

483rd WE-Heraeus Seminar  
Extrasolar Planets:  
**Towards Comparative Planetology beyond the Solar System**

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**Tides  
on Earth,  
in the Solar System  
and Beyond.....**

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Abteilung Planetenforschung  
Köln







IDENTIFYING THE "PROGRESSIVE" HORSESHOE CRAB







**tide levels for Hamburg, Sankt Pauli:**

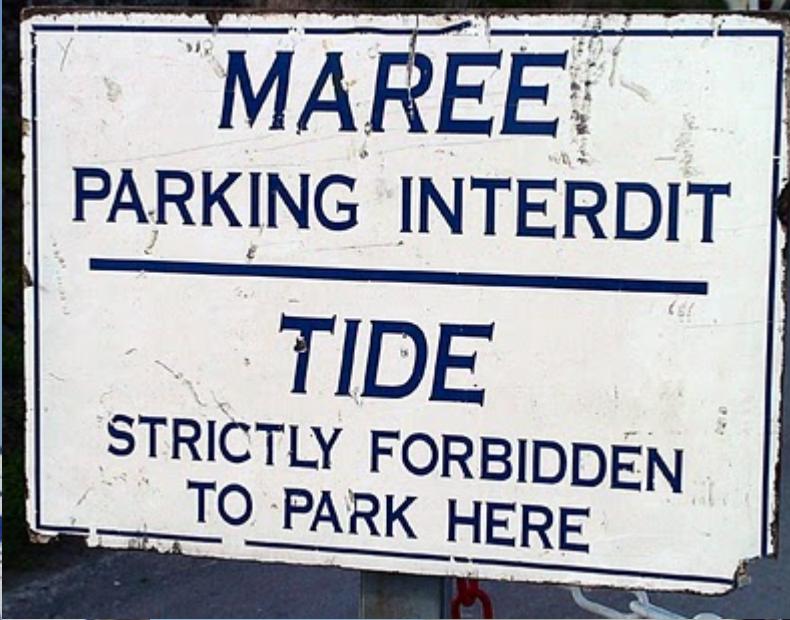
**Tuesday, 7th June 2011 (CEST)**

Low tide	04:03	0.3 m
High tide	09:07	3.9 m
Low tide	16:07	0.4 m
High tide	21:16	4.3 m

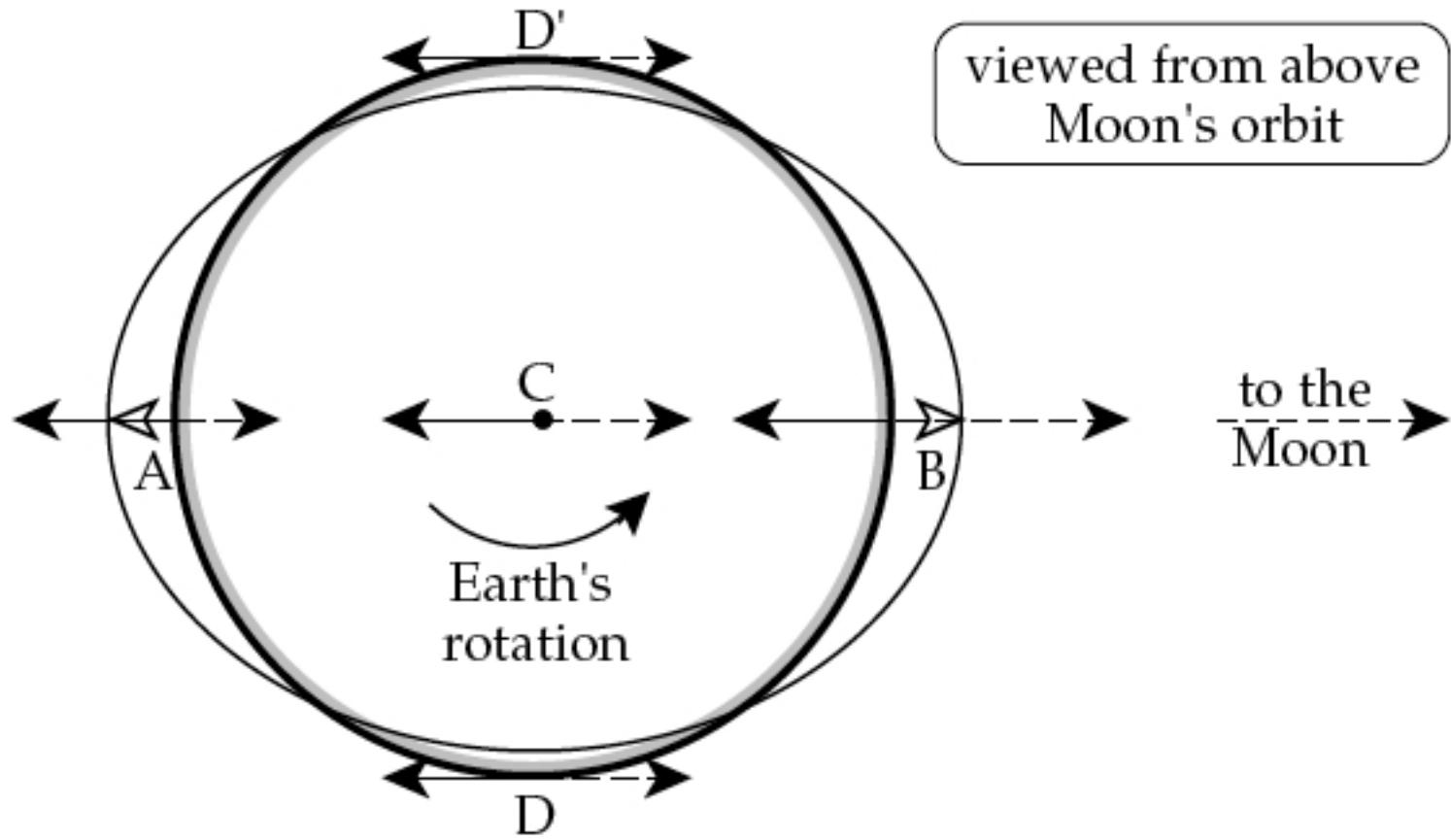
Bundesamt für Seeschifffahrt und Hydrographie



Mont St. Michel, Bretagne

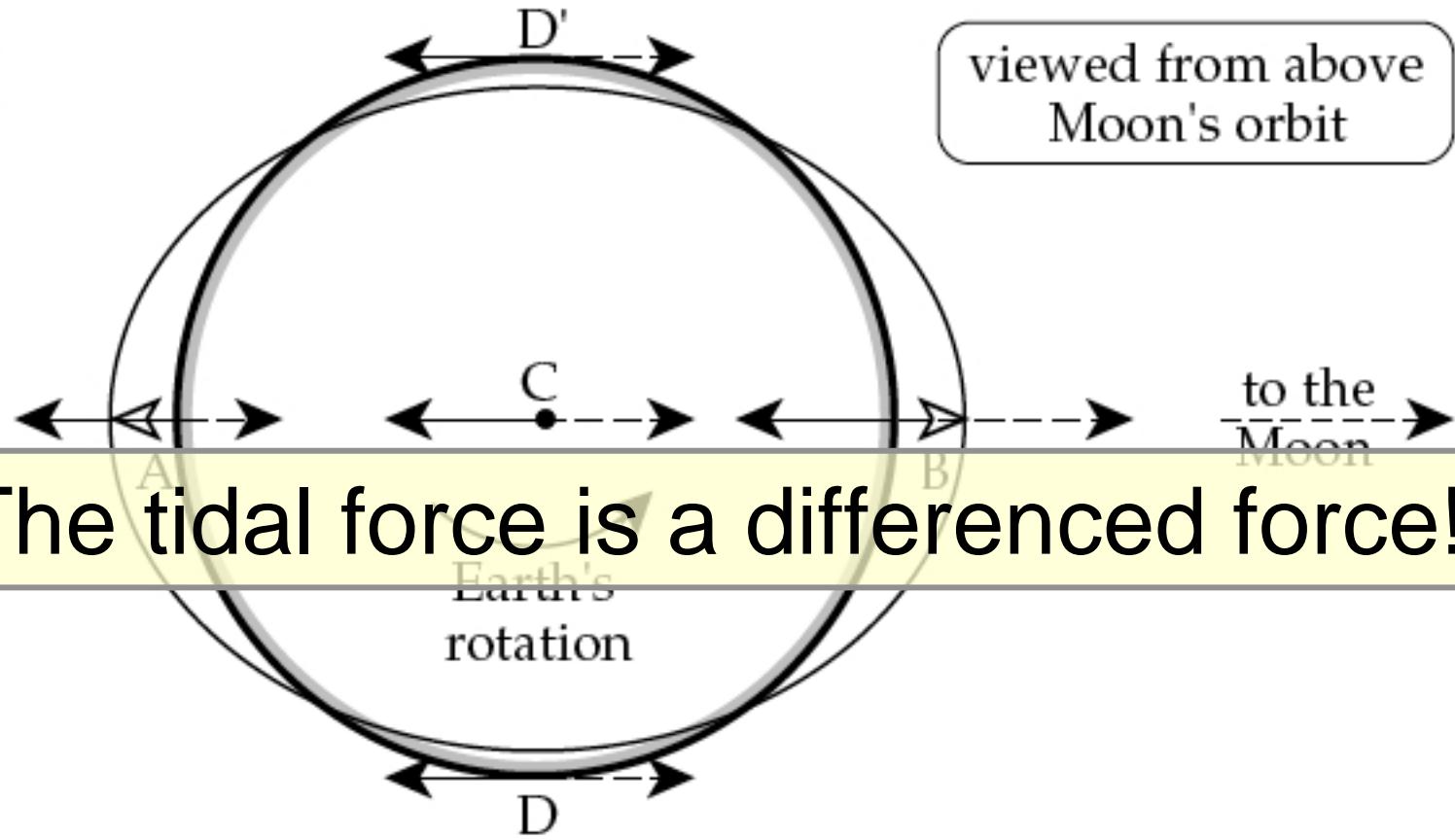


(a)



- $a_L$   $\leftarrow$  constant centrifugal acceleration
- $a_G$   $\rightarrow$  variable lunar gravitation
- $a_T$   $\rightarrow$  residual tidal acceleration

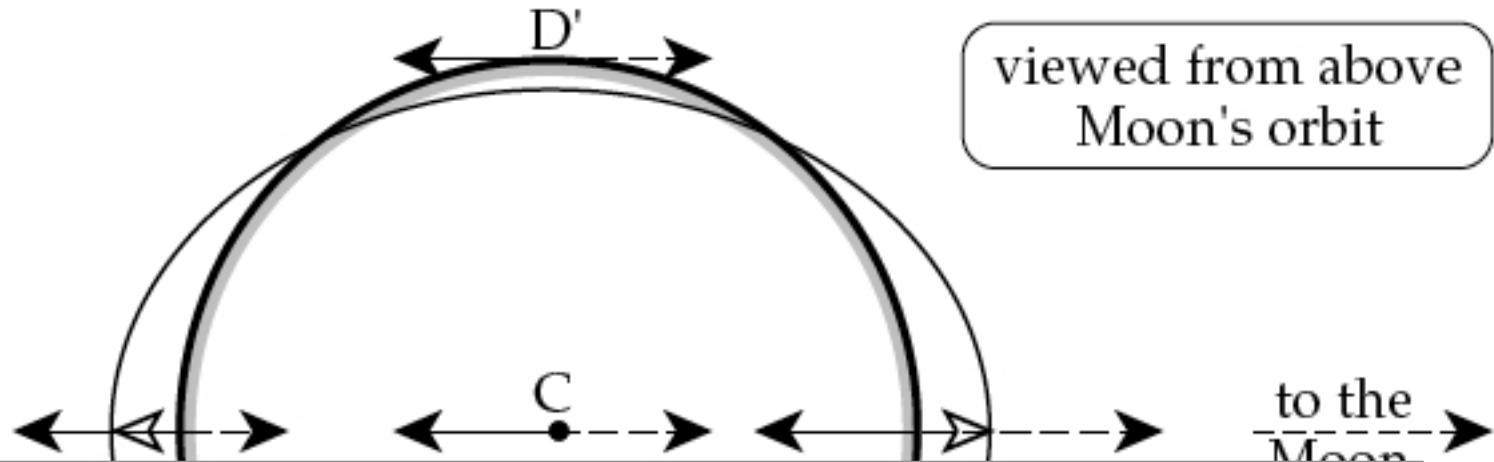
(a)



The tidal force is a differenced force!

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- $a_T$   $\rightarrow$  residual tidal acceleration

(a)



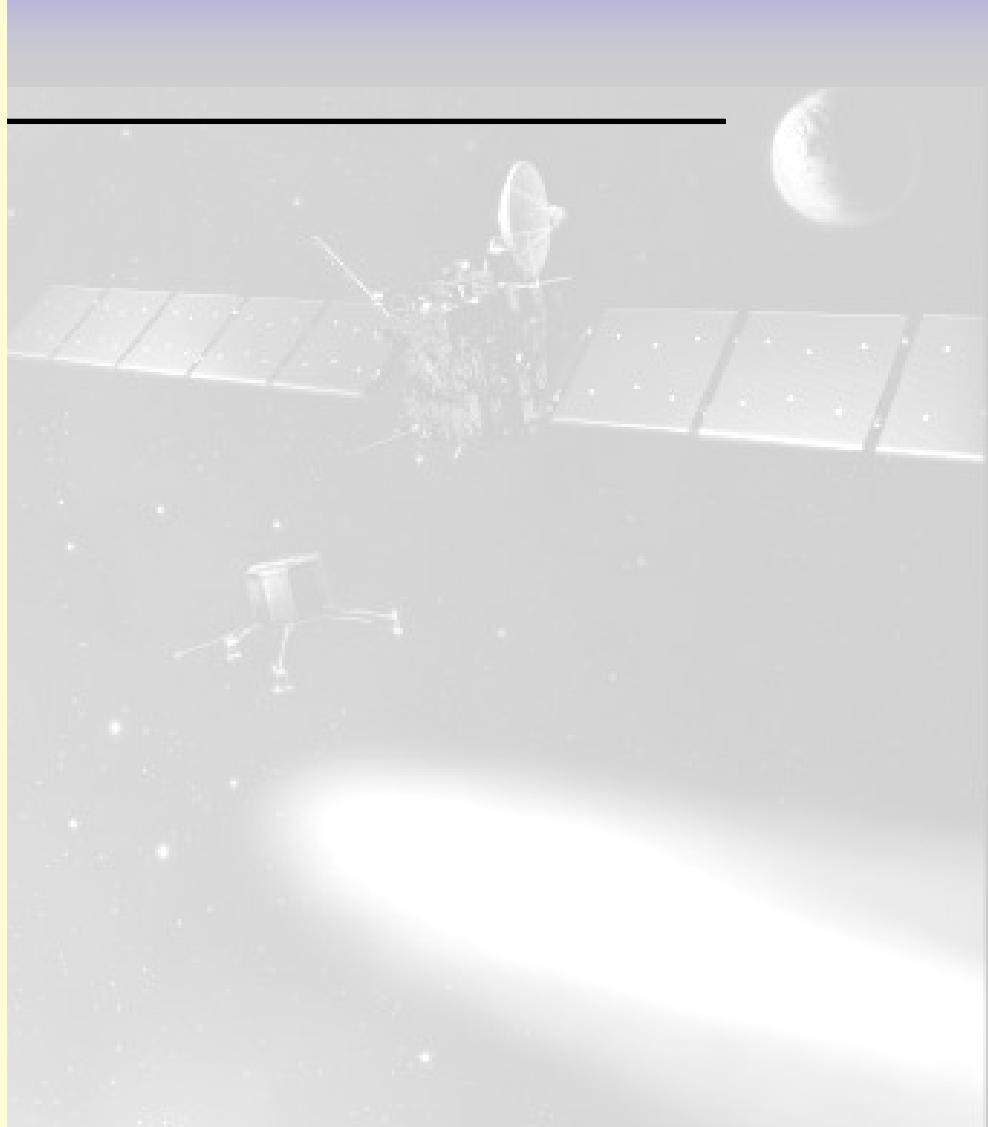
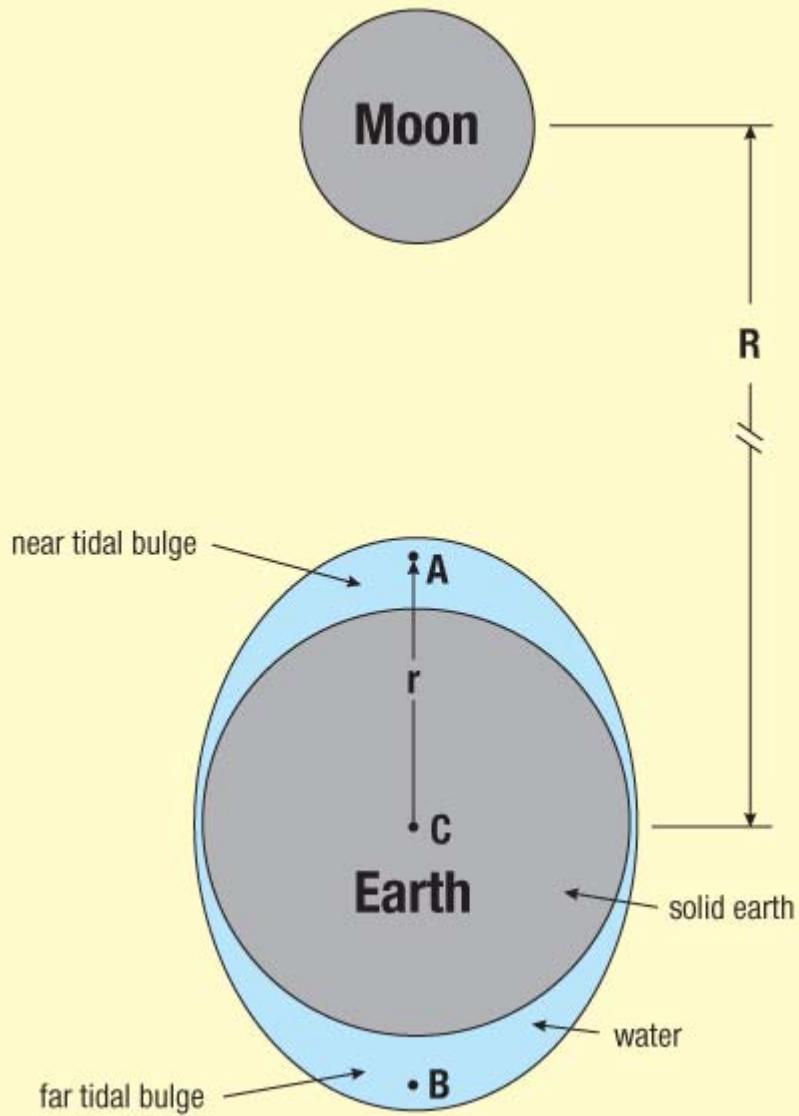
viewed from above  
Moon's orbit

The formation of the tidal bulge is  
usually known as the (high) tide

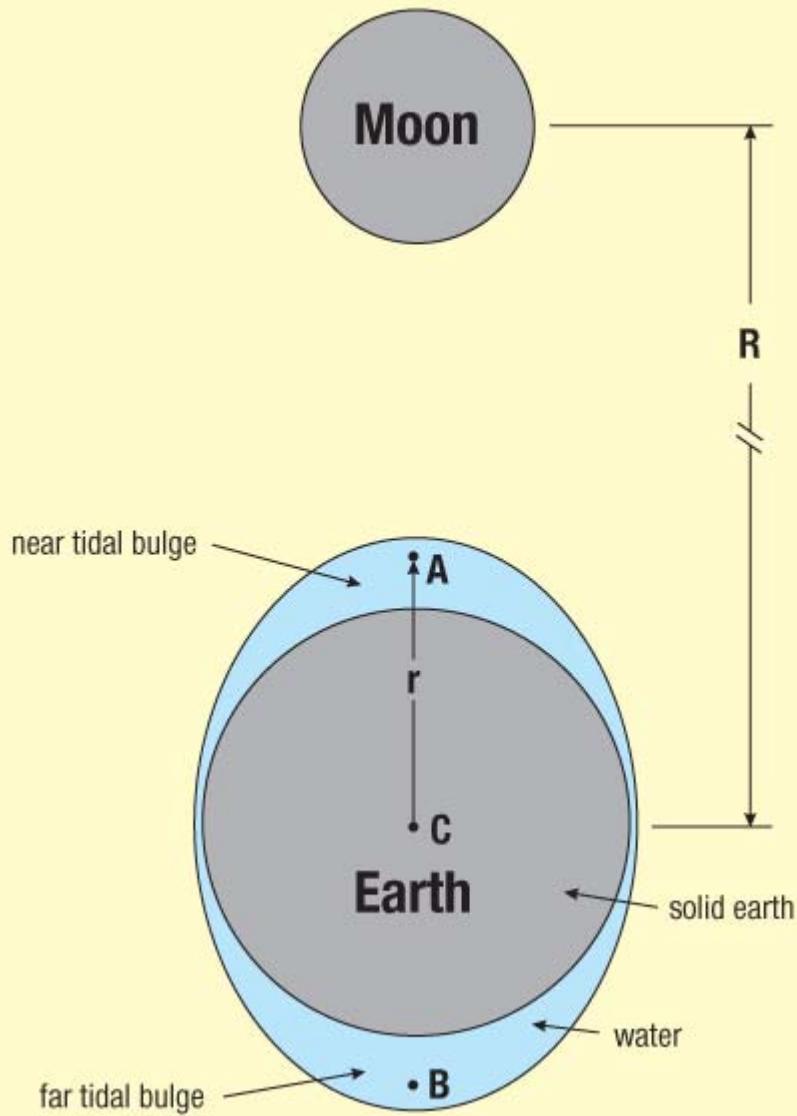


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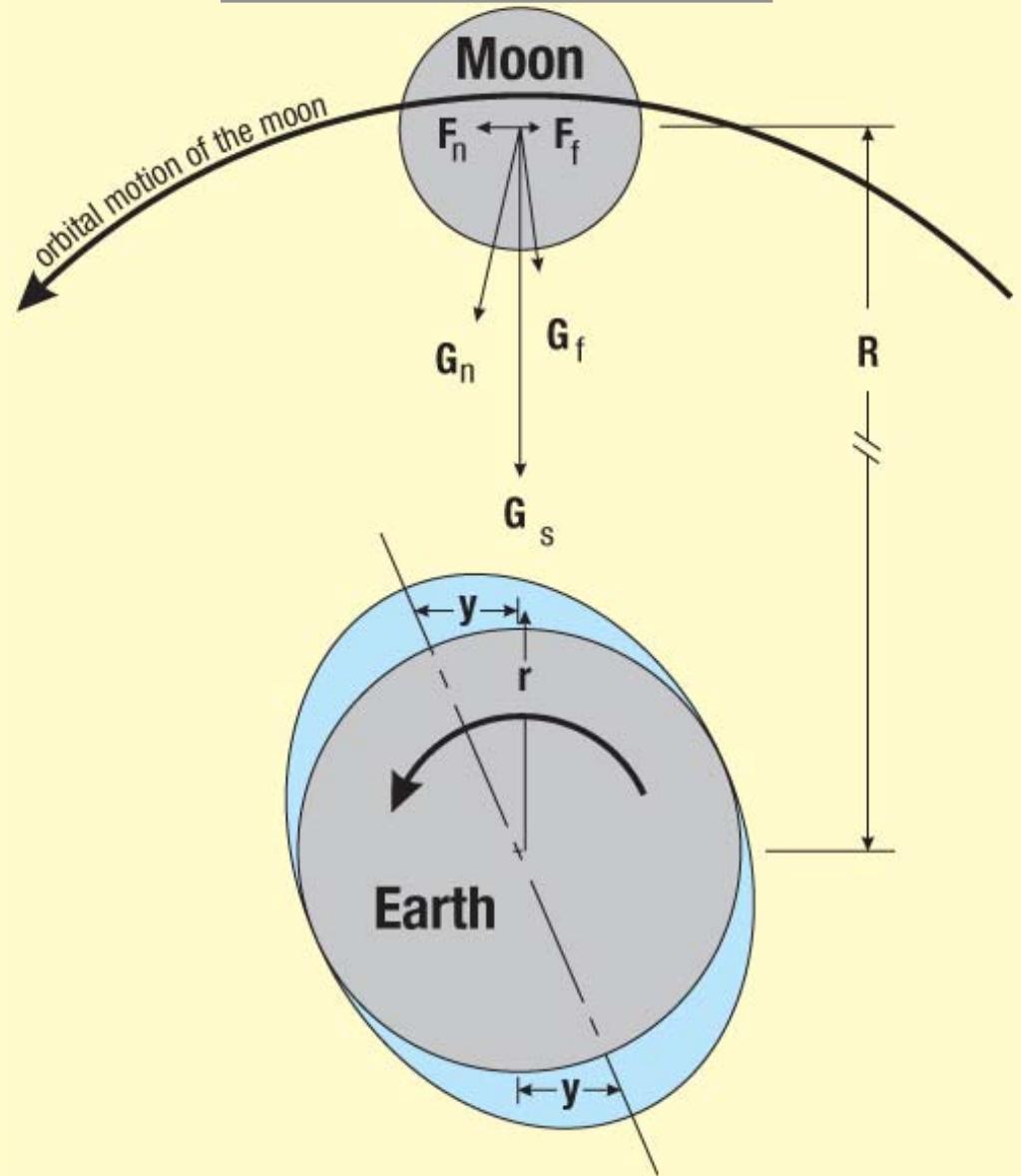
## Revolution without rotation



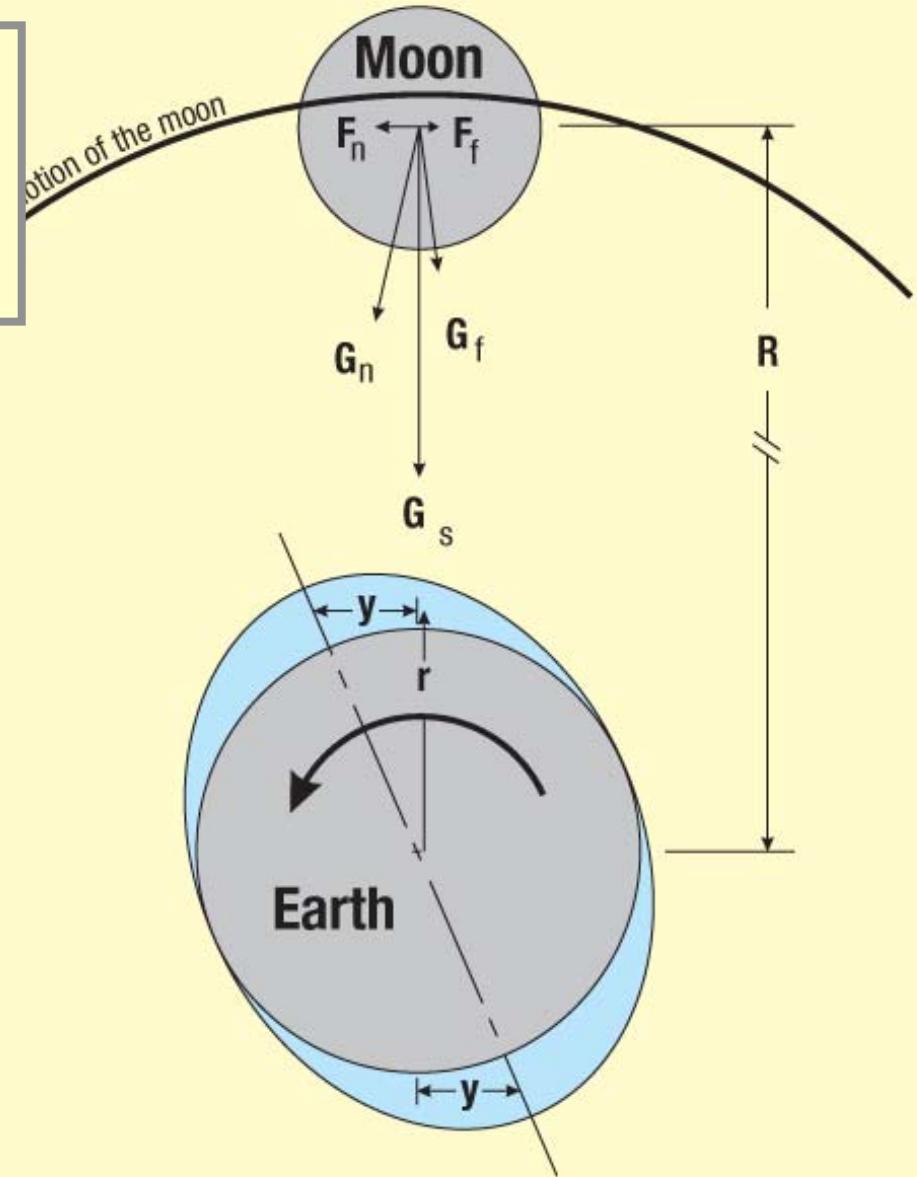
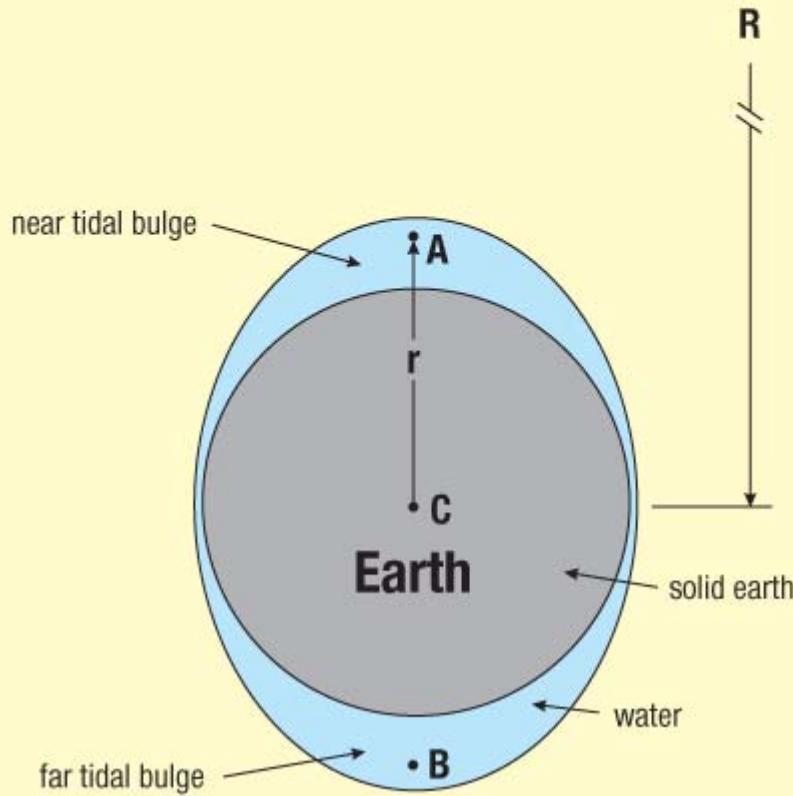
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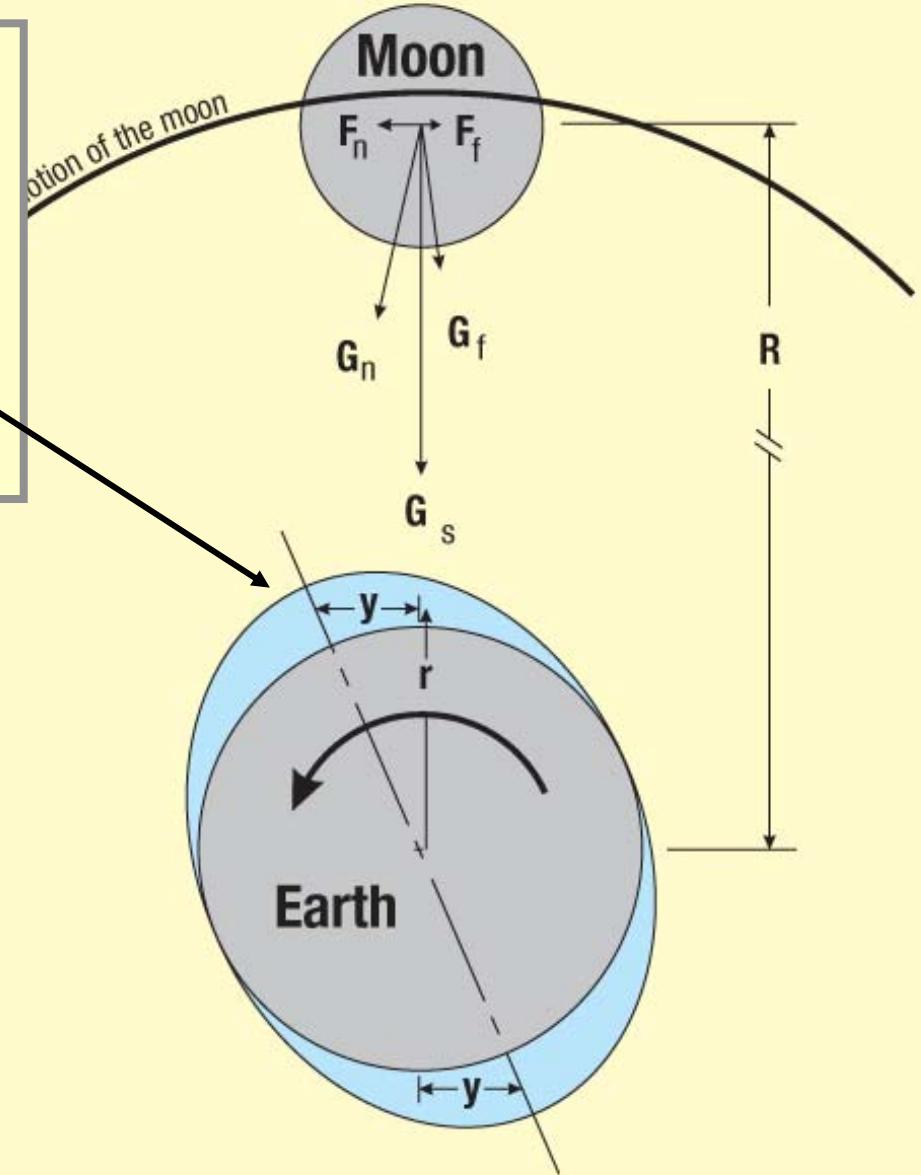
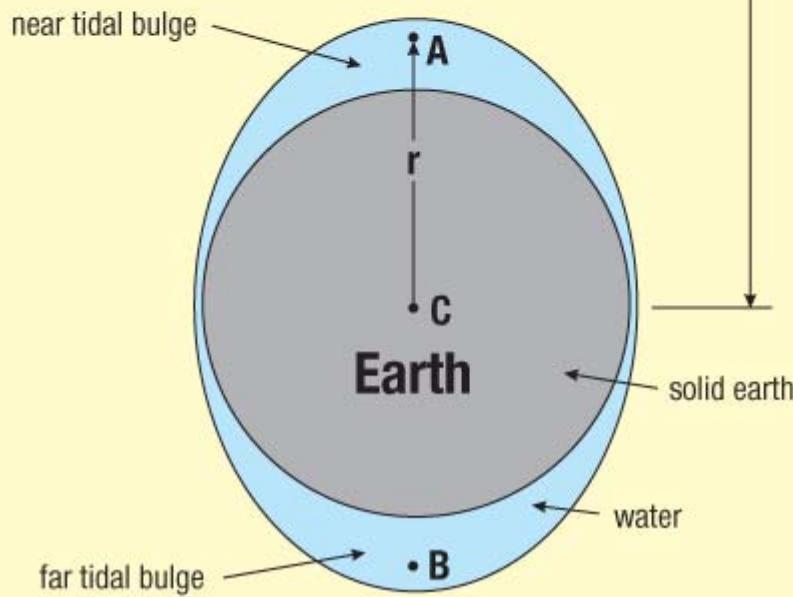
## But the Earth rotates!



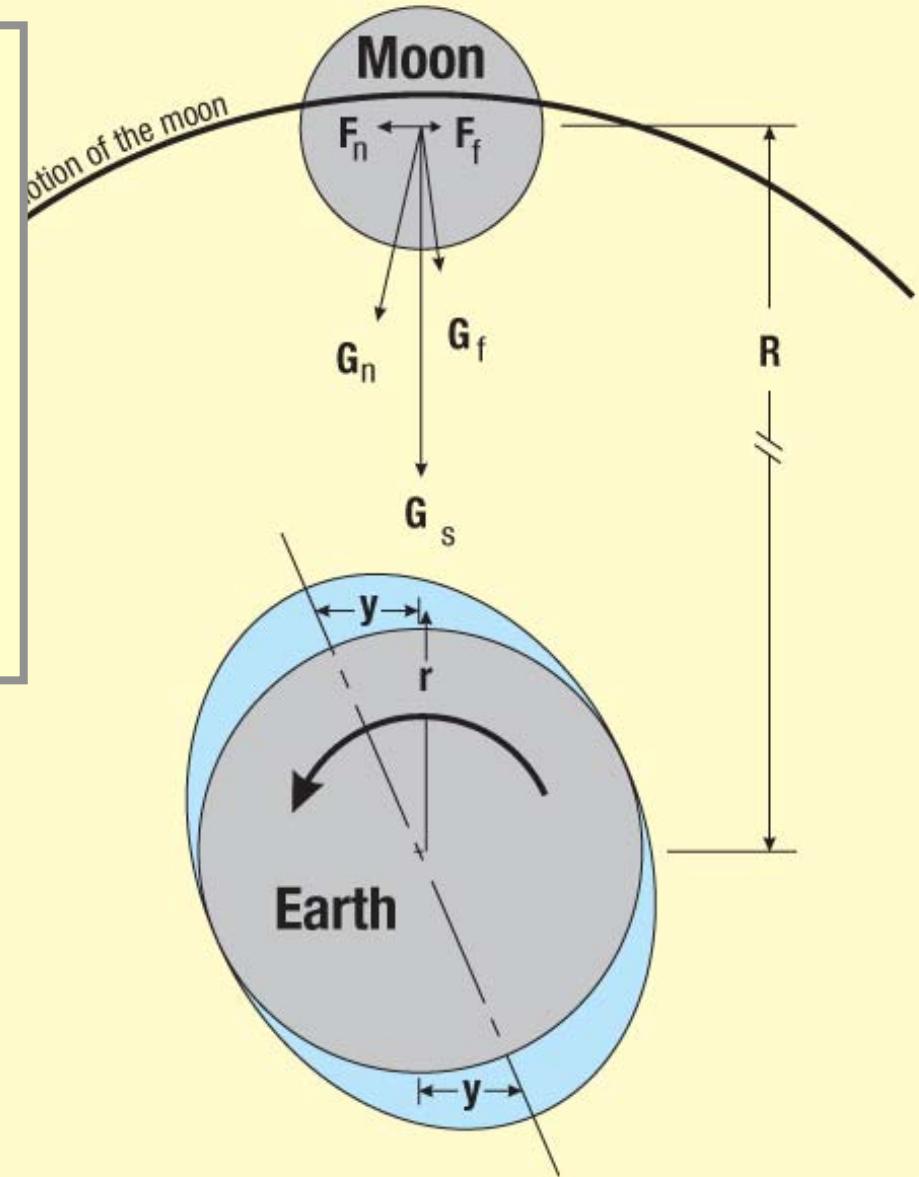
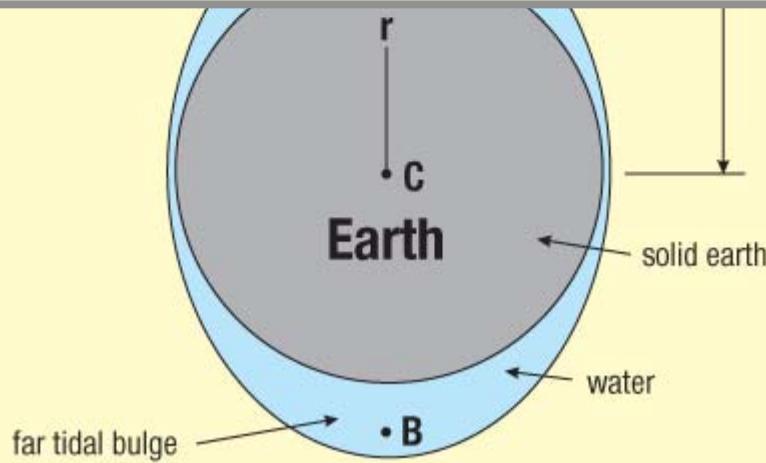
- The Earth rotates faster than the Moon revolves



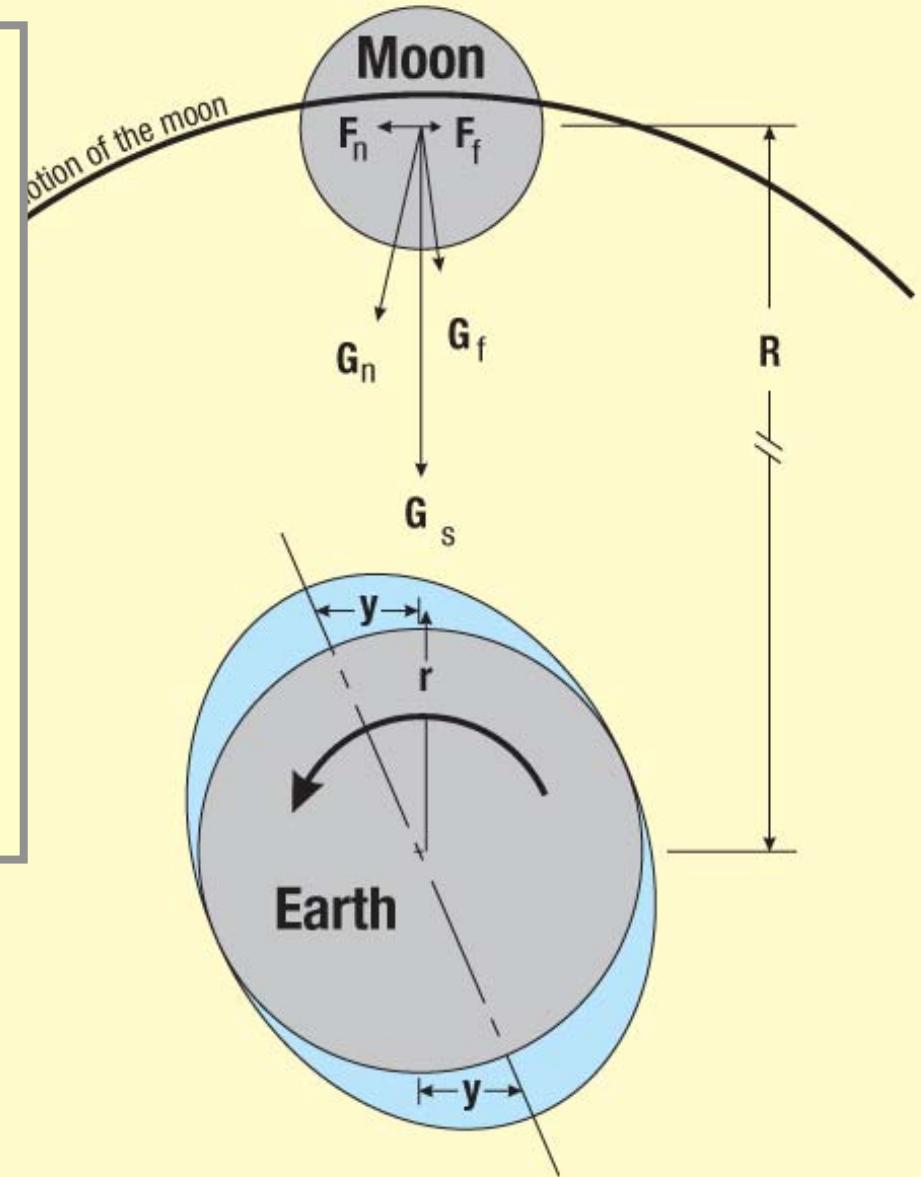
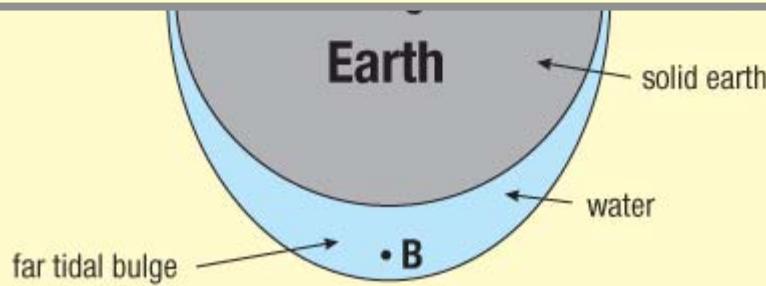
- The Earth rotates faster than the Moon revolves
- The tidal bulge is advancing the direct LOS



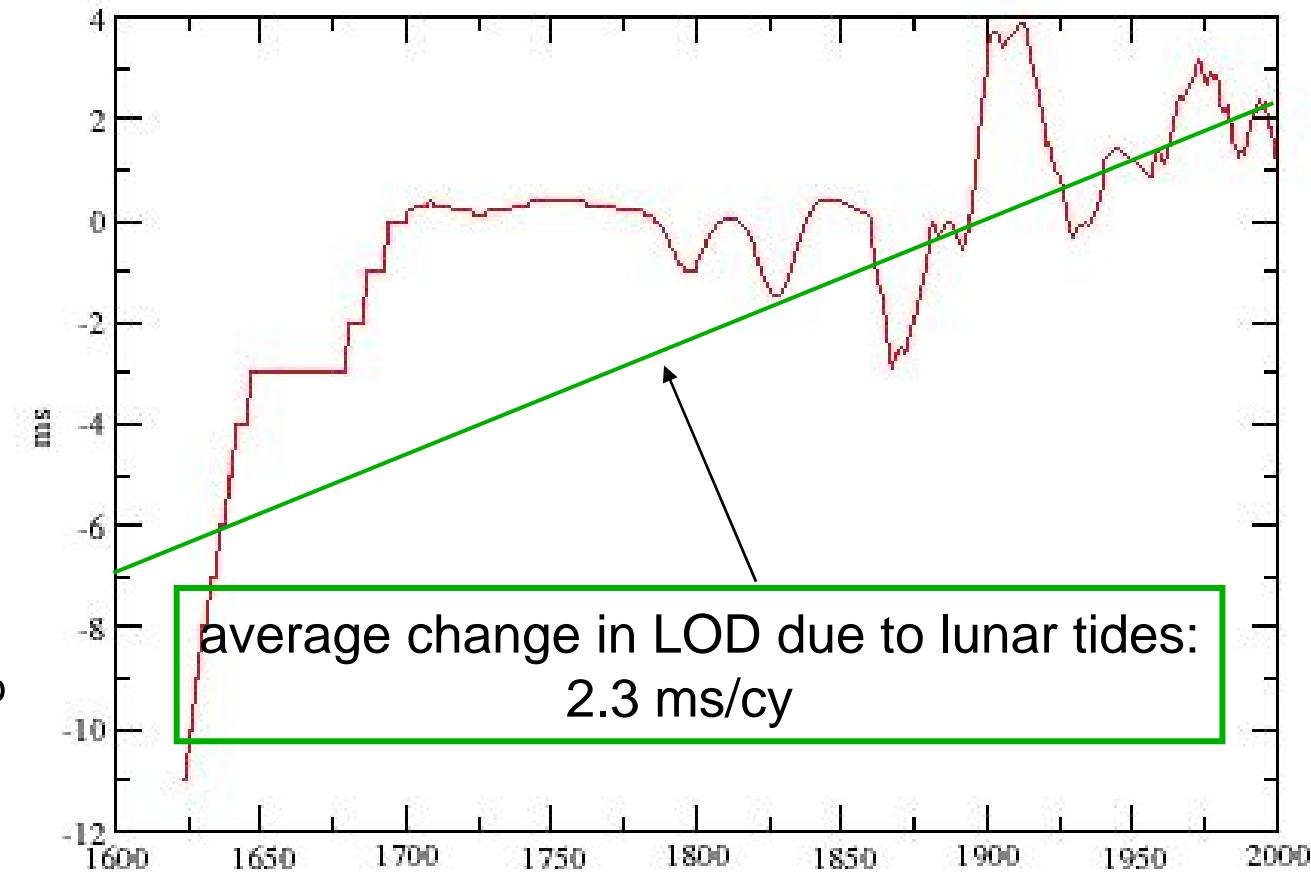
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- The tidal bulge is advancing the direct LOS
- Torques try to pull the tidal bulge back into LOS



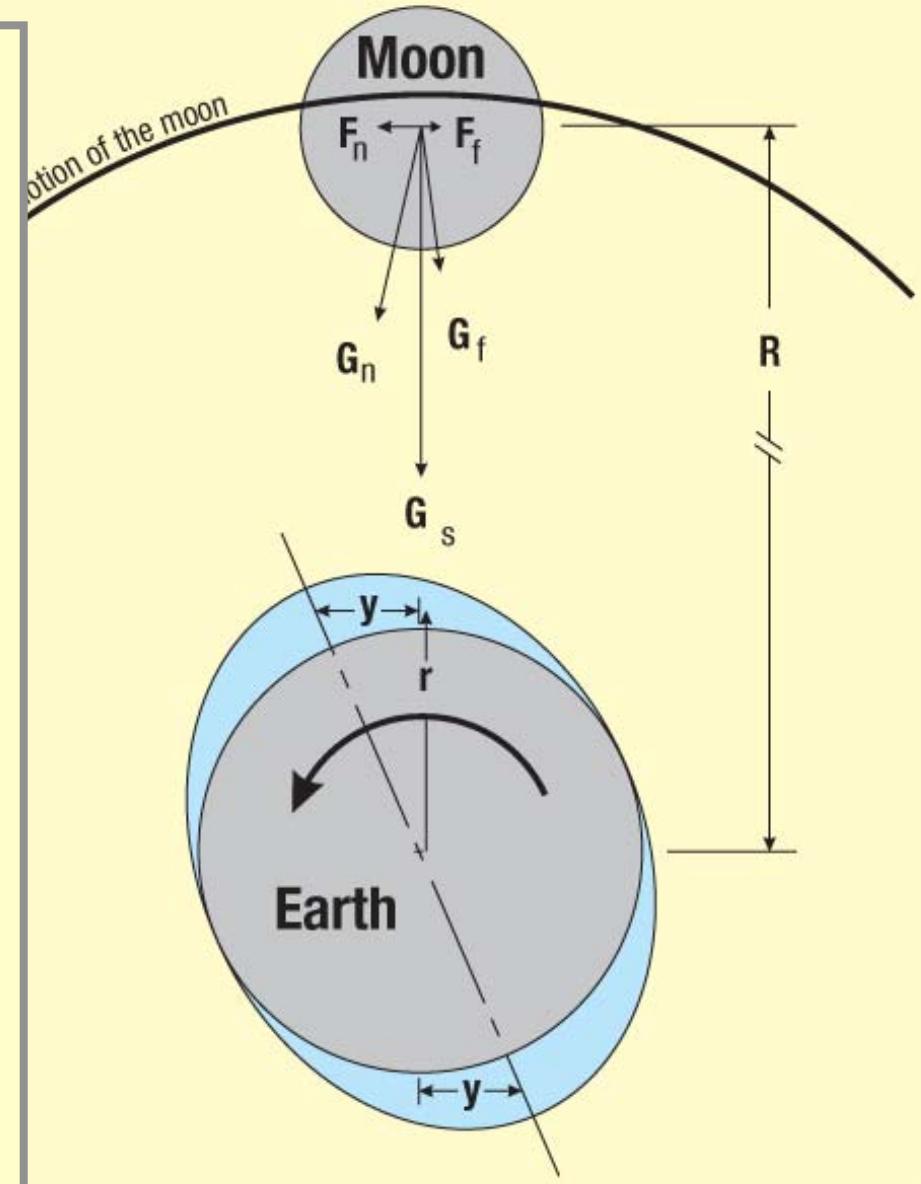
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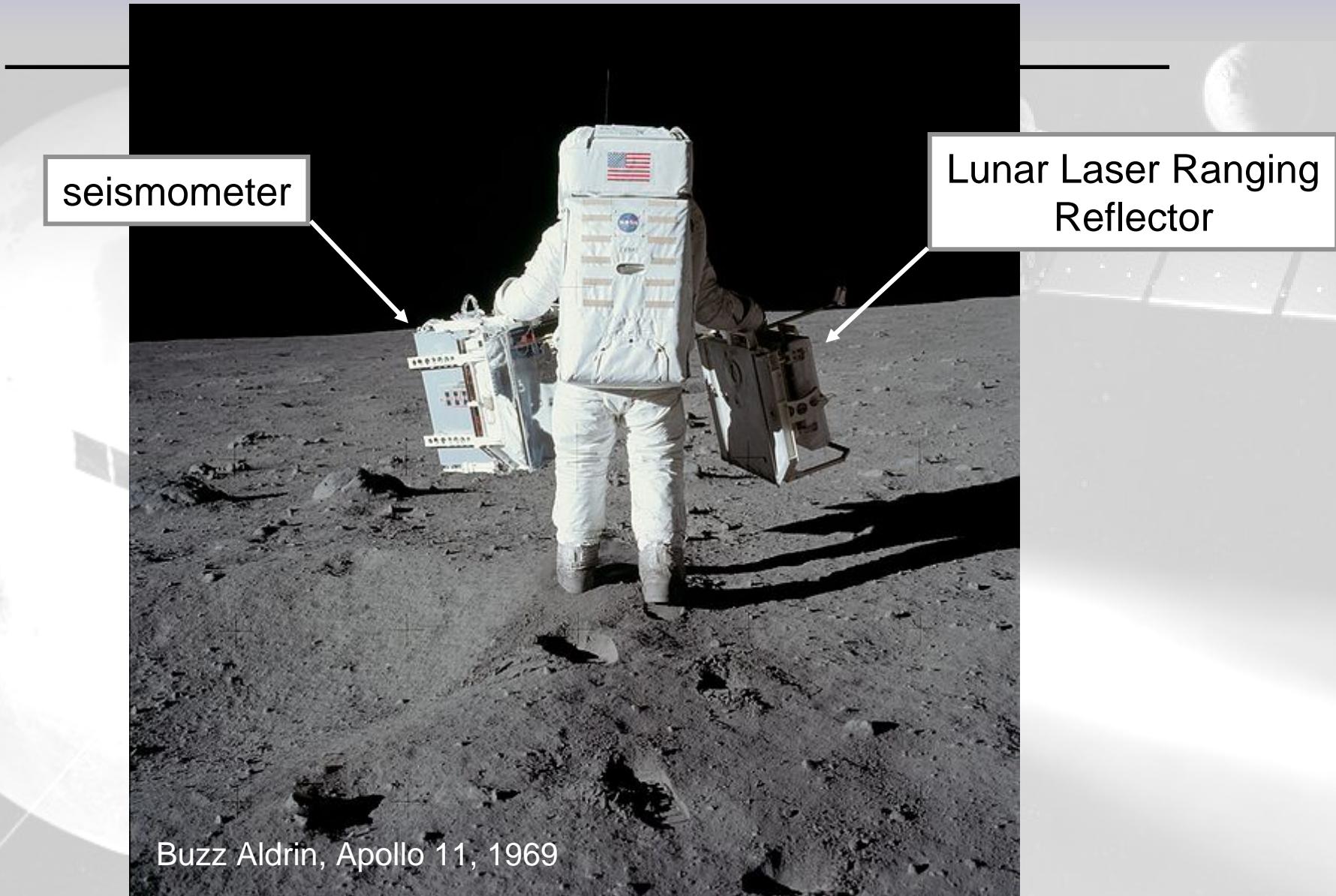
change in LOD relative to 86400 s



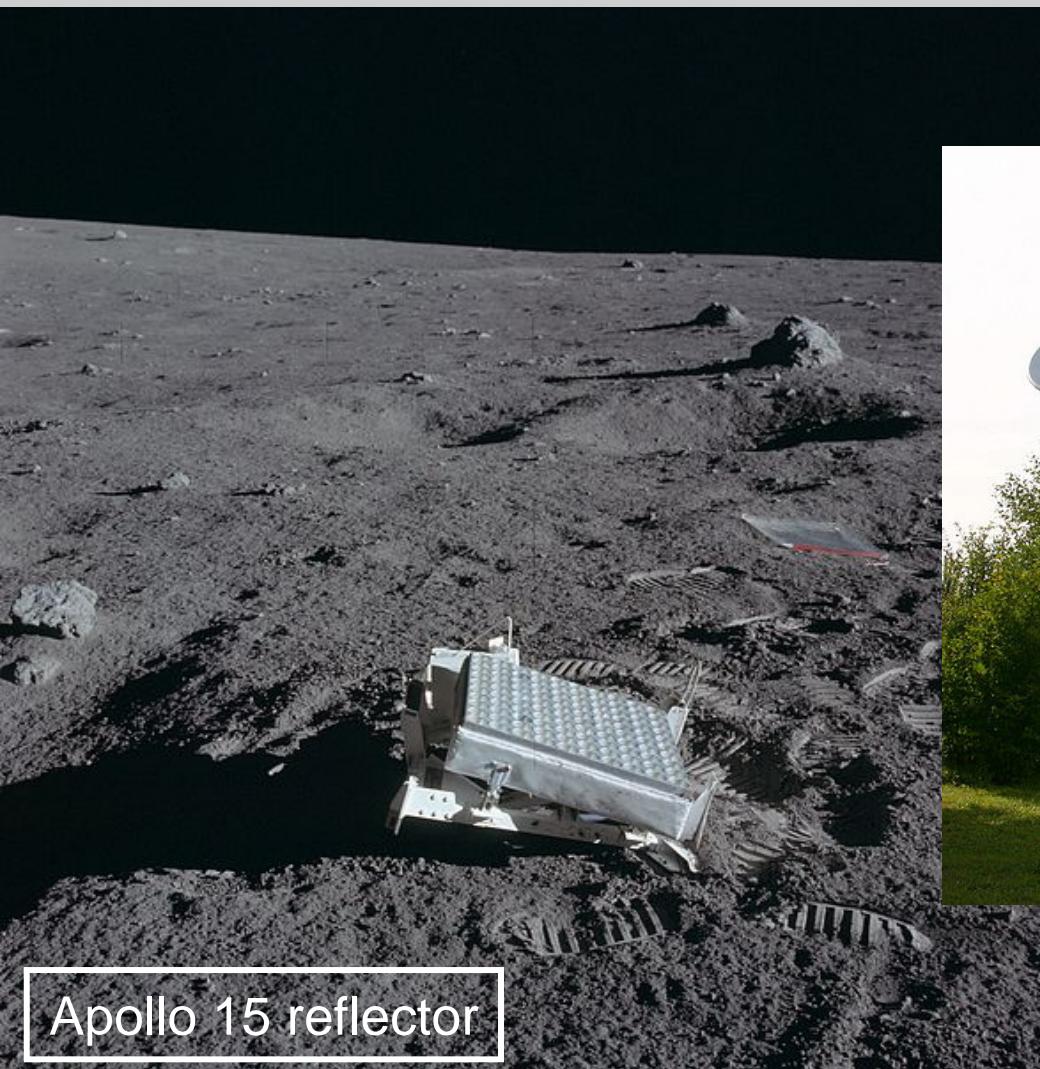
- The Earth rotates faster than the Moon revolves
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- Consequence is a slow down of Earth rotation (tidal friction)
- another consequence from conservation of angular momentum: the Moon recedes from the Earth



# The first extraterrestrial applied space geophysicist



# Lunar Laser Ranging



Apollo 15 reflector

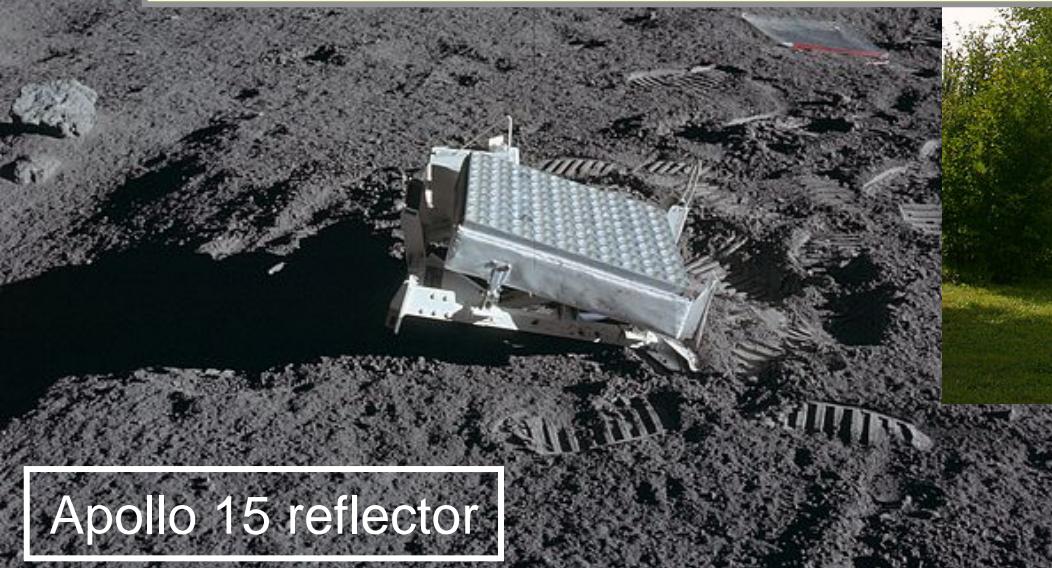


Wettzell Lunar Laser Station

# Lunar Laser Ranging



The Moon recedes from Earth by 4m/cy

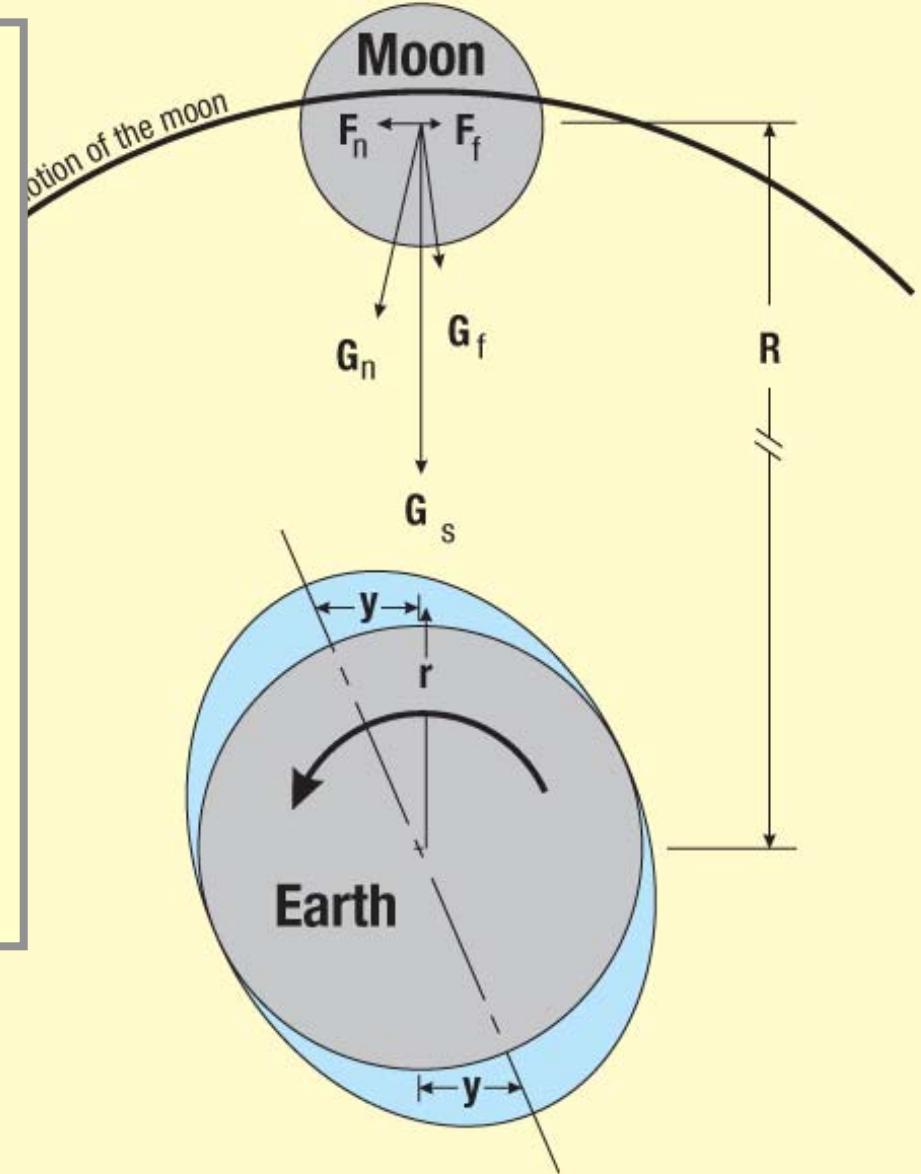
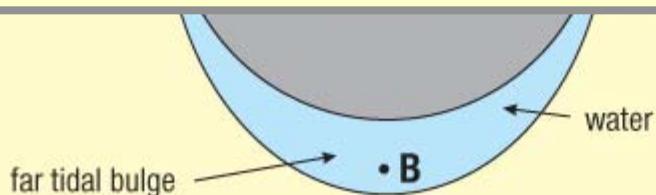


Apollo 15 reflector



Wettzell Lunar Laser Station

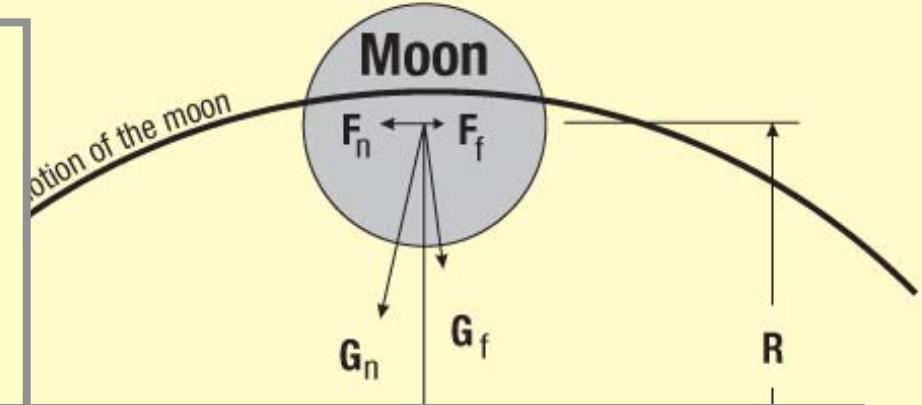
- The Earth rotates faster than the Moon revolves
- The **tidal bulge is advancing the direct LOS by 3°**
- Earth rotates below the tidal bulge
- A location on the surface experiences periodically high and low tides



Ocean tides are the directly visible manifestation of tidal forces

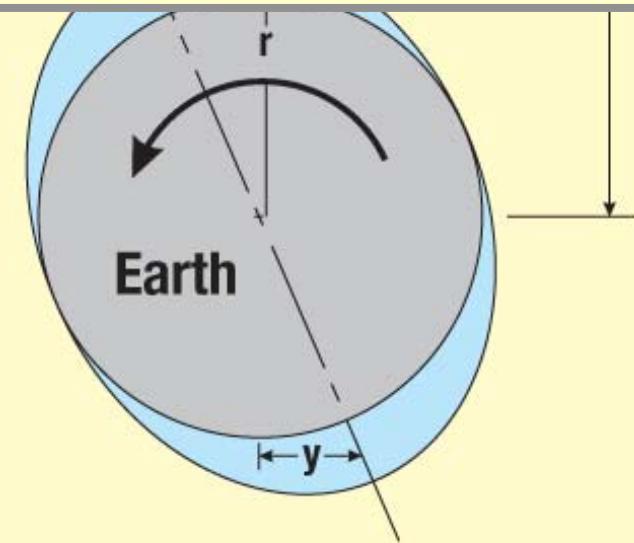


- The Earth rotates faster than the Moon revolves
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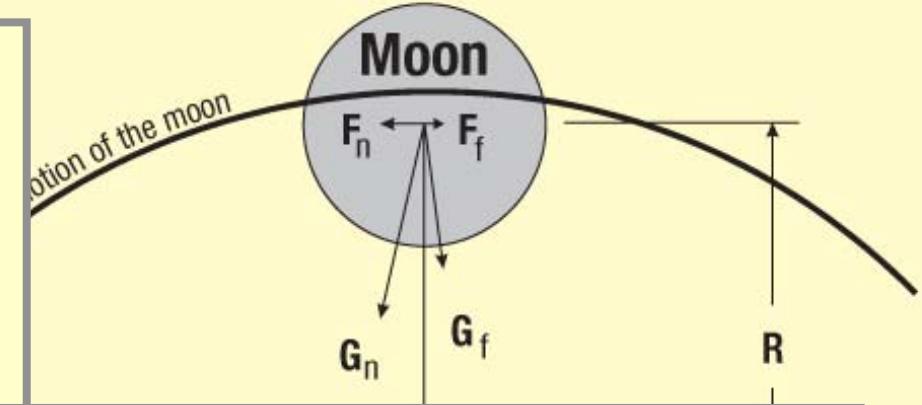


- Is there a case in the solar system where a moon revolves faster than the planet rotates?

- consequence of tidal forces due to the rotation of Earth (tidal friction)
- another consequence from conservation of angular momentum: the Moon recedes from the Earth



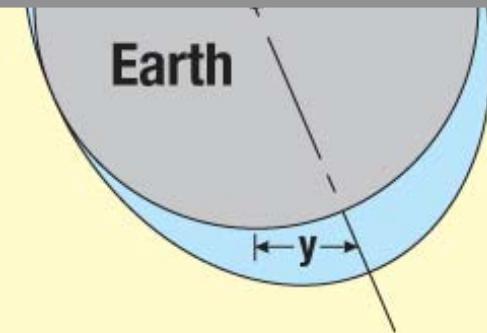
- The Earth rotates faster than the Moon revolves
- The tidal bulge is advancing the direct LOS



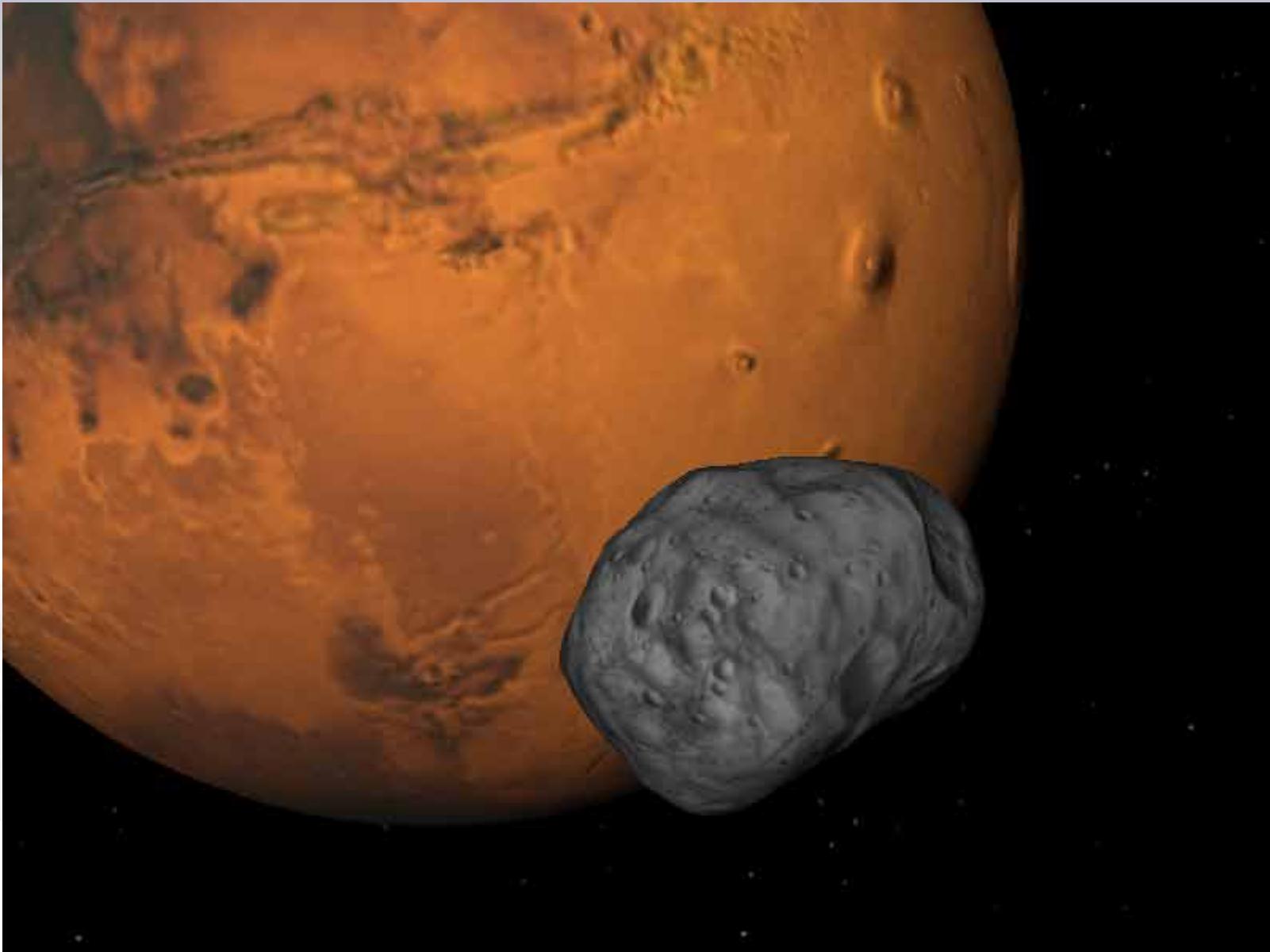
- Is there a case in the solar system where a moon revolves faster than the planet rotates?

YES, there is!

- another consequence from conservation of angular momentum: the Moon recedes from the Earth



# The Mars/Phobos case



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Phobos

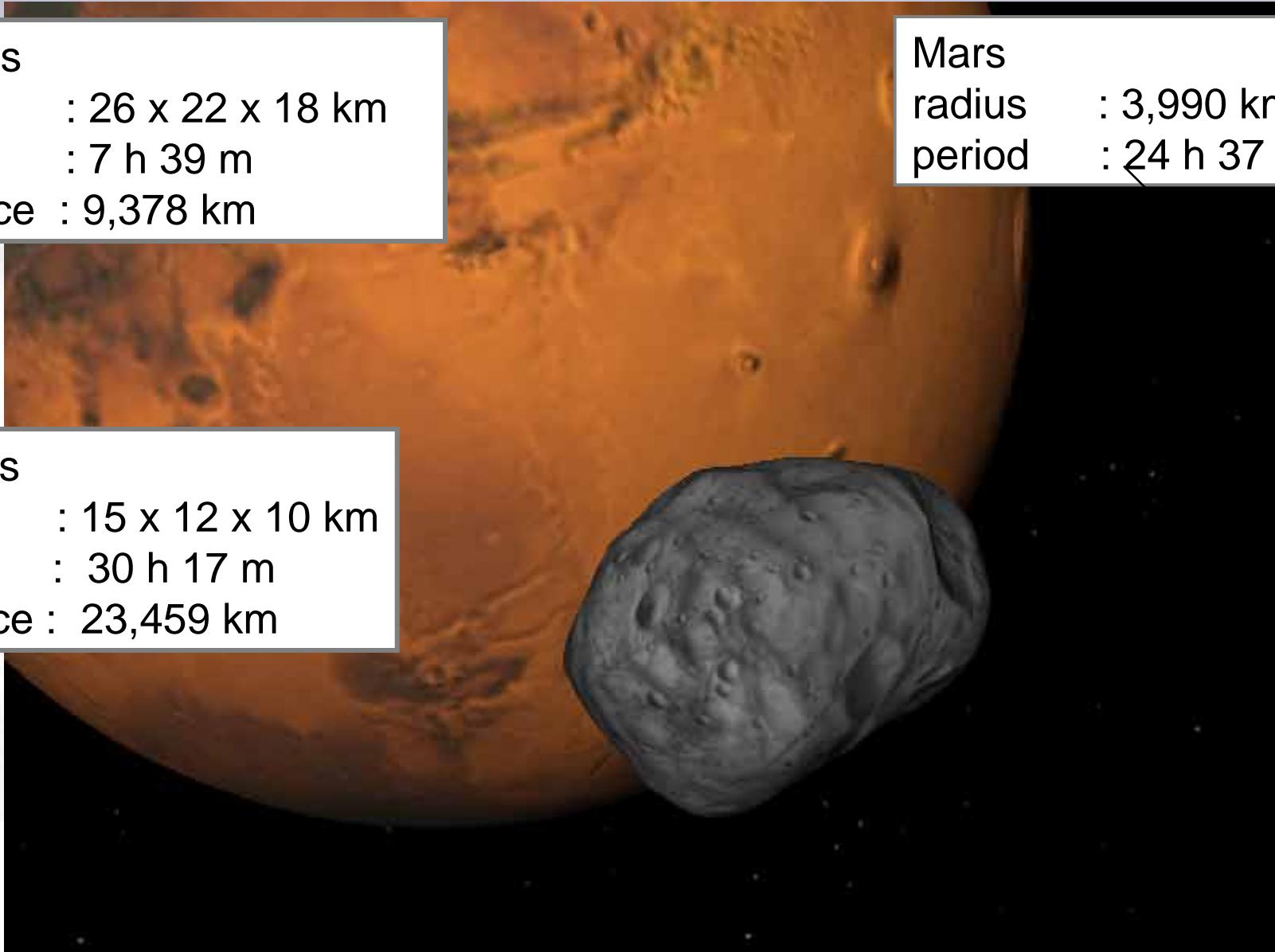
size : 26 x 22 x 18 km  
period : 7 h 39 m  
distance : 9,378 km

Mars

radius : 3,990 km  
period : 24 h 37 m

Deimos

size : 15 x 12 x 10 km  
period : 30 h 17 m  
distance : 23,459 km



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size : 15 x 12 x 10 km

period : 30 h 17 m

distance : 23,459 km

synchronous orbit:

planetary rotation = moon revolution

Mars

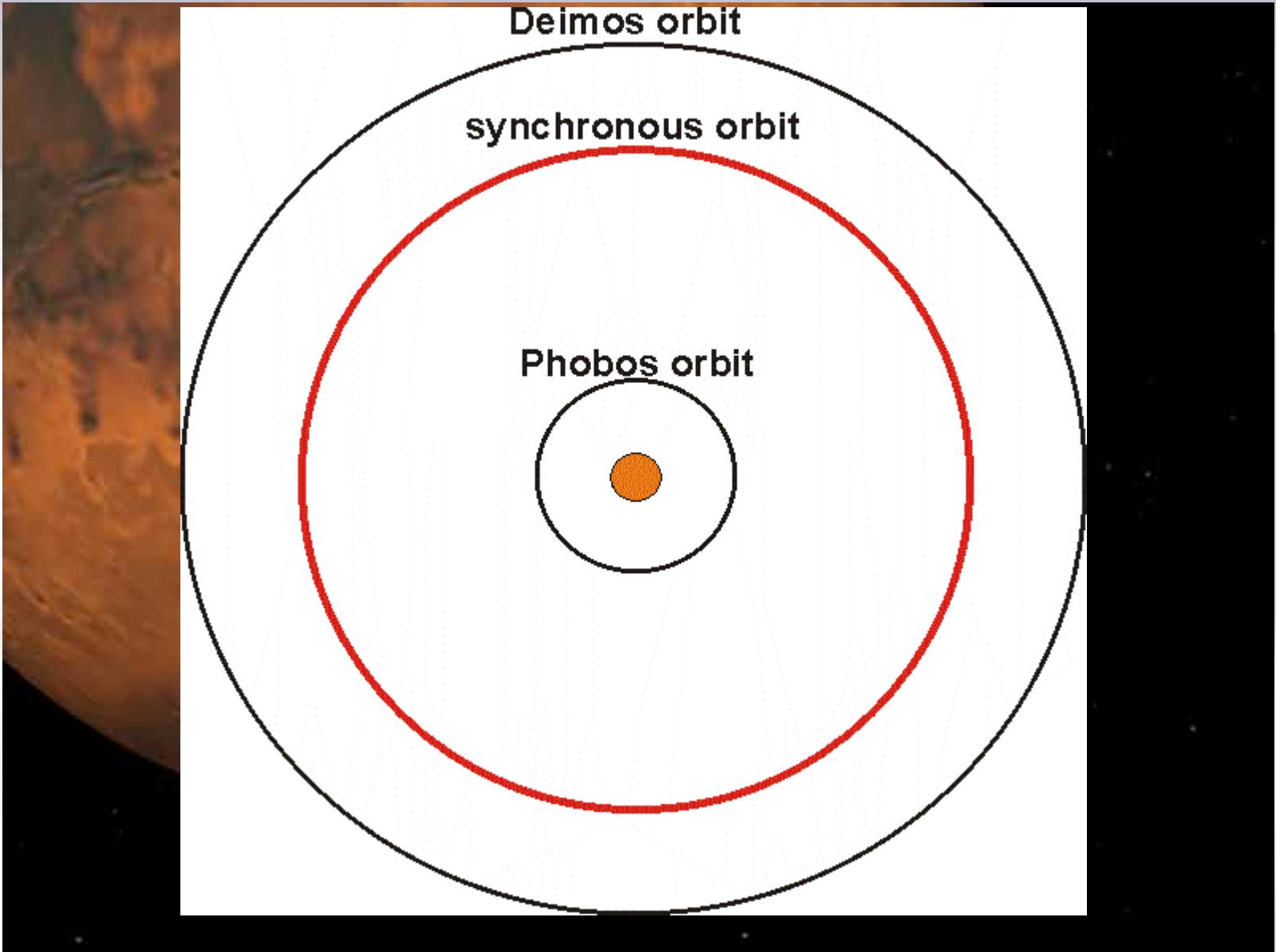
24 h 37 m

21,000 km distance

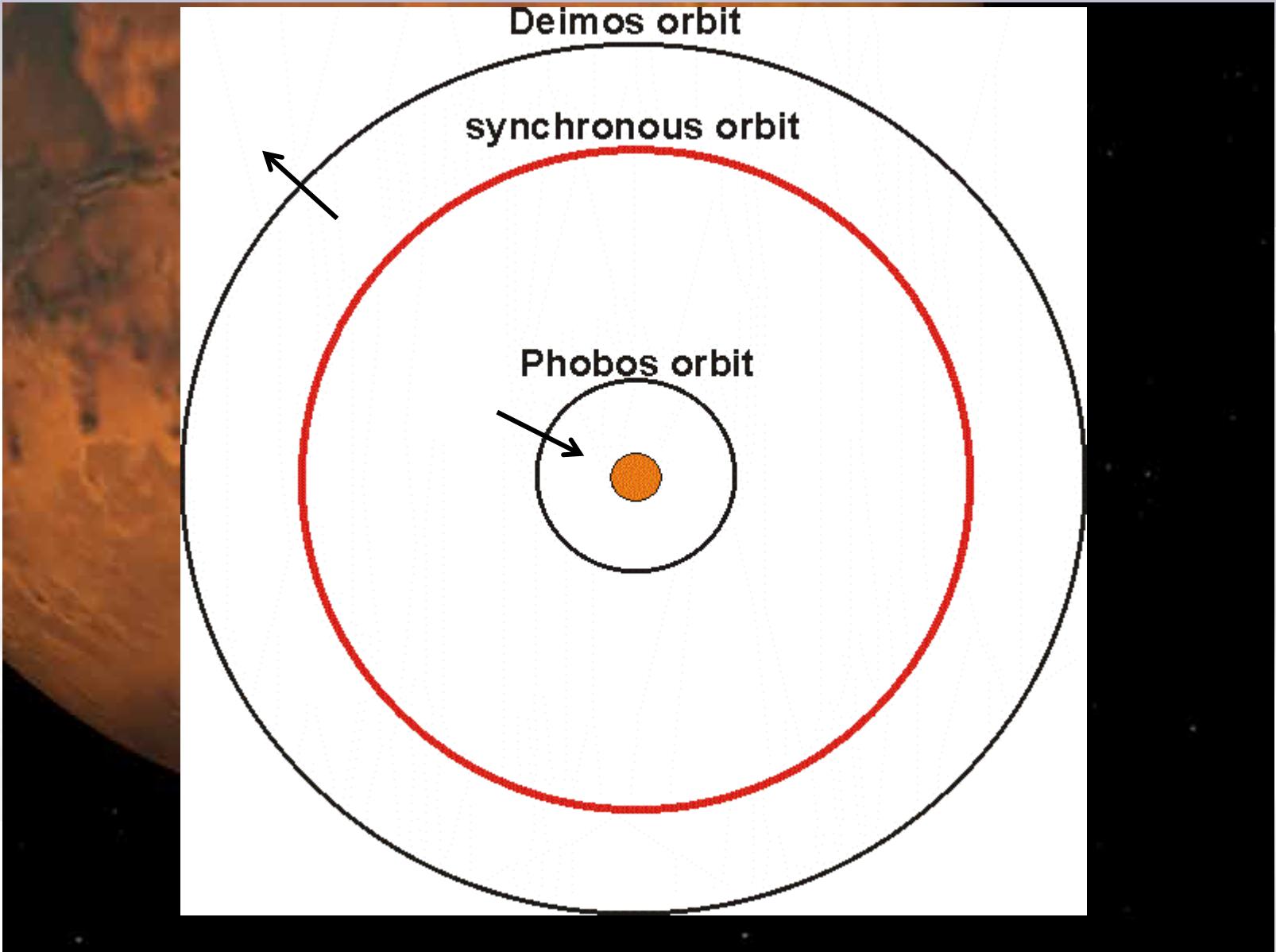
Phobos: inside

Deimos: outside

# The Mars/Phobos case

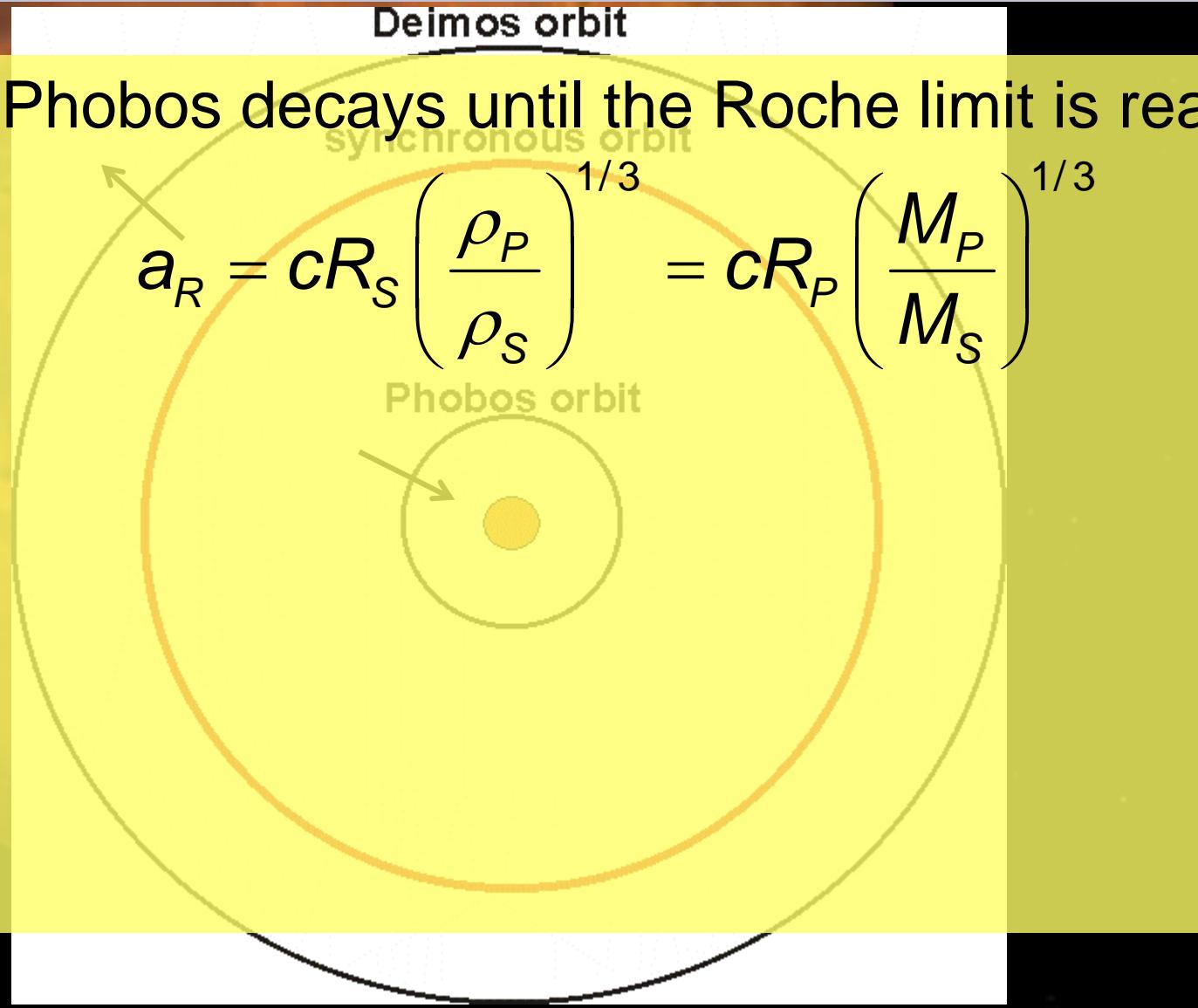


# The Mars/Phobos case



## The Mars/Phobos case

The orbit of Phobos decays until the Roche limit is reached.



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Roche limit:

$$a_R = cR_S \left( \frac{\rho_P}{\rho_S} \right)^{1/3} = cR_P \left( \frac{M_P}{M_S} \right)^{1/3}$$

Proportionality factor  $c$  depends on the nature of the secondary:  
fluid  $c = 2.44 \Rightarrow a_R = 10,500 \text{ km}$   
solid  $c = 1.26 \Rightarrow a_R = 5,500 \text{ km}$   
rubble pile  $c = 1.54 \Rightarrow a_R = 6,700 \text{ km}$

## The Mars/Phobos case

Deimos orbit

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Phobos orbit

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$$\text{rubble pile} \quad c = 1.54 \Rightarrow a_R = 6,700 \text{ km}$$

Phobos is highly porous => „rubble pile“ (Andert et al., 2010)

When will Phobos reach the Roche limit?

$$\tau_{Roche} = \frac{\frac{2}{13} (a^{13/2} - a_R^{13/2})}{3 \frac{k_{2,Mars}}{Q_{Mars}} \frac{M_{Ph}}{M_{Mars}} R_{Mars}^5 \sqrt{GM_{Mars}}}$$

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## Other tidal phenomena in the solar system

---

- forcing Mercury in a 3/2 resonance
- forcing all moons in the solar system into synchronous rotation
- deforming the Jupiter moon Io periodically => tidal heating => volcanic activity
- and the non-plus-ultra:

## Other tidal phenomena in the solar system

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- forcing Mercury in a 3/2 resonance
- forcing all moons in the solar system into synchronous rotation
- deforming the Jupiter moon Io periodically => tidal heating => volcanic activity
- and the non-plus-ultra: synchronizing the Pluto system

# Tides in extrasolar systems

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- studying tides in extrasolar planetary systems requires the knowledge of mass and size of the planet and the star
  - feasible with transiting planets combined with radial velocity observations and spectroscopy
- 
- orbital decay => survivability
  - stellar spin-up
  - spin-orbit coupling => synchronization of rotation and revolution
  - circularization =>  $e \rightarrow 0$

# The tidal potential or the Doodson constant

two tidal potentials:

- a) The primary acting on the secondary
- b) The secondary acting on the primary

$$D_{ps} = \frac{GM_p}{a^3} R_s^2$$

$$D_{sp} = \frac{GM_s}{a^3} R_p^2$$

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$$D_{ps} = \frac{GM_p}{a^3} R_s^2$$

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Earth/Moon system:

Earth on Moon

$$D_{ps} = 20.9 \text{ m}^2/\text{s}^2$$

Moon on Earth

$$D_{sp} = 3.5 \text{ m}^2/\text{s}^2$$

Sun on Earth

$$D_{sp} = 1.6 \text{ m}^2/\text{s}^2$$

When will the planet reach the stellar Roche limit?

$$\tau_{Roche} = \frac{\frac{2}{13} (a^{13/2} - a_R^{13/2})}{3 \frac{k_{2,star}}{Q_{star}} \frac{M_P}{M_{star}} R_{star}^5 \sqrt{GM_{star}}}$$

When will the planet reach the stellar Roche limit?

$$\tau_{Roche} = \frac{\frac{2}{13} (a^{13/2} - a_R^{13/2})}{3 \frac{k_{2,star}}{Q_{star}} \frac{M_P}{M_{star}} R_{star}^5 \sqrt{GM_{star}}}$$

stellar property factor

When will the planet reach the stellar Roche limit?

$$\tau_{Roche} = \frac{\frac{2}{13} (a^{13/2} - a_R^{13/2})}{3 \frac{k_{2,star}}{Q_{star}} \frac{M_P}{M_{star}} R_{star}^5 \sqrt{GM_{star}}}$$

- 
- tidal forces affect the evolution of extrasolar planetary systems
    - if  $a < 0.1$  AU
    - if  $M_p$  is large
    - may force orbits towards  $e \rightarrow 0$
    - may lead to the loss of companions
    - may force massive companions into double-synchronous rotation
      - but the companion is not safe....
    - may spin-up stellar rotation (look for old fast rotating stars)

...book your next boat tour  
when the tide is in!

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