

**Extrasolar Planets:
Towards Comparative Planetology
beyond the Solar System**

Bad Honeff, June, 7th 2011



**Dynamical Evolution of
Exoplanet Systems**

Rudolf Dvorak, AstroDynamicsGroup, Vienna

OUTLINE

- i) Exoplanetary systems (confirmed, unconfirmed)**
- ii) Stability criteria (Hill)**
- iii) Resonances (Mean Motion, Secular, Kozai)**
- iv) Confirmed systems (some examples)**
- v) Kepler candidates (multiple transits)**
- vi) EPILOG: The 1:1 Resonance**

What is the dynamical evolution of planets in multiplanetary systems (MPS)?

A) Confirmed MPS: 51 (Exoplanet.eu)

MAIN QUESTION:

**IS THE SYSTEM STABLE
with the derived parameters**

-- mass

Mass unknown to $m \cdot \sin i$ for RV but known for transit observations

A) Confirmed MPS:

2 Planets:	34
3 Planets:	9
4 Planets:	5
5 Planets:	1 (55 Cnc)
6 Planets:	1 (Kepler 11)
7 Planets:	1 (HD 10180)

J. Schneider

<http://exoplanet.eu>

B) Kepler candidates

Lissauer et al, **Architecture and Dynamics of Kepler's Candidate Multiple Transiting Planet Systems**, ApJ, submitted

2 Planets:	115
3 Planets:	45
4 Planets:	8
5 Planets:	1
6 Planets:	1

Stability criteria:

Hill stability

2 (3) planets in Mean Motion Resonance (MMR)

Secular stability (Laplace-Lagrange)

Kozai stability

Stable according to Numerical Integrations

Regular (stable) with respect to Chaos indicators



The Hill Radius:

Two planets should be at least separated with respect to their semimajor axes according to the Hill radius; it assures that the orbits will not cross.

$$R_{H_{j,j+1}} = \left[\frac{M_j + M_{j+1}}{3M_\star} \right]^{1/3} \frac{(a_j + a_{j+1})}{2}$$

A numerical estimate for 2 planets (Gladman, 1993) gives

$$\Delta \equiv \frac{a_o - a_i}{R_H} > 2\sqrt{3} \approx 3.46$$

Smith and Lissauer, 2009 for more planets (3 and 5) with comparable masses

$$\Delta_{\text{crit}} \approx 9$$

A) Confirmed MPS:

2 Planets: 34

3 Planets: 9

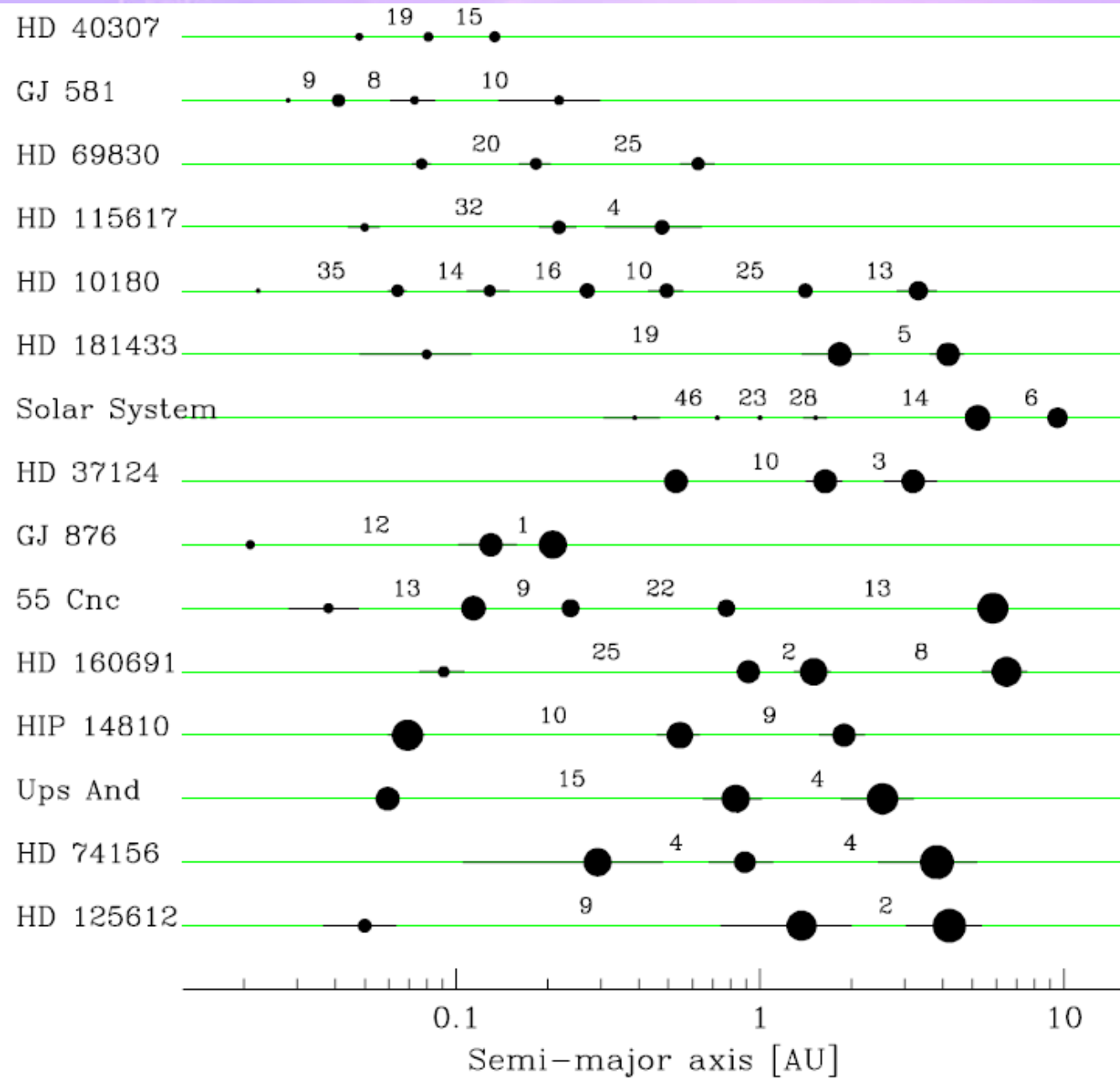
4 Planets: 5

5 Planets: 1 (55 Cnc)

6 Planets: 1 (Kepler 11)

7 Planets: 1 (HD 10180)





15 MPS with at least 3 planets. The numbers indicate the separation in Hill radii. Size of the planet proportional to the determined $\log m \sin(i)$

After Lovis et al. A&A, 2011

Well-characterized Multiplanet System

System	Components	Period Ratio
GJ 876	<i>e, c, b</i>	4:2:1
HD 82943	<i>b, c</i>	2:1
HD 37124	<i>c, d</i>	2:1
HD 128311	<i>c, b</i>	2:1
HD 73526	<i>c, b</i>	2:1
μ Ara	<i>b, e</i>	2:1
KOI 152 ^a	2,3	2:1
KOI 877 ^a	2,1	2:1
24 Sex	<i>c, b</i>	2:1
Kepler-9	<i>c, b</i>	2:1
PSR B1257+12	<i>B, C</i>	3:2
HD 45364	<i>c, b</i>	3:2
HD 200964	<i>c, b</i>	4:3
55 Cnc	<i>c, b</i>	3:1
HD 10180	<i>d, e</i>	3:1
HD 60532	<i>c, b</i>	3:1
HD 108874	<i>c, b</i>	4:1
Solar	h, g	5:2
HD 10180	<i>e, f</i>	5:2
KOI 896 ^a	1,2	5:2
HD 202206	<i>c, b</i>	5:1

J.T.Wright et al,
2011, April 1, ApJ
730-93

Name	Gliese 876 b	Gliese 876 c	Gliese 876 d	Gliese 876 e
Discovered in	2000	2000	2005	2010
Mass	2.2756 (\pm 0.0045) M_J ref.	0.7142 (\pm 0.0039) M_J ref.	0.021 (\pm 0.001) M_J ref.	0.046 (\pm 0.005) M_J ref.
Semi major axis	0.208317 (\pm 2e-05) AU ref.	0.12959 (\pm 2.4e-05) AU ref.	0.02080665 (\pm 1.5e-07) AU ref.	0.3343 (\pm 0.0013) AU ref.
Orbital period	61.1166 (\pm 0.0086) days ref.	30.0881 (\pm 0.0082) days ref.	1.93778 (\pm 2e-05) days ref.	124.26 (\pm 0.7) days ref.
Eccentricity	0.0324 (\pm 0.0013) ref.	0.25591 (\pm 3e-05) ref.	0.207 (\pm 0.055) ref.	0.055 (\pm 0.012) ref.
ω	50.3 (\pm 3.2) deg. ref.	48.76 (\pm 0.7) deg. ref.	234 (\pm 20) deg. ref.	239.22 deg. ref.
Inclination	84 (\pm 6) deg. ref.	48.07 (\pm 2.06) deg. ref.	50 deg. ref.	59.5 deg. ref.
Update	11/05/11	14/12/10	14/12/10	23/06/10

R.V Baluev, CMDA, 2011:

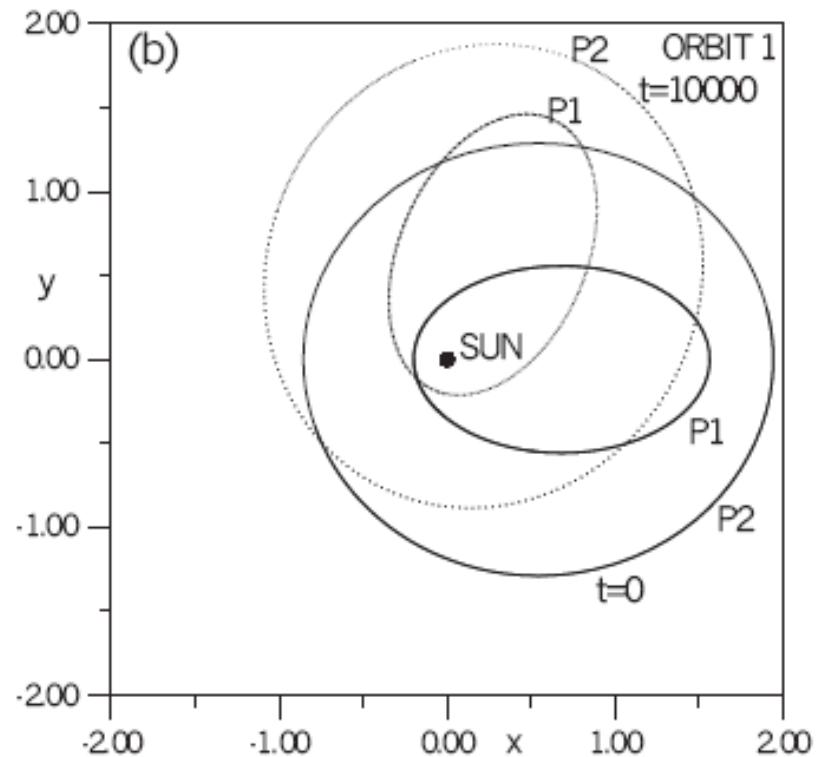
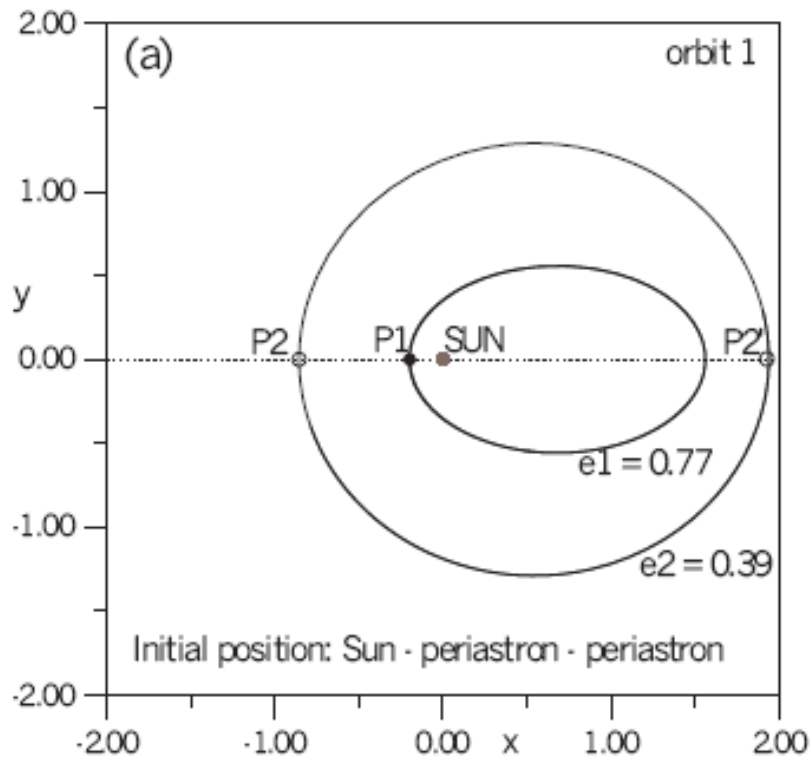
Orbital structure of the GJ876 MPS, based on the latest Keck and HARPS RV data

Laplace Resonance between the Jupiter moons Ganymed-Europa-Io: 4:2:1

The 2:1 Mean Motion Resonance

Table 1. All possible phases at $t = 0$ and $t = T/2$

<i>Type 1:</i>	Sun - P_1 (per) - P_2 (per)	\rightarrow	P_2 (ap) - Sun - P_1 (per)
<i>Type 2:</i>	Sun - P_1 (ap) - P_2 (ap)	\rightarrow	P_2 (per) - Sun - P_1 (ap)
<i>Type 3:</i>	Sun - P_1 (per) - P_2 (ap)	\rightarrow	P_2 (per) - Sun - P_1 (per)
<i>Type 4:</i>	Sun - P_1 (ap) - P_2 (per)	\rightarrow	P_2 (ap) - Sun - P_1 (ap)



Dvorak, R. 2008 Chaotic Dynamics in Planetary Systems
In 'Chaos in Astronomy' p. 255-268, eds: Contopoulos and Voglis

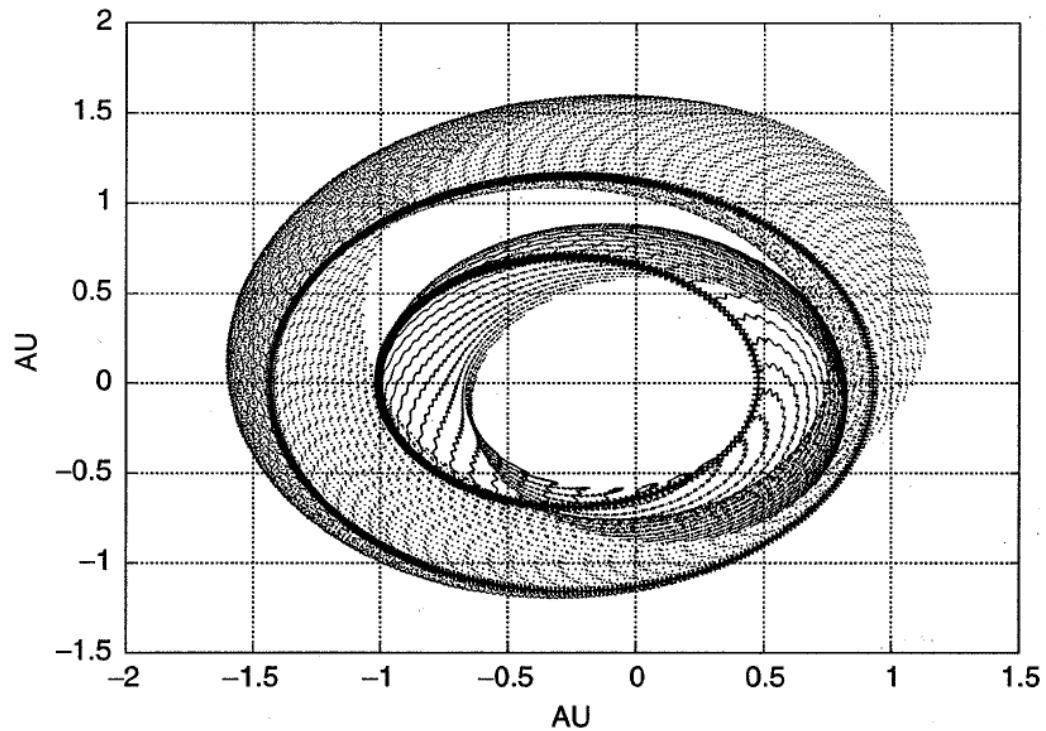
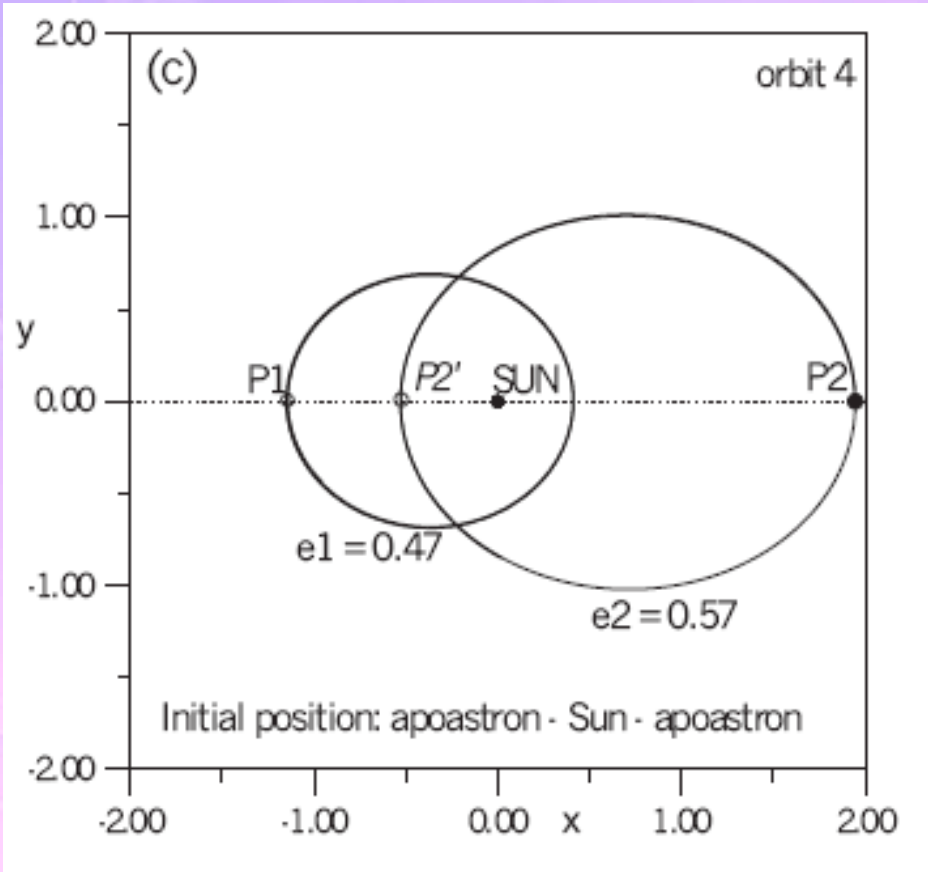


Fig. 9. The orbits of the two planets in HD 82943 in the 2:1 resonance locked in apsidal alignment



Although the orbits cross the planets are protected against close encounters because of the 2:1 MMR

Hadjidemetriou, J.D. & Dionyssia Psychoyos, Dynamics of Extrasolar Planetary Systems: 2/1 Resonant Motion, LNP 626, 412-432

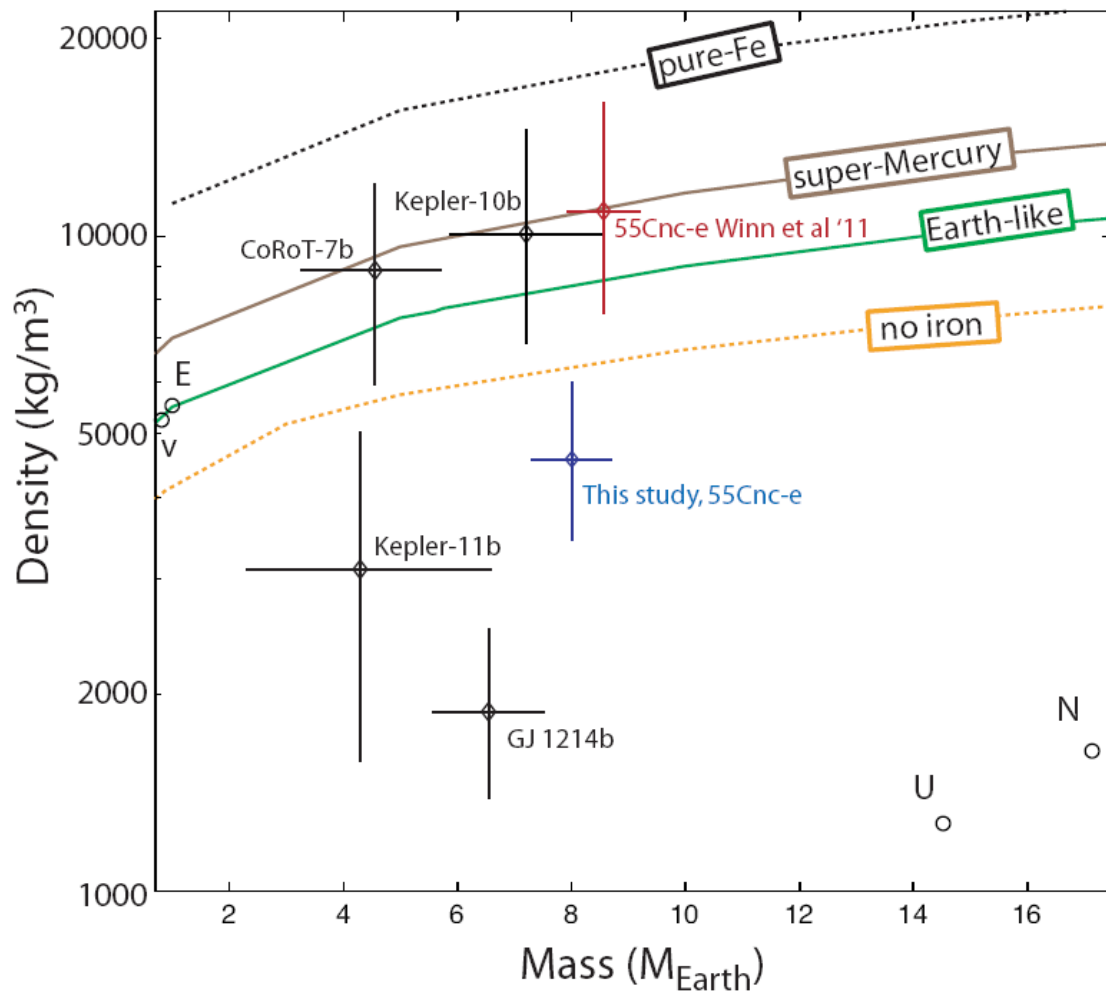
Name	55 Cnc
Distance	13.02 (\pm 0.4) pc ref.
Spectral Type	G8 V
Apparent Magnitude V	5.95
Mass	0.96 (\pm 0.1) M_{sun} ref.
Age	5.5 Gyr ref.
Effective Temperature	5243 (\pm 93) K ref.
Radius	0.95 (\pm 0.02) R_{sun} ref.
Metallicity [Fe/H]	0.31 (\pm 0.04) ref.
Right Asc. Coord.	08 52 37
Decl. Coord.	+28 20 02

Demory et al, A&A, 2011:
Detection of a transit of a
super-Earth 55 Cnc e with
Warm Spitzer

Winn et al. ApJ 2011, A
Super Earth transiting a
naked-eye star (detected
with the MOST Space
telescope

3:1 MMR

Name	55 Cnc b	55 Cnc c	55 Cnc d	55 Cnc e	55 Cnc f
Discovered in	1996	2002	2002	2004	2007
Mass	0.824 (\pm 0.007) M_J ref.	0.169 (\pm 0.008) M_J ref.	3.835 (\pm 0.08) M_J ref.	0.027 (\pm 0.002) M_J ref.	0.144 (\pm 0.04) M_J ref.
Semi-major axis	0.115 (\pm 0.0001) AU ref.	0.24 (\pm 4.5e-05) AU ref.	5.77 (\pm 0.11) AU ref.	0.01564 (\pm 0.00028) AU ref.	0.785 (\pm 0.007) AU ref.
Orbital period	14.651262 (\pm 0.0007) days ref.	44.3446 (\pm 0.007) days ref.	5218 (\pm 230) days ref.	0.73654 (\pm 3e-06) days ref.	260.7 (\pm 1.1) days ref.
Eccentricity	0.0159 (\pm 0.008) ref.	0.053 ref.	0.025 (\pm 0.03) ref.	0.057 (-0.041 ^{+0.064}) ref.	0.0002 ref.
ω	164 deg. ref.	57.4 (\pm 11) deg. ref.	181.3 (\pm 32) deg. ref.	170 (-130 ⁺⁹⁰) deg. ref.	205 (\pm 60) deg. ref.
Radius	-	-	-	0.19 (\pm 0.013) R_J ref.	-
$T_{transit}$	-	-	-	2455568.026 (-0.0006 ^{+0.0012}) ref.	-
$T_{sec-transit}$	-	-	-	2455568.364 (\pm 0.028)	-
T_{peri}	2450002.94749 (\pm 1.2) ref.	2449989.3385 (\pm 3.3) ref.	2452500.6 (\pm 230) ref.	2449999.83643 (\pm 0.0001) ref.	2450080.9108 (\pm 1.1) ref.
Inclination	-	-	53 (\pm 6.8) deg. ref.	83.4 (\pm 1.7) deg. ref.	-
Update	04/05/11	14/12/10	04/05/11	04/05/11	14/12/10



KOZAI RESONANCE

A coupling between the inclinations and eccentricities allows large eccentricities for the planets

$$L_i = \kappa_i \sqrt{a}$$

$$\cancel{X}_i = L_i \sqrt{(1 - e_i^2)}$$

$$H_i = G_i \cos i_i$$

$$l_i = M_i$$

$$\cancel{X} = \omega_i$$

$$h_i = \Omega_i$$

Delaunay elements

For 2 planets the equations can be reduced to a 2-degree of freedom system

Kozai resonance in extrasolar systems

A.-S. Libert^{1,2,*} and K. Tsiganis^{2,**}

A&A 493, 677–686 (2009)

5 systems NOT in MMR:

HD 12661, HD 169830, HD74156, HD 155358
and υ Andromedae

Parameter study varying the unknown mutual
inclinations and nodal longitudes

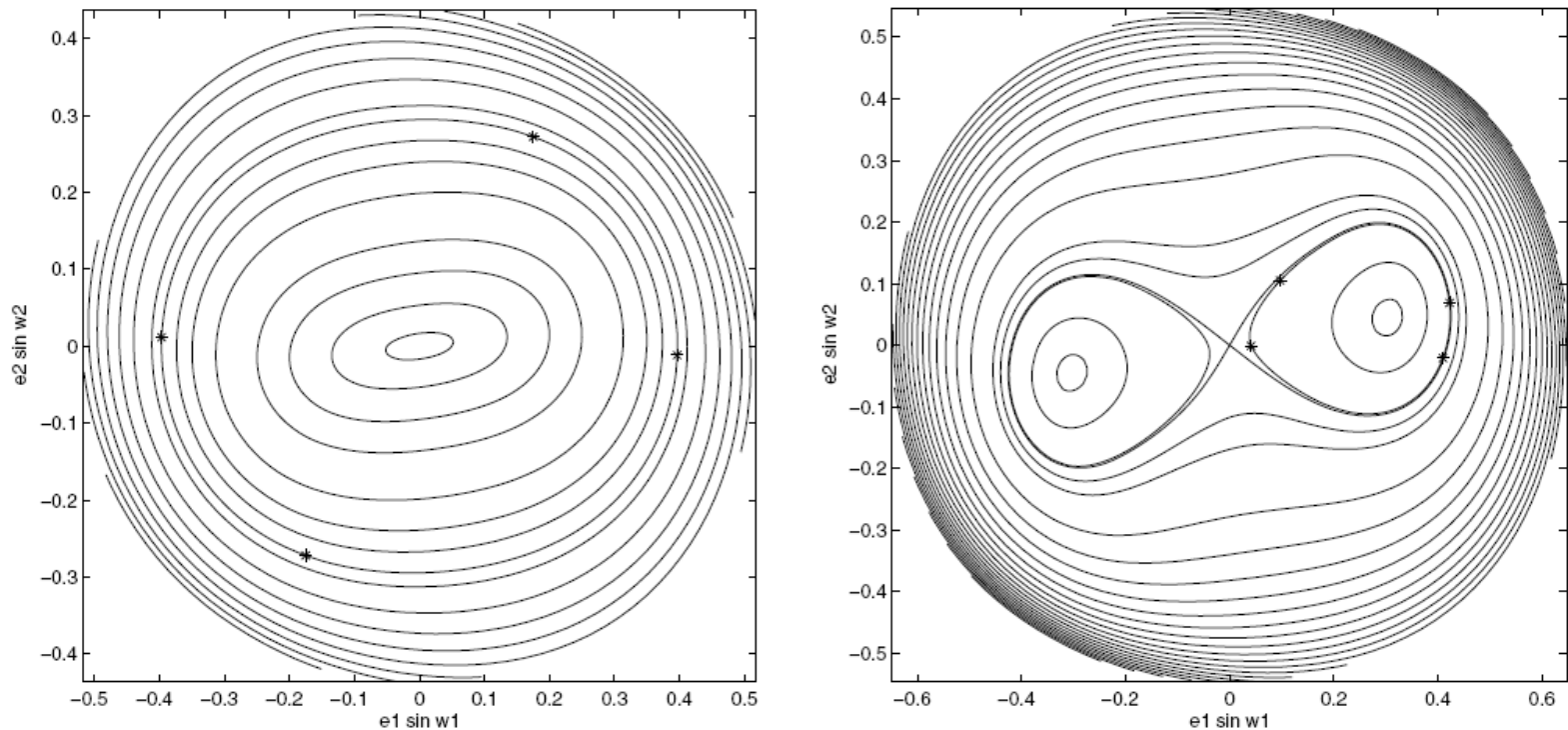
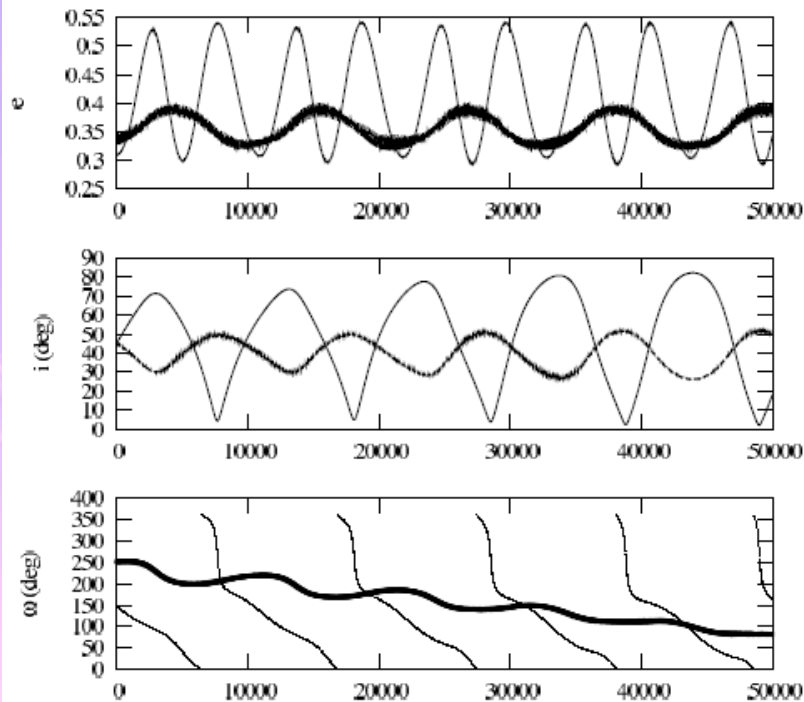


Fig. 4. Representative planes ($e'_1 \sin w'_1, e'_2 \sin w'_2$) for the HD 12661 system with $I_{\text{mut}} = 30^\circ$ ($a_1/a_2 = 0.294$, $m_1/m_2 = 1.279$, $\Sigma = 0.0440$) on the left and with $I_{\text{mut}} = 45^\circ$ ($a_1/a_2 = 0.284$, $m_1/m_2 = 1.279$, $\Sigma = 0.0714$) on the right. The difference in the dynamics of the two cases is obvious.



$i_1 = i_2 = 45 \text{ deg}$

$I_{\text{mut}} = 55 \text{ deg}$

Integration over 50000 yrs

Name	HD 169830 b	HD 169830 c
Discovered in	2000	2003
Mass	$2.88 M_J$ ref.	$4.04 M_J$ ref.
Semi major axis	0.81 AU ref.	3.6 AU ref.
Orbital period	$225.62 (\pm 0.22) \text{ days}$ ref.	$2102 (\pm 264) \text{ days}$ ref.
Eccentricity	$0.31 (\pm 0.01)$ ref.	$0.33 (\pm 0.02)$ ref.
ω	$148 (\pm 2) \text{ deg.}$ ref.	$252 (\pm 8) \text{ deg.}$ ref.
T_{peri}	$51923 (\pm 1) \text{ JD } 2.450.000$ ref.	$52516 (\pm 25) \text{ JD } 2.450.000$ ref.
Update	23/12/10	23/12/10

Secular Perturbations:

Theory de Lagrange -Laplace

Without strong MMR the variations of the Keplerian elements can be described by the secular equations; which are obtained after averaging over the longitudinal motion of the planets (**a=constant!**)

$$h_j = e_j \sin \tilde{\omega}_j$$

$$k_j = e_j \cos \tilde{\omega}_j$$

$$p_j = i_j \sin \Omega_j$$

$$q_j = i_j \cos \Omega_j$$

→

$$\dot{h}_j = +\frac{1}{n_j a_j^2} \frac{\partial F_j}{\partial k_j}$$

$$\dot{k}_j = -\frac{1}{n_j a_j^2} \frac{\partial F_j}{\partial h_j}$$

$$\dot{p}_j = +\frac{1}{n_j a_j^2} \frac{\partial F_j}{\partial q_j}$$

$$\dot{q}_j = -\frac{1}{n_j a_j^2} \frac{\partial F_j}{\partial p_j}$$

→

$$h_j = \sum_{i=1}^2 e_{ji} \sin (g_i t + \beta_i)$$

$$k_j = \sum_{i=1}^2 e_{ji} \cos (g_i t + \beta_i)$$

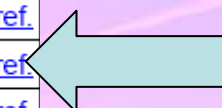
$$p_j = \sum_{i=1}^2 I_{ji} \sin (f_i t + \gamma_i)$$

$$q_j = \sum_{i=1}^2 I_{ji} \cos (f_i t + \gamma_i)$$

Lovis et al, A&A, 528, 2011

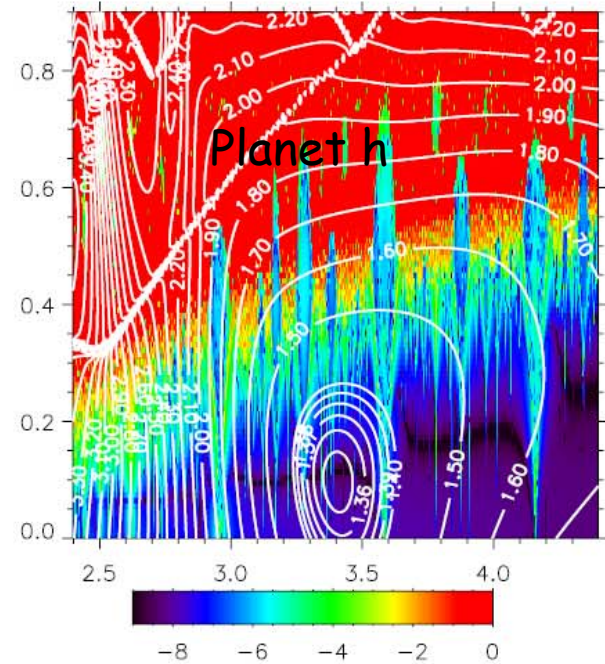
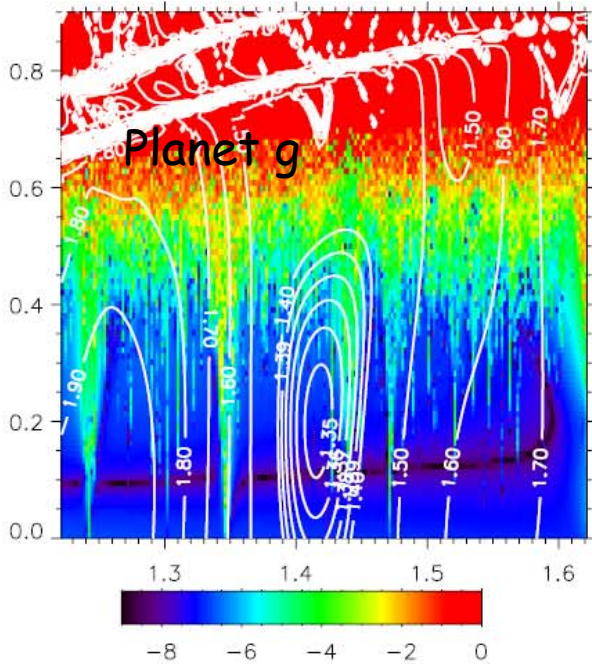
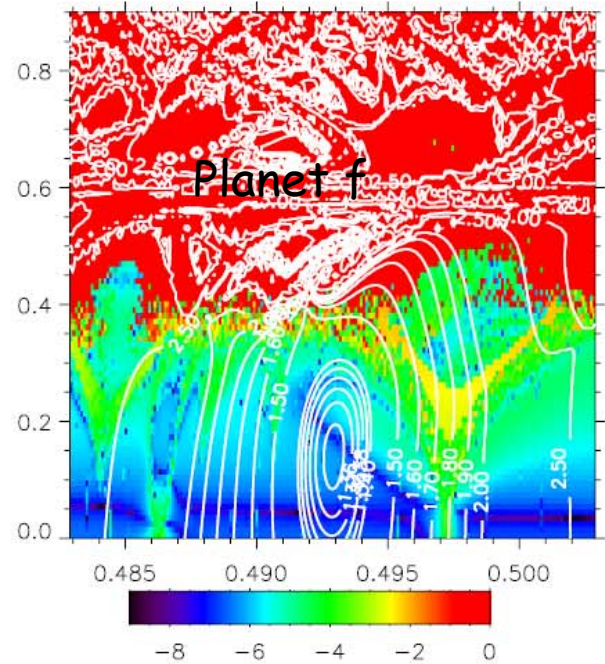
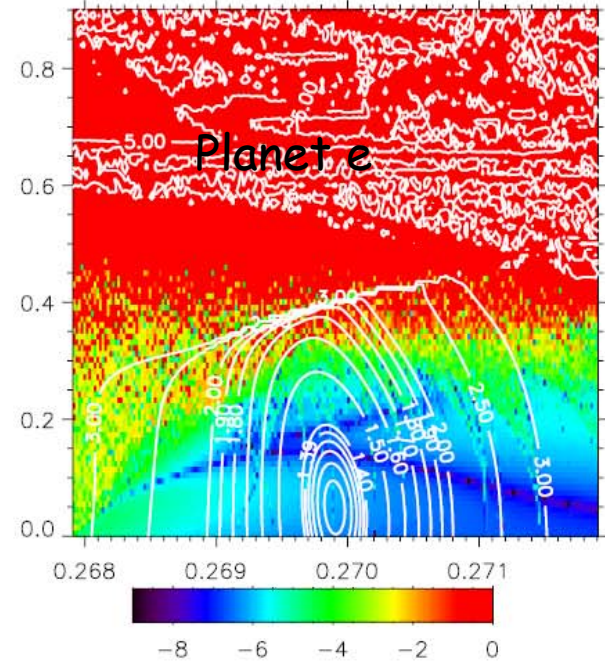
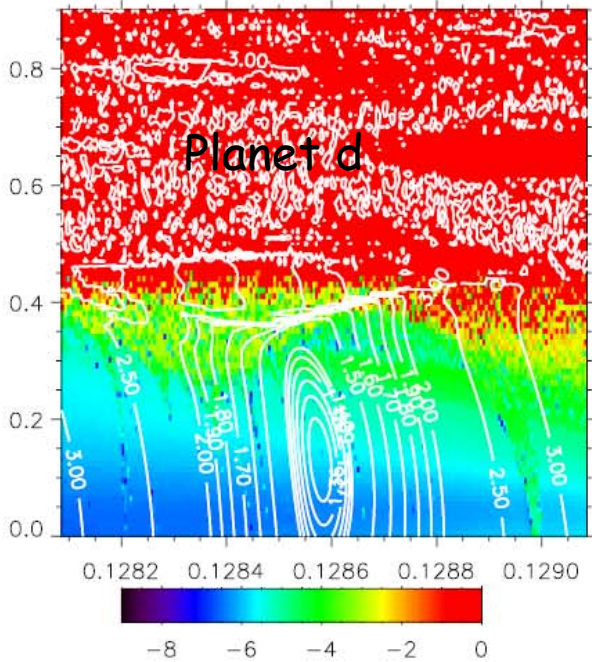
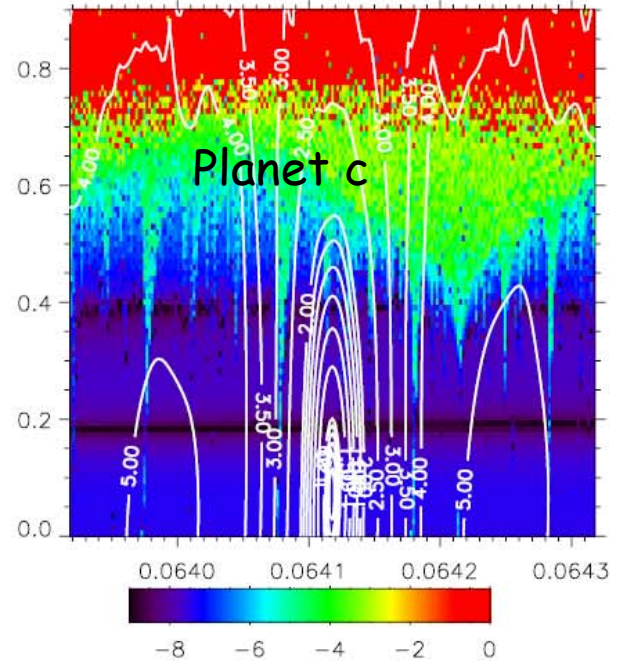
**XXVIII. Up to seven planets orbiting HD 10180:
probing the architecture of low-mass planetary systems**

3:1 MMR

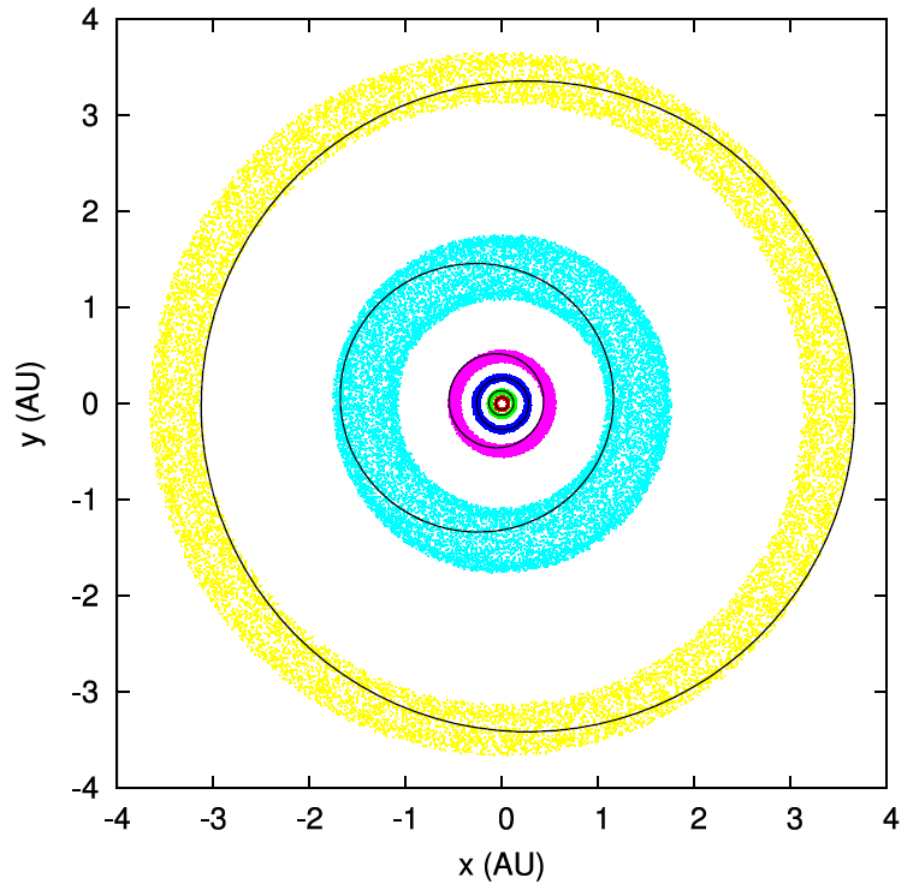


	HD 10180 b (<i>unconfirmed</i>)	HD 10180 c	HD 10180 d	HD 10180 e
Discovered in	2010	2010	2010	2010
Mass	0.00424755 M_J	ref. 0.041217 M_J	ref. 0.03696945 M_J	ref. 0.07897304 M_J
Semi major axis	0.02225 (\pm 0.00035) AU	ref. 0.0641 (\pm 0.001) AU	ref. 0.1286 (\pm 0.002) AU	ref. 0.2699 (\pm 0.0042) AU
Orbital period	1.17768 (\pm 0.0001) days	ref. 5.75979 (\pm 0.00062) days	ref. 16.3579 (\pm 0.038) days	ref. 49.745 (\pm 0.022) days
Eccentricity	0	ref. 0.045 (\pm 0.026)	ref. 0.088 (\pm 0.041)	ref. 0.026 (\pm 0.036)
ω	-	ref. 332 (\pm 43) deg.	ref. 315 (\pm 33) deg.	ref. 166 (\pm 110) deg.
Update	07/12/10	07/12/10	07/12/10	07/12/10

HD 10180 f	HD 10180 g	HD 10180 h
2010	2010	2010
0.07519743 M_J	ref. 0.06733159 M_J	ref. 0.202624 M_J
0.4929 (\pm 0.0078) AU	ref. 1.422 (\pm 0.026) AU	ref. 3.4 (\pm 0.11) AU
122.76 (\pm 0.17) days	ref. 601.2 (\pm 8.1) days	ref. 2222 (\pm 91) days
0.135	ref. 0.19 (\pm 0.14)	ref. 0.08 (\pm 0.07)
332 deg.	ref. 347 (\pm 49) deg.	ref. 174 (\pm 74) deg.
07/12/10	07/12/10	07/12/10



HD 10180 variation of eccentricity versus semimajor axes



HD 10180 for $1.5 \cdot 10^8$ years

k	a_{\min}	a_{\max}	e_{\min}	e_{\max}
1	0.022253	0.022253	0.000	0.082
2	0.064114	0.064122	0.010	0.203
3	0.128536	0.128626	0.000	0.179
4	0.269814	0.270092	0.000	0.156
5	0.492348	0.493184	0.023	0.137
6	1.419645	1.424347	0.188	0.242
7	3.387207	3.402716	0.044	0.081

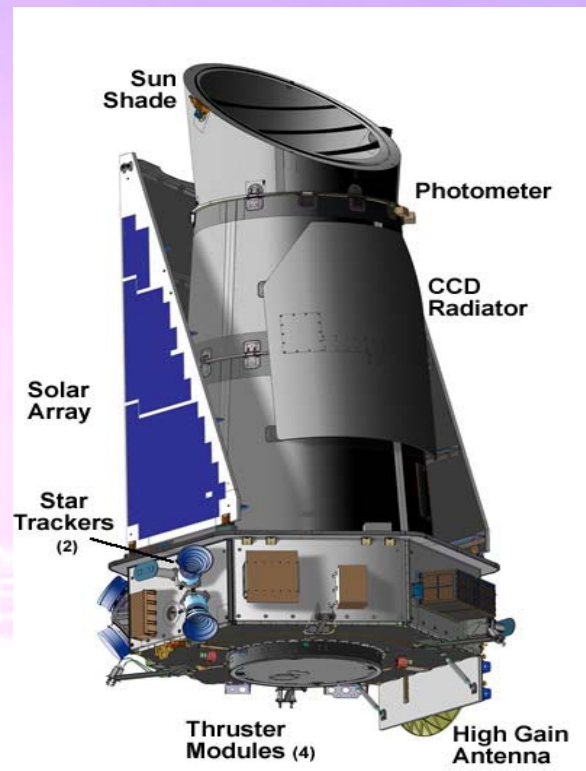
k	Period yr	g_k (num) deg yr ⁻¹	g_k (ana) deg yr ⁻¹
1	1029.34	0.349739	0.358991
2	1453.39	0.247696	0.245229
3	3020.08	0.119202	0.118471
4	4339.70	0.082955	0.079644
5	13 509.96	0.026647	0.025290
6	61 517.43	0.005852	0.005581
7	473 061.76	0.000761	0.000663

Perihelion motion

Kepler-11 f	Kepler-11 g
2011	2011
$0.007237 (-0.00378^{+0.00692}) M_J$	$< 0.95 M_J$
$0.25 (\pm 0.009) AU$	$0.462 (\pm 0.016) AU$
$46.68876 (\pm 0.00074) days$	$118.37774 (\pm 0.00112) days$
0	0
$0.2335 (\pm 0.0224) R_J$	$0.3274 (\pm 0.0313) R_J$
$2454964.6487 (\pm 0.0059)$	$2455120.2901 (\pm 0.0022)$
$89.4 (\pm 0.2) deg.$	$89.8 (\pm 0.2) deg.$
02/02/11	02/02/11

←

**5:2
MMR**



Name	Kepler-11 b	Kepler-11 c	Kepler-11 d	Kepler-11 e
Discovered in	2011	2011	2011	2011
Mass	$0.01353 (-0.00629^{+0.00692}) M_J$	$0.0425 (-0.0198^{+0.0151}) M_J$	$0.01919 (-0.005349^{+0.009754}) M_J$	$0.02643 (-0.005978^{+0.007866}) M_J$
Semi major axis	$0.091 (\pm 0.003) AU$	$0.106 (\pm 0.004) AU$	$0.159 (\pm 0.005) AU$	$0.194 (\pm 0.007) AU$
Orbital period	$10.30375 (\pm 0.00016) days$	$13.02502 (\pm 8e-05) days$	$22.68719 (\pm 0.00021) days$	$31.9959 (\pm 0.00028) days$
Eccentricity	0	0	0	0
Radius	$0.1762 (\pm 0.017) R_J$	$0.28175 (\pm 0.0268) R_J$	$0.3068 (\pm 0.02862) R_J$	$0.4043 (\pm 0.0385) R_J$
T _{transit}	$2454971.5052 (\pm 0.0077)$	$2454971.1748 (\pm 0.0031)$	$2454981.455 (\pm 0.0044)$	$2454987.159 (\pm 0.0037)$
Inclination	$88.5 (-0.6^{+1}) deg.$	$89 (-0.6^{+1}) deg.$	$89.3 (-0.4^{+0.6}) deg.$	$88.8 (\pm 0.2) deg.$
Update	02/02/11	02/02/11	02/02/11	02/02/11



B) Kepler candidates

2 Planets: 115

3 Planets: 45

4 Planets: 8

5 Planets: 1

6 Planets: 1

KOI #	$R_{p,1}$ (R_{\oplus})	$R_{p,2}$ (R_{\oplus})	P_2/P_1	$\Delta_{1,2}$	$R_{p,3}$ (R_{\oplus})	P_3/P_2	$\Delta_{2,3}$
85	3.24	1.67	2.719393	28.2	2.01	1.387583	9.2
94	12.64	4.01	2.143302	9.0	6.89	4.052306	14.9
111	2.46	2.25	2.071194	21.5	2.51	4.373353	40.8
137	6.04	8.56	2.180367	14.4	2.32	1.944495	8.9
148	2.12	2.97	2.024651	18.7	2.04	4.434225	37.6
152	4.92	2.54	2.032345	20.4	2.41	1.900850	13.5
156	1.90	1.64	1.549841	14.0	2.77	1.464437	10.0
168	3.69	1.86	1.395701	11.9	2.00	1.511764	10.9
248	2.86	2.49	2.795806	23.1	1.99	1.515099	9.1
250	3.63	3.61	3.465802	25.5	1.28	1.404620	6.1
284	2.53	1.96	1.038334	1.3	1.87	2.807616	31.5
351	8.50	6.02	3.522872	23.1	1.95	1.575869	6.2
377	5.73	6.20	12.089890	40.6	1.04	2.020508	10.7
398	8.61	3.49	2.417105	21.8	1.92	12.403143	29.2
408	3.63	2.87	1.701564	12.5	2.56	2.454333	23.6
481	2.47	1.65	4.922978	45.4	2.97	4.478249	36.0
520	3.06	1.96	2.348524	22.6	2.75	2.018139	17.2
528	3.14	3.43	2.146104	18.2	3.19	4.703530	33.6
567	2.89	2.29	1.899657	17.5	2.23	1.429507	10.9
571	1.70	1.97	1.869767	17.7	1.81	1.836067	16.7
623	2.04	1.74	1.848406	21.8	1.80	1.514823	15.0
665	2.28	1.19	1.905525	33.9	0.84	1.910436	23.9

4 planet systems (candidates)

KOI #	$R_{p,1}$ (R_{\oplus})	$R_{p,2}$ (R_{\oplus})	P_2/P_1	$\Delta_{1,2}$	$R_{p,3}$ (R_{\oplus})	P_3/P_2	$\Delta_{2,3}$	$R_{p,4}$ (R_{\oplus})	P_4/P_3	$\Delta_{3,4}$
70	2.27	1.60	1.649977	22.6	1.97	1.779783	20.8	0.60	7.150242	53.5
117	2.39	1.32	1.541373	19.4	1.29	1.623476	25.1	0.68	1.853594	22.3
191	11.56	2.81	3.412824	35.8	1.39	6.350795	20.2	1.49	1.258207	2.9
707	3.36	2.64	1.652824	13.4	2.48	1.459577	9.8	2.19	1.290927	7.3
730	3.09	2.29	1.333812	9.4	2.45	1.001034	0.0	1.83	1.499447	10.7
834	4.87	1.85	2.943873	44.2	1.39	2.149832	28.5	1.39	1.787503	12.5
880	4.90	5.76	2.476866	25.6	2.76	4.480164	28.7	2.01	1.948715	11.1
952	2.27	2.32	2.037706	19.5	2.44	1.483153	9.2	1.14	2.602734	21.3

MMR	# Total Pairs	# Adjacent Pairs
2:1	90	74
3:2	24	21
4:3	7	5
5:4	3	2
3:1	80	54
5:3	15	11
7:5	5	5
9:7	3	3

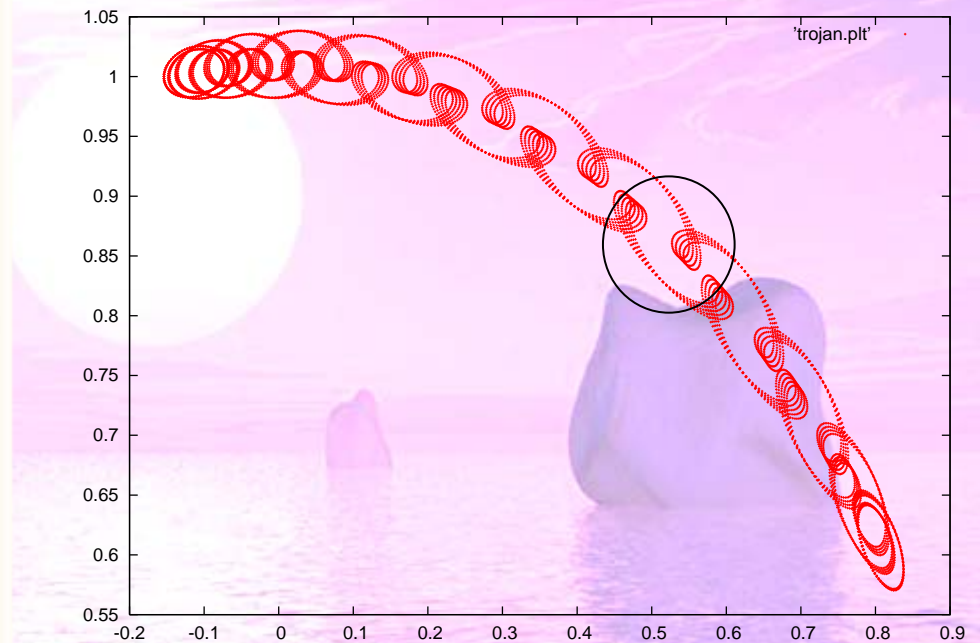
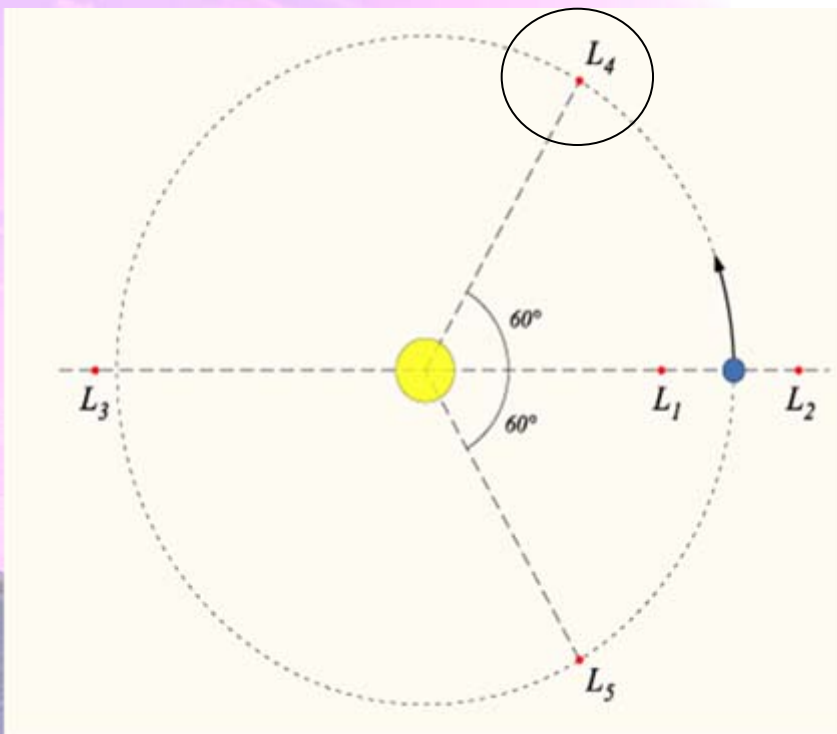
property	KOI-500	KOI-157 (nominal Kepler-11)	Kepler-11 (Lissauer et al. 2011)
$R_{p,1}$ (R_{\oplus})	2.75	1.70	1.97 ± 0.19
$R_{p,2}$ (R_{\oplus})	2.83	3.03	3.15 ± 0.30
P_2/P_1	3.113327	1.264079	1.26410
Δ_{12}	40.5	6.8	7.0
$R_{p,3}$ (R_{\oplus})	1.47	3.50	3.43 ± 0.32
P_3/P_2	1.512077	1.741808	1.74182
Δ_{23}	12.7	13.1	15.9
$R_{p,4}$ (R_{\oplus})	2.06	4.21	4.52 ± 0.43
P_4/P_3	1.518394	1.410300	1.41031
Δ_{34}	10.4	7.3	10.9
$R_{p,5}$ (R_{\oplus})	1.23	2.22	2.61 ± 0.25
P_5/P_4	1.349929	1.459232	1.45921
Δ_{45}	6.8	8.8	13.3
$R_{p,6}$ (R_{\oplus})	—	3.24	3.66 ± 0.35
P_6/P_5	—	2.535456	2.53547
Δ_{56}	—	24.1	—

Orbits in 1:1 MMR:

Trojan-like orbits

Horseshoe orbits - Exchange orbits type I

Exchange orbits type II



Funk et al, 2010,
Exchange orbits: a
possible application to
extrasolar planetary
systems? MNRAS 410

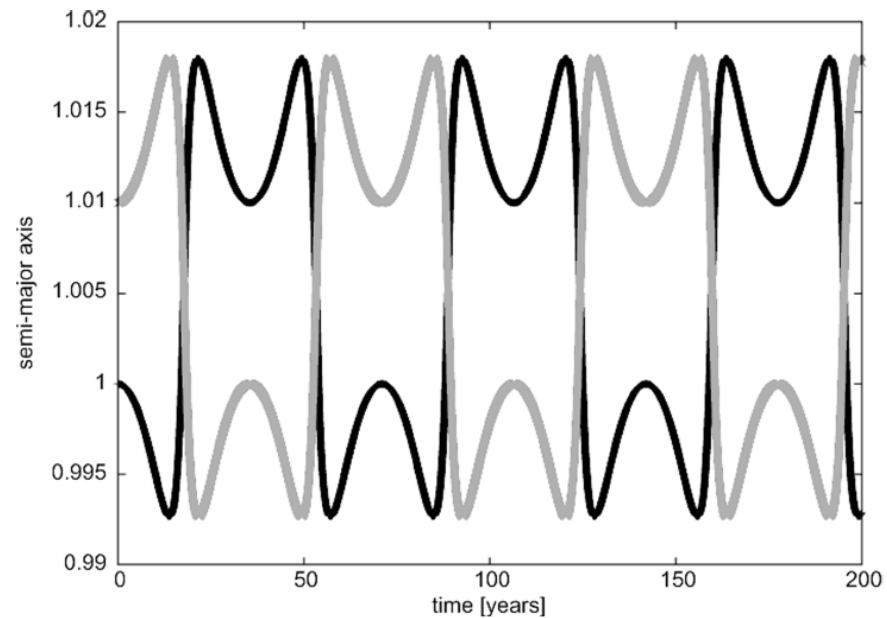
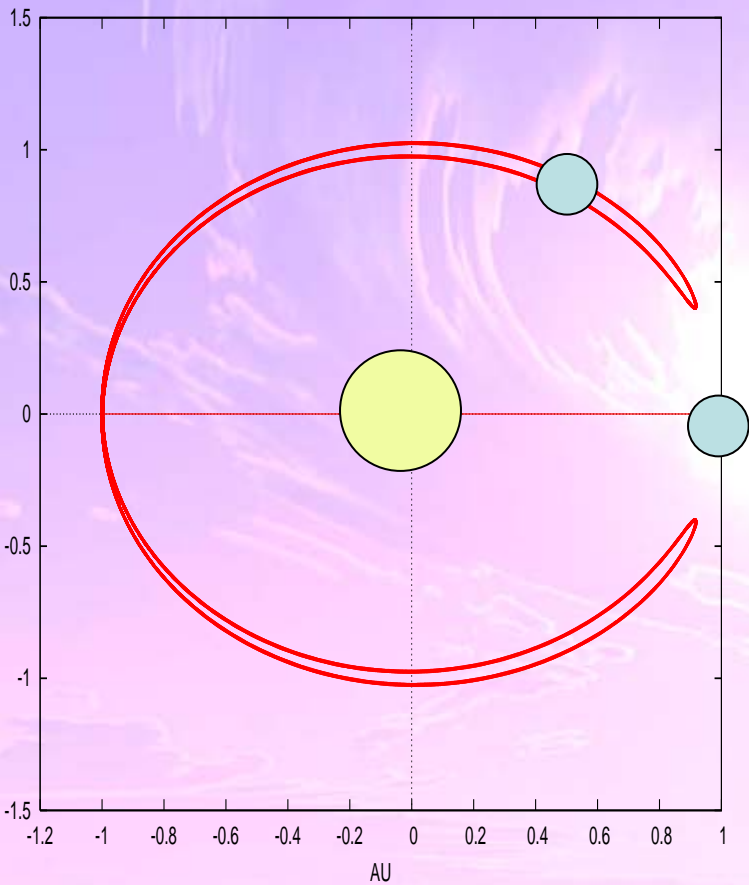


Figure 1. Exchange-*a* configuration: in this example both planets have a mass of $10^{-5} M_{\odot}$ and move on circular orbits in the same plane. The mean anomaly of planet 1 is 0° and that of planet 2 is 180° . The semi-major axis of planet 1 (black line) initially equals 1 au, while the semi-major axis of planet 2 (grey line) is initially 1.001 au.

MOVIE

Horseshoe orbits

Exchange-*a* orbits

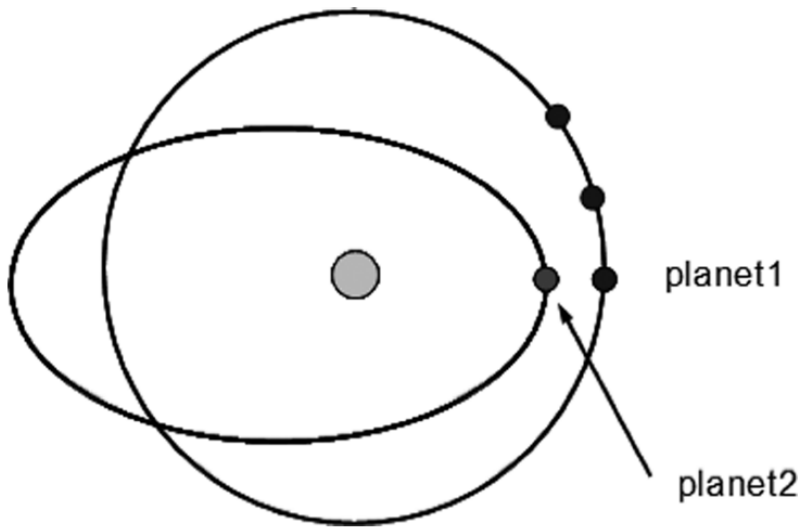
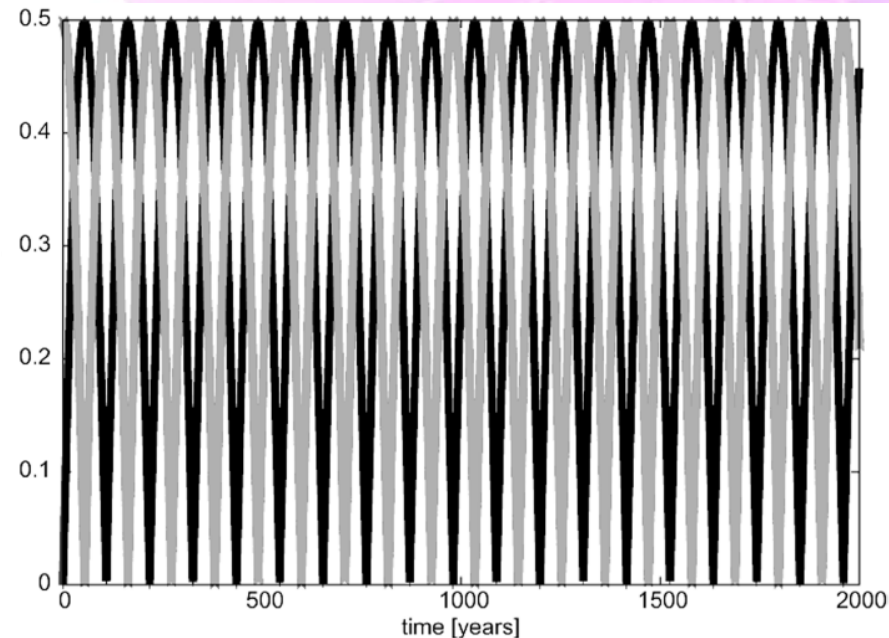
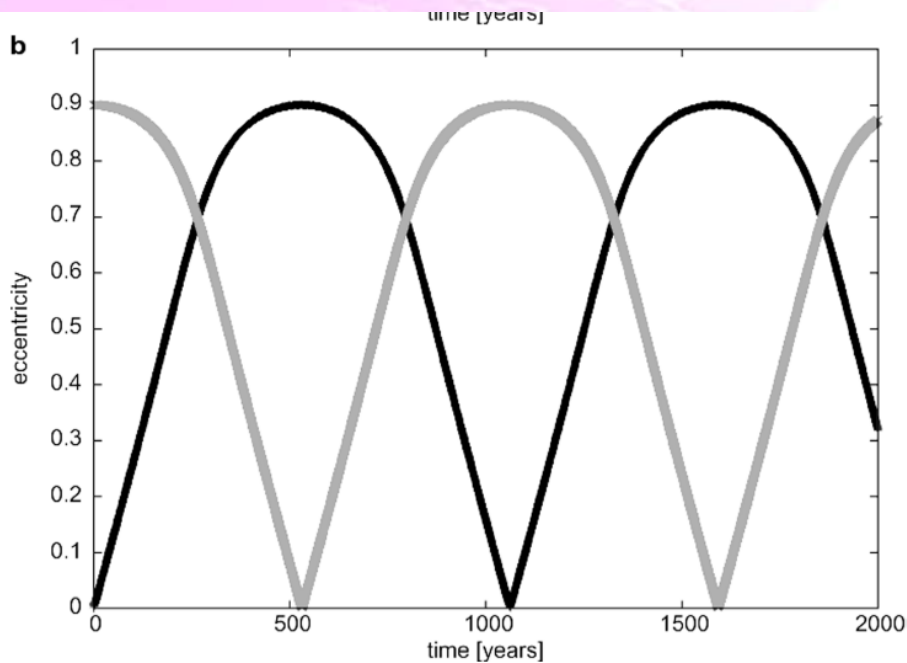


Figure 3. Schematic plot for the initial exchange- e configuration: planet 1 was always started on a circular orbit with different mean anomalies, while planet 2 was initially placed on different eccentric orbits with a zero mean anomaly.

Exchange- e orbit

$$e_1=0, e_2=0.5$$

$$e_1=0, e_2=0.9$$



Resume

Funk et al. 2010 The stability of ultra-compact systems, A&A 516: 10 planets with $17 m_{\text{Earth}}$ inside 0.26 AU have stable orbits

iii) Kozai Resonances lead to large eccentricities and inclinations

iv) Dense Systems may exist

The study of MPS is a new challenge for Celestial Mechanics
AND also a challenge for Astronomy and Astrophysics

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