- Require box-size of ~ 8 - 10 scale-heights for stochastic forces to converge

- Planetesimal velocity dispersion shows minimum value of ~ 20 m/s for bodies of size ~ 50m in disc with $\alpha=0.03$

- For fully turbulent discs, velocity dispersion of > 1 km sized planetesimals $\sigma(v) \geq 200$ m/s
  $\rightarrow$ collisional breakup and quenching of runaway and oligarchic growth

- Macroscopic bodies experience orbital diffusion on scales of a few AU within expected gas disc life times

May be possible to use solar system data to constrain strength of midplane turbulence and size of the dead zone (e.g. composition gradients in asteroid belt)

- Stochastic forces can probably overcome standard type I migration only for ~ Mars mass planets, but turbulence must be present to prevent saturation of corotation torques

- Future work: stratified discs (global and shearing box), with and without dead zones, to examine velocity dispersion and radial migration as function of dead zone size, and influence of turbulence on corotation torques

There are two competing models for explaining how giant planets form:

Gravitational instability model

- the protostellar disc fragments to form giant planets directly

Core accretion model

- a large core composed of rock + ice forms first, and then accretes a massive gaseous envelope

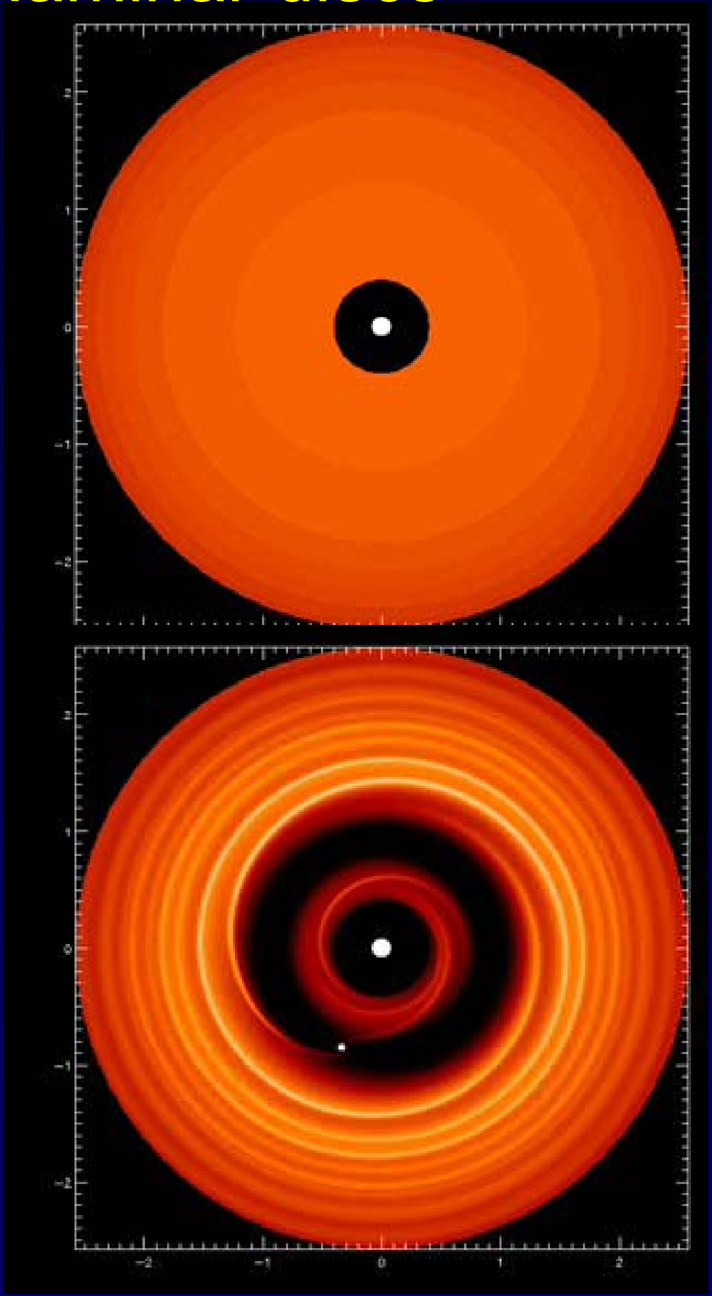
High mass protoplanets in laminar discs

- When planets grow to ~ Jovian mass they open gaps:

  (i) The waves they excite become shock waves when $R_{Hill} > H$

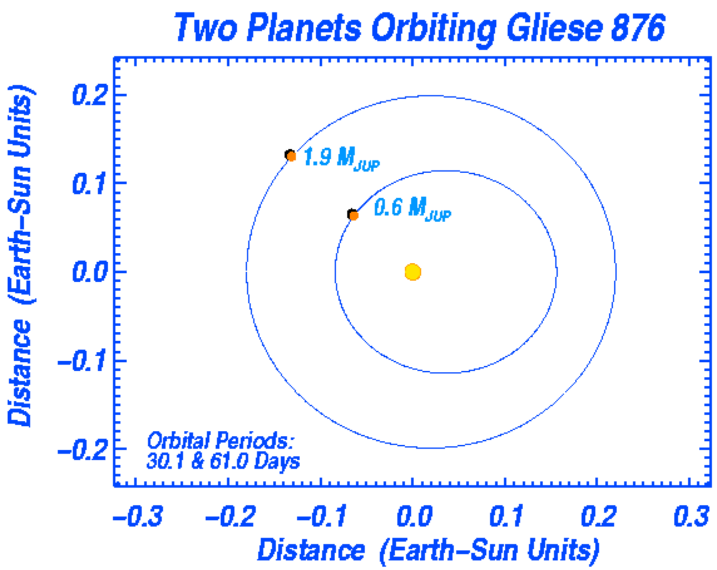
  (ii) Planet tidal torques exceed viscous torques

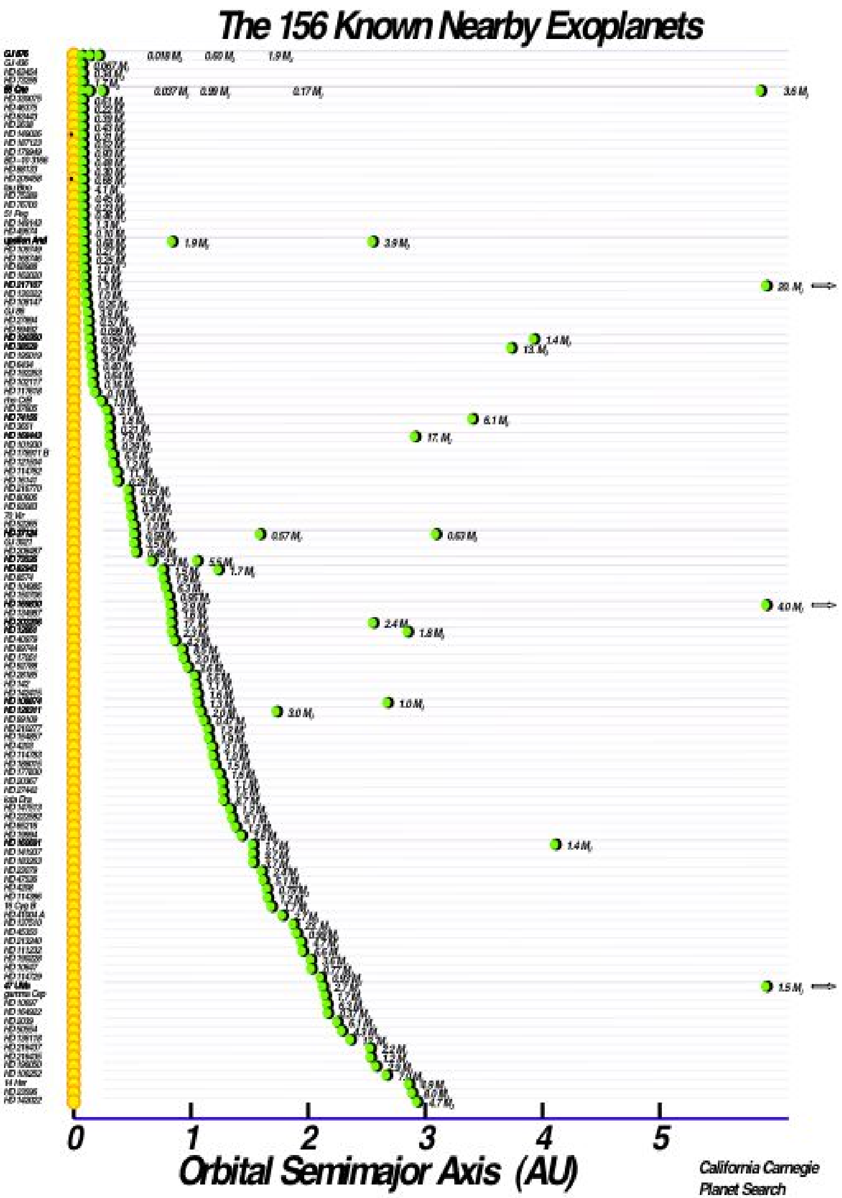
- Inward migration occurs on viscous evolution time scale of the disk

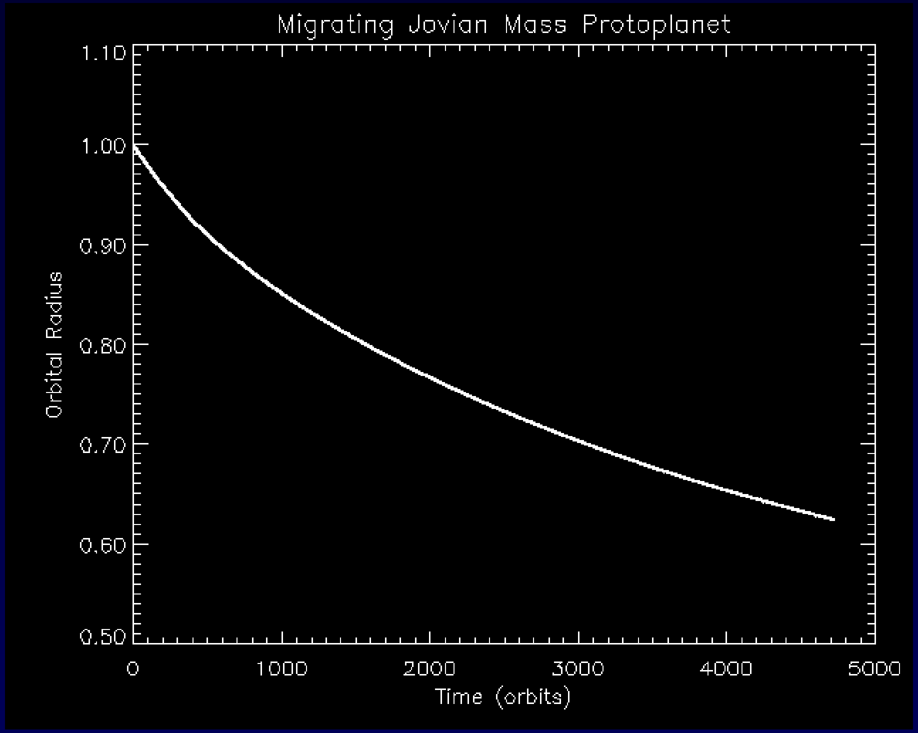


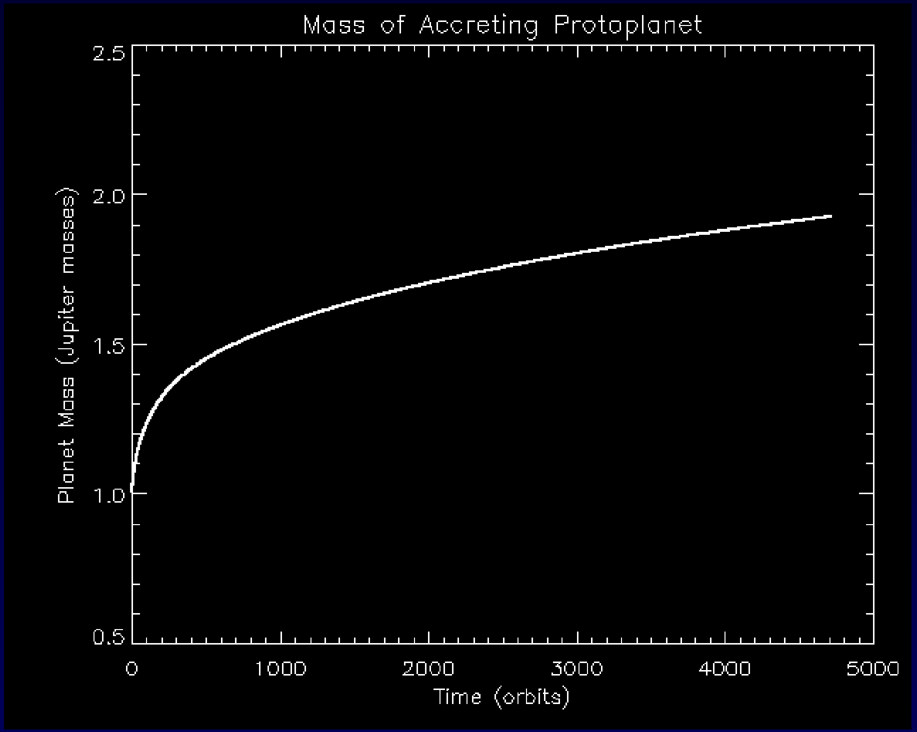
Evidence for type II migration

- Existence of short period planets (Hot Jupiters)
- Resonant multiplanet systems:
  GJ876 – 2:1









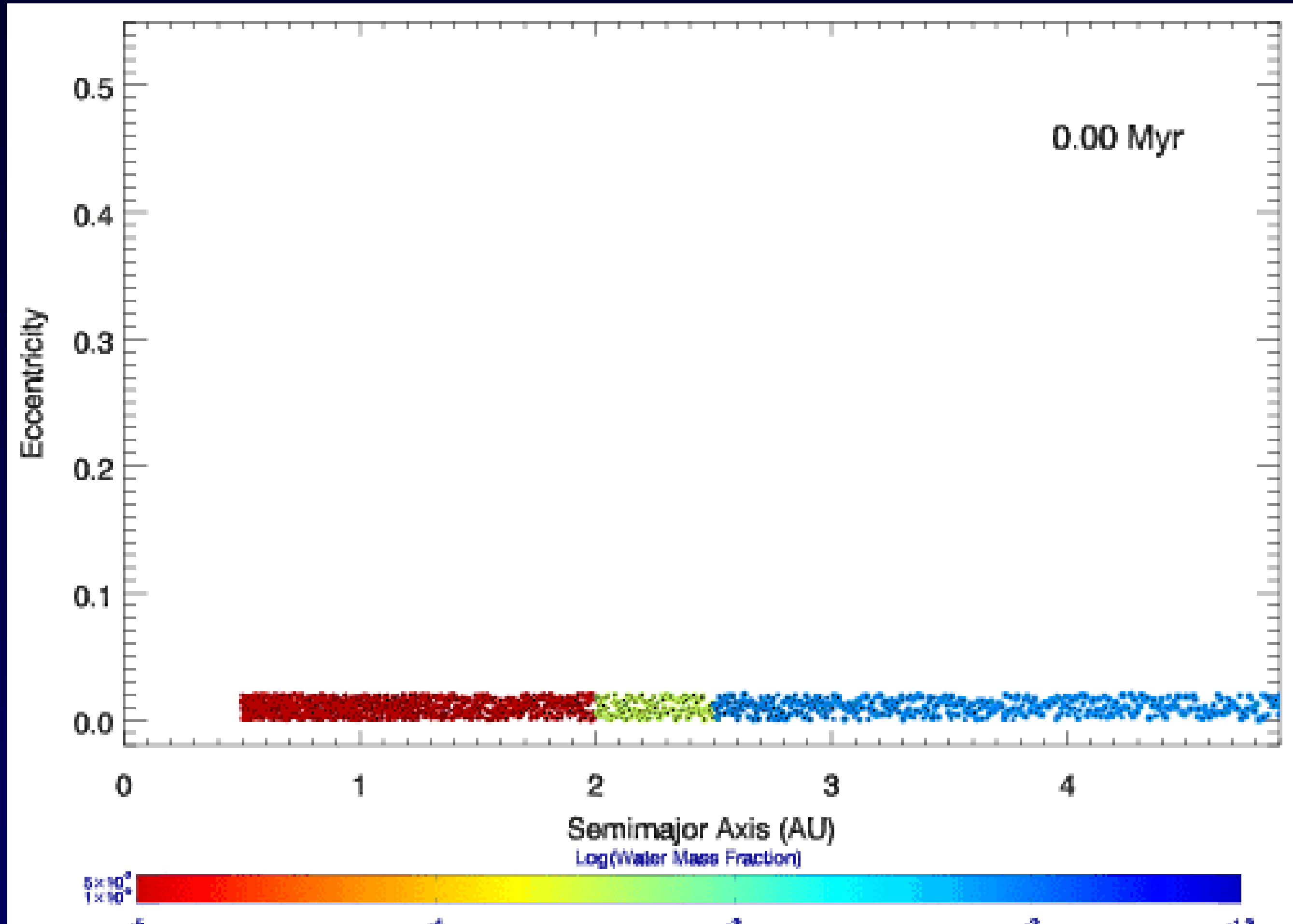
- Inward migration occurs on time scale of ~ few x 10^5 year
- Jovian mass planets remain on ~ circular orbits
- Heavier planets migrate more slowly than viscous rate due to their inertia
- A 1 M_J planet accretes additional 2 – 3 M_J during migration time of ~ few x 10^5 yr

- Once planetesimals have formed, runaway growth of the most massive planetesimals leads to formation of planetary embryos

- These cease accreting when they reach their isolation mass ~ lunar mass at 1 AU

- Final phase of terrestrial planet formation involves giant impacts between the embryos

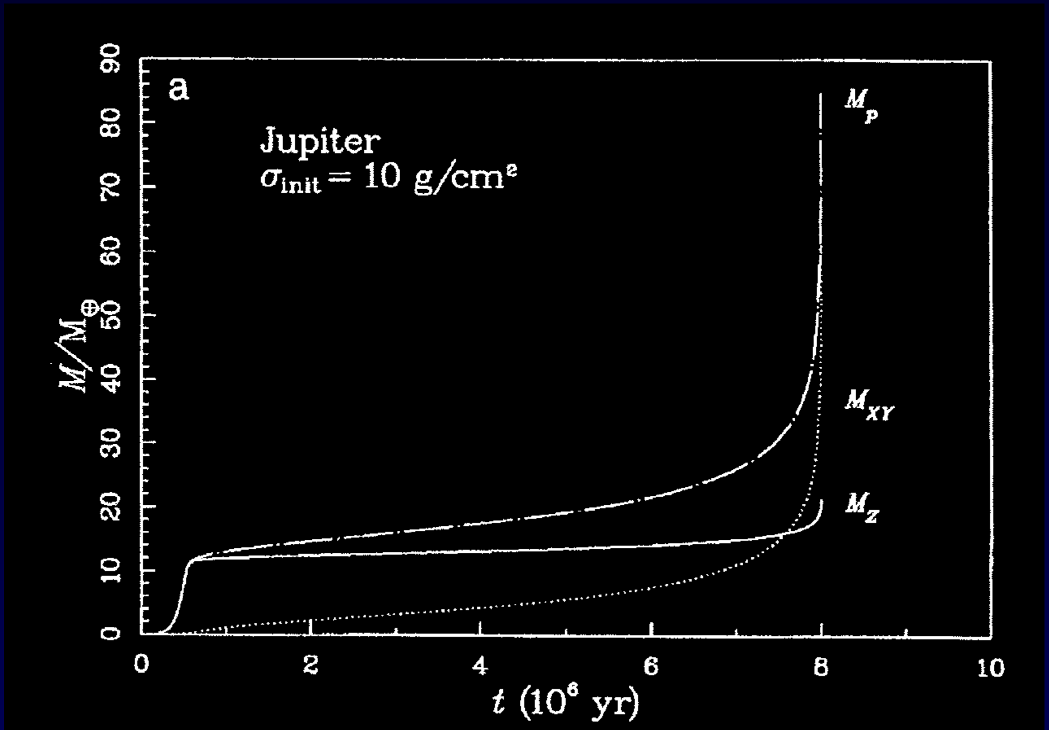
Raymond, Mandel & Sigurdson (2006)



The giant impact phase of terrestrial planet formation requires ~ 100 Myr to complete – in good agreement with the radio-dating of lunar rock samples

But current N-body models tend to form a Mars analogue which is too big, and the final planetary eccentricities are too large – simple hit+stick model for accretion too crude

Giant planet formation



Pollack et al. (1996)

The diagram to the left shows a computer model for the formation of Jupiter via the core instability model.

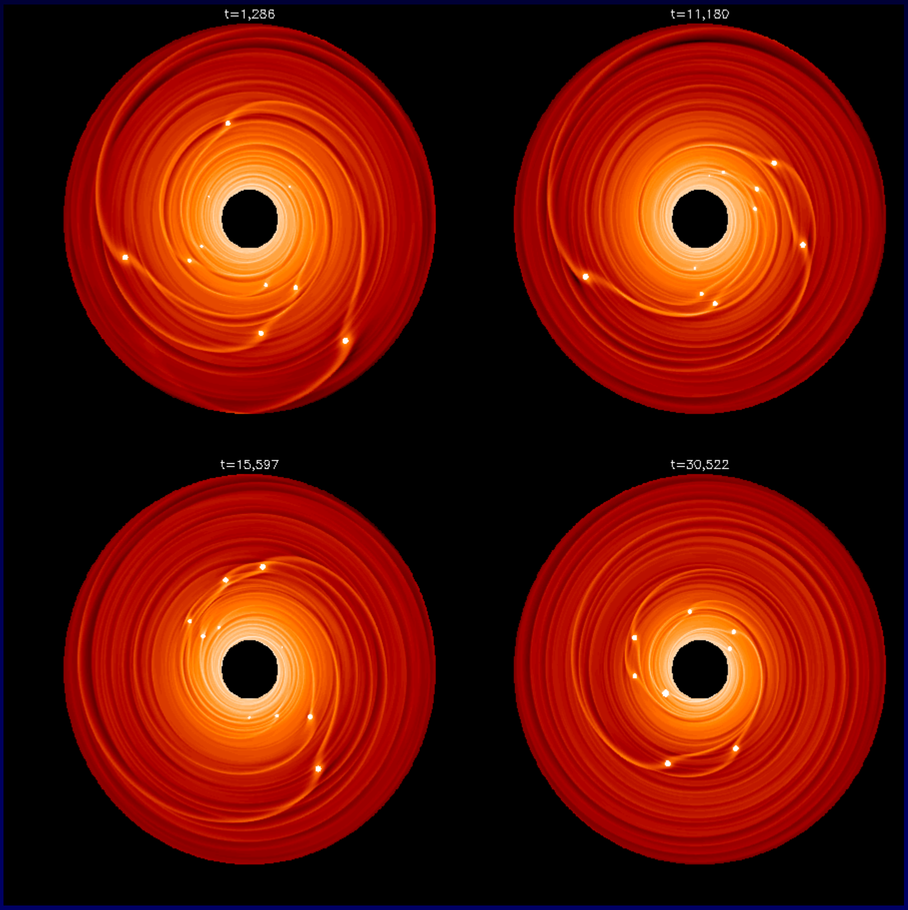
Solid line - formation of the solid rock+ice core.
Dotted line - the gas envelope
Dot-dashed line - Total planet mass

- Stage 1: Form rock + ice core from planetesimal accretion
- Stage 2: Low mass gaseous atmosphere accumulates onto core
- Stage 3: Core exceeds critical core mass value and gas envelope settles onto core on the Kelvin-Helmholtz time scale as thermal energy is radiated
- Stage 4: Accumulation of gas becomes more rapid once envelope and core mass are approximately equal
- Stage 5: Gas accretion enters runaway phase - final gas envelope is accumulated within a few thousand years

Stopping/slowing type I migration

- MHD Turbulence will generate a random walk - this may be able to overcome type I migration

- Corotation torques may slow/stop planet migration (Masset et al 2006; Paardekooper & Mellema 2007; Paardekooper & Papaloizou 2008)



Terrestrial Planet Formation