

# Disk-Planet Interaction

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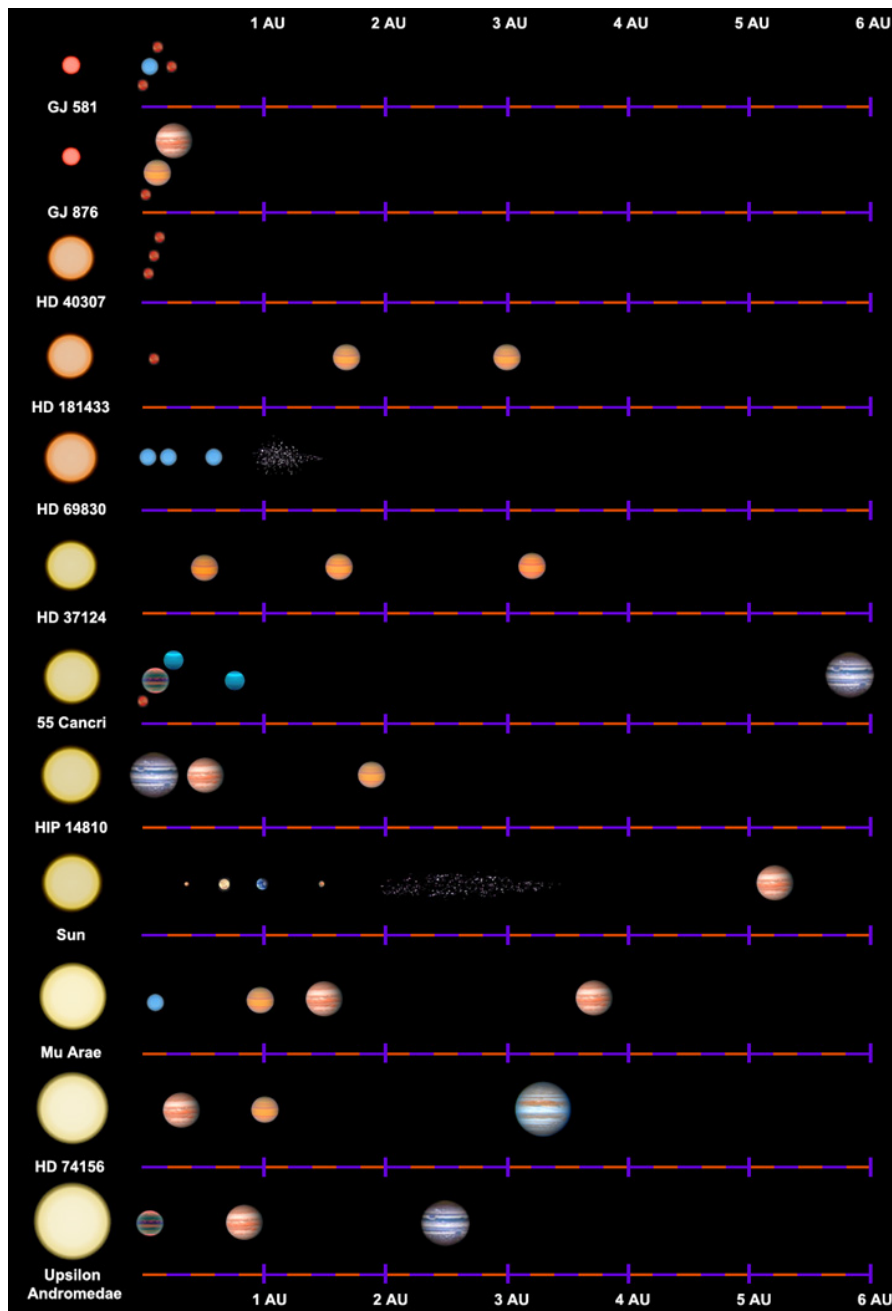


7. June, 2011

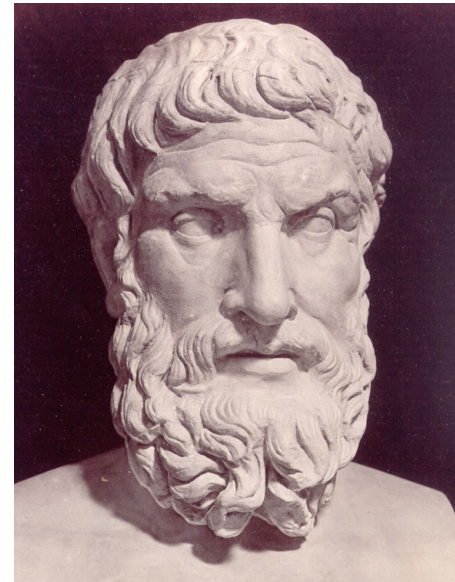


- Introduction
- Migration
  - Principle
  - Disk Thermodynamics
  - Dynamical evolution
- Eccentricity & Inclination
- Summary

(A. Crida)



Epikur (ca. 341-270 BC)  
 “There is an infinite number of worlds, some similar to ours some very different.”

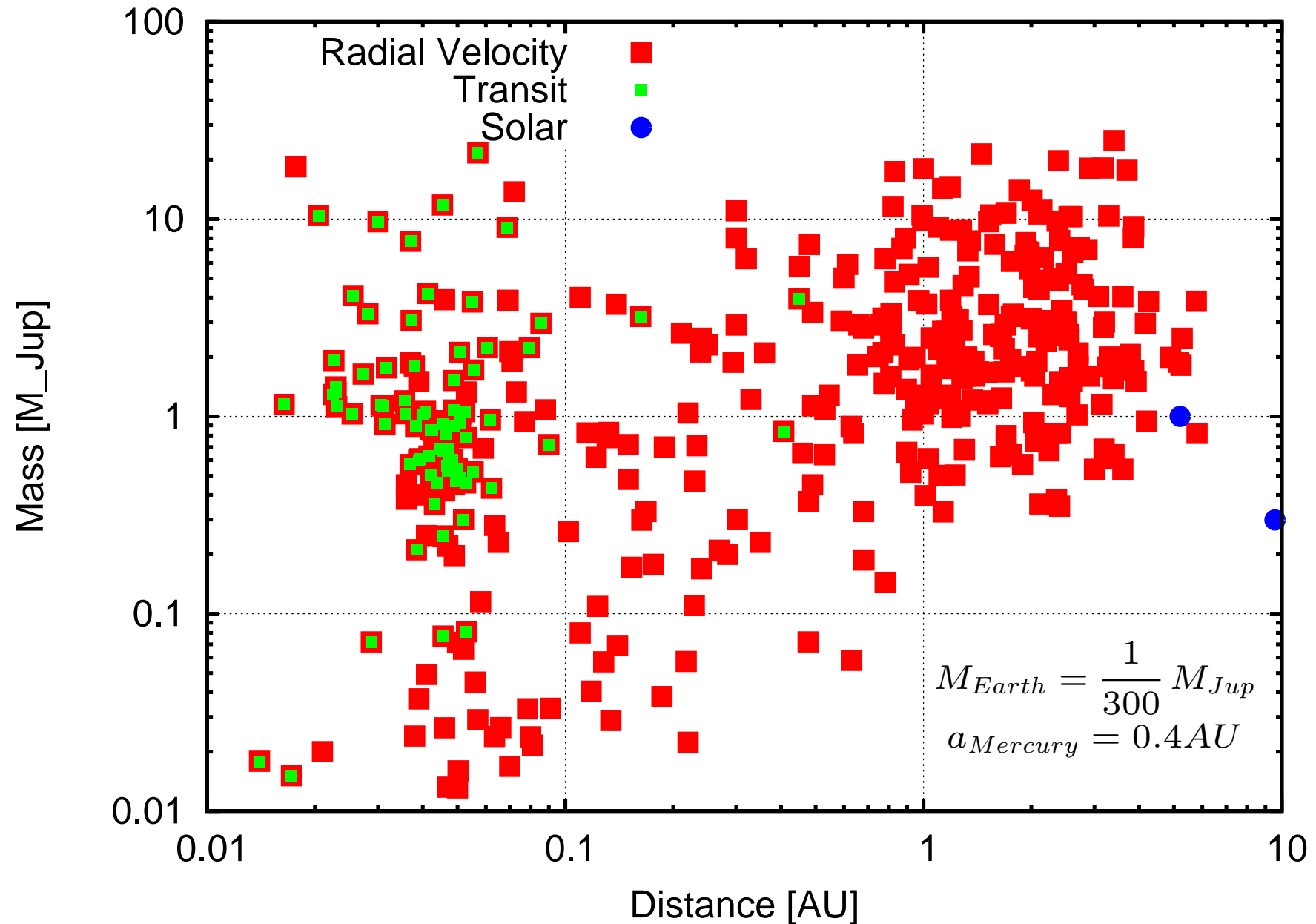


Architecture shaping by  
**Disk-Planet** interaction!



Small distances (hot Jupiters) &amp; large masses

(Data: exoplanet.eu)





- Not possible to form hot Jupiters in situ
  - disk too hot for material to condense
  - not enough material
- Difficult to form massive planets
  - gap formation

But planets grow in disks:

⇒ Have a closer look at planet-disk interaction



## Two contributions:

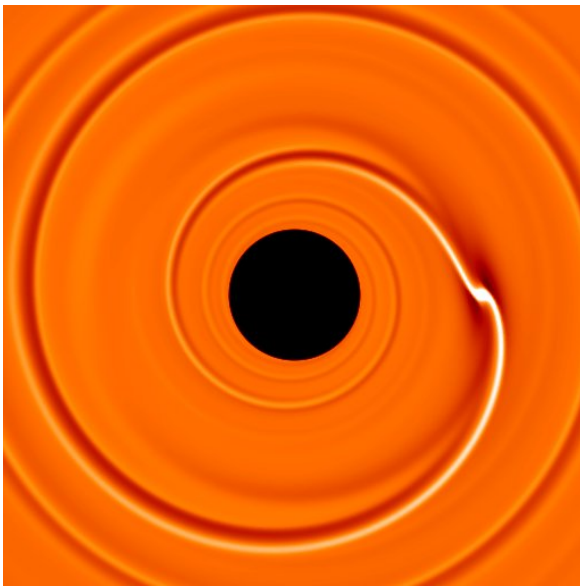
- Spiral Arms (Lindblad torques)
- Horse Shoe (Corotation torques)

(A. Crida)

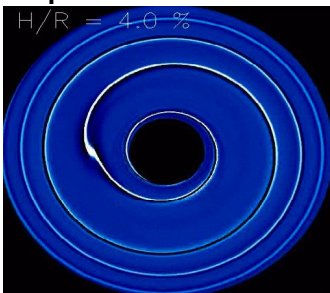


Planet with  $20 M_{Earth}$   
in protoplanetary Disk  
Hydrodynamical Simulation  
Disk with constant density

(Masset, 2002)



Dependence on Temperature,  $c_s$



(Masset, 2002)

Planet generates spiral waves  
in the density of the disk

Spirals are maxima of density

Gravitational interaction with planet

Inner Spiral

- pulls planet forward:
- positive torque

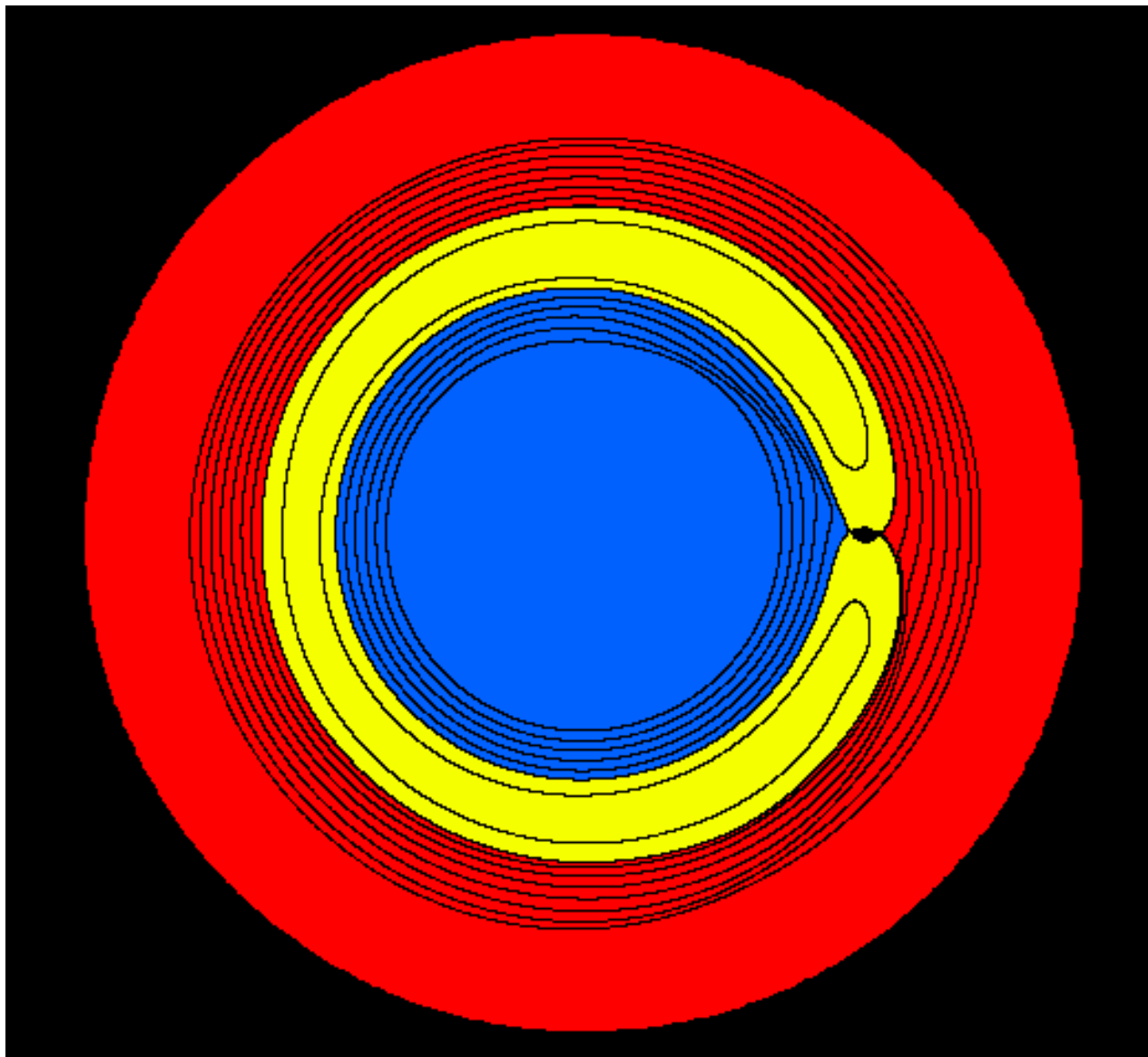
Outer Spiral

- pulls planet backward:
- negative torque

→ Net Torque

⇒ Migration

Most important:  
Strength & Direction ?



3 Regions

Outer disk (spiral)

Inner disk (spiral)

⇒ Lindblad torques

Horseshoe (coorbital)

⇒ Corotation Torques

Efficiency:

- Difference:  
inward-outward kick

Scaling with:

- Vortensity gradient
- Entropy gradient

(F. Masset)





Axisymmetric, constant density disk, differential rotation with  $\Omega(r)$

Decompose the planet potential

$$\psi_p(r, \varphi, t) = \sum_{m=0}^{\infty} \psi_m(r) \cos\{m[\varphi - \varphi_p(t)]\}$$

$\varphi_p = \Omega_p t$  Azimuth angle of the planet

$\psi_m(r)$ :  $m$ -folded potential, rotating with pattern-speed  $\Omega_p$

Frequency of potential in matter frame  $\omega = m(\Omega(r) - \Omega_p)$

**Response** when  $\omega$  matches either 0 or  $\pm\kappa$

( $\kappa$  epicyclic frequency)

$\omega = \pm\kappa$ : Outer or Inner **Lindblad** Resonance (Spirals)

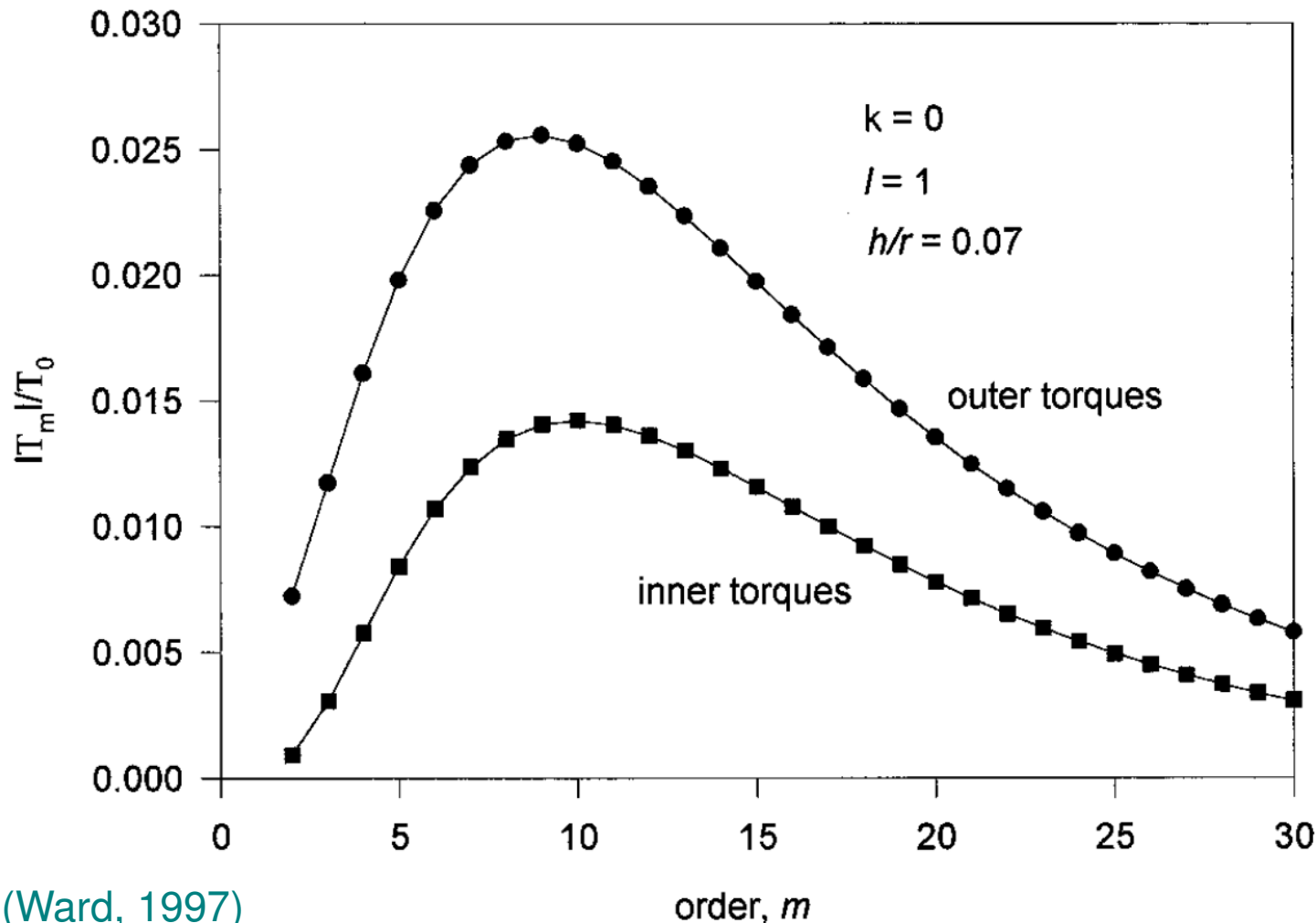
$\omega = 0$ : **Corotation** Resonance (Horseshoe)

**Linearize hydrodynamic equations** (Goldreich & Tremaine; Lin & Papaloizou)



$$\Gamma_{tot} = \int_{disk} \Sigma(\vec{r} \times \nabla \psi_p) df = \int_{disk} \sum_m \Sigma \frac{\partial \psi_m}{\partial \varphi} df = \sum_m \Gamma_m$$

Absolute value of Torque  $|\Gamma_m|$  due to **spirals** for each mode  $m$



Outer spiral wins  
**Inward migration**

(Ward, 1997)



Migration Timescale  $\tau_M$  from  $\dot{L} = \Gamma$  with  $L_P = m_P \sqrt{GM_* a_P}$

$$\frac{1}{a_P} \frac{da_P}{dt} = \frac{1}{\tau_M} = 2 \frac{\Gamma}{L_P} \quad (1)$$

Lindblad torques: 3D results from spirals (Tanaka et al. 2002)

$$\Gamma_L = -(2.34 - 0.1\alpha_\Sigma) \Gamma_0 \quad \text{with} \quad \Gamma_0 = \left(\frac{m_P}{M_*}\right)^2 \left(\frac{H}{r}\right)^{-2} \Sigma_P a_P^4 \Omega_P^2 \quad (2)$$

$$\text{with density slope} \quad \Sigma \propto r^{-\alpha_\Sigma} \quad (3)$$

Time scale:  $1 M_{\text{Earth}}$  at 1 AU:  $10^5$  Years (shorter than growth time)



$$\Gamma_{\text{CR}} \propto \frac{d}{dr} \left( \frac{\Sigma}{B} \right) \quad (4)$$

Where  $B$  is the 2nd Oort constant. Note:  $B/\Sigma$  is specific vorticity

Corotation torques: from horseshoe region (Tanaka et al. 2002)

$$\Gamma_{\text{CR}} = 1.36 \left( \frac{3}{2} - \alpha_{\Sigma} \right) \Gamma_0 \quad (5)$$

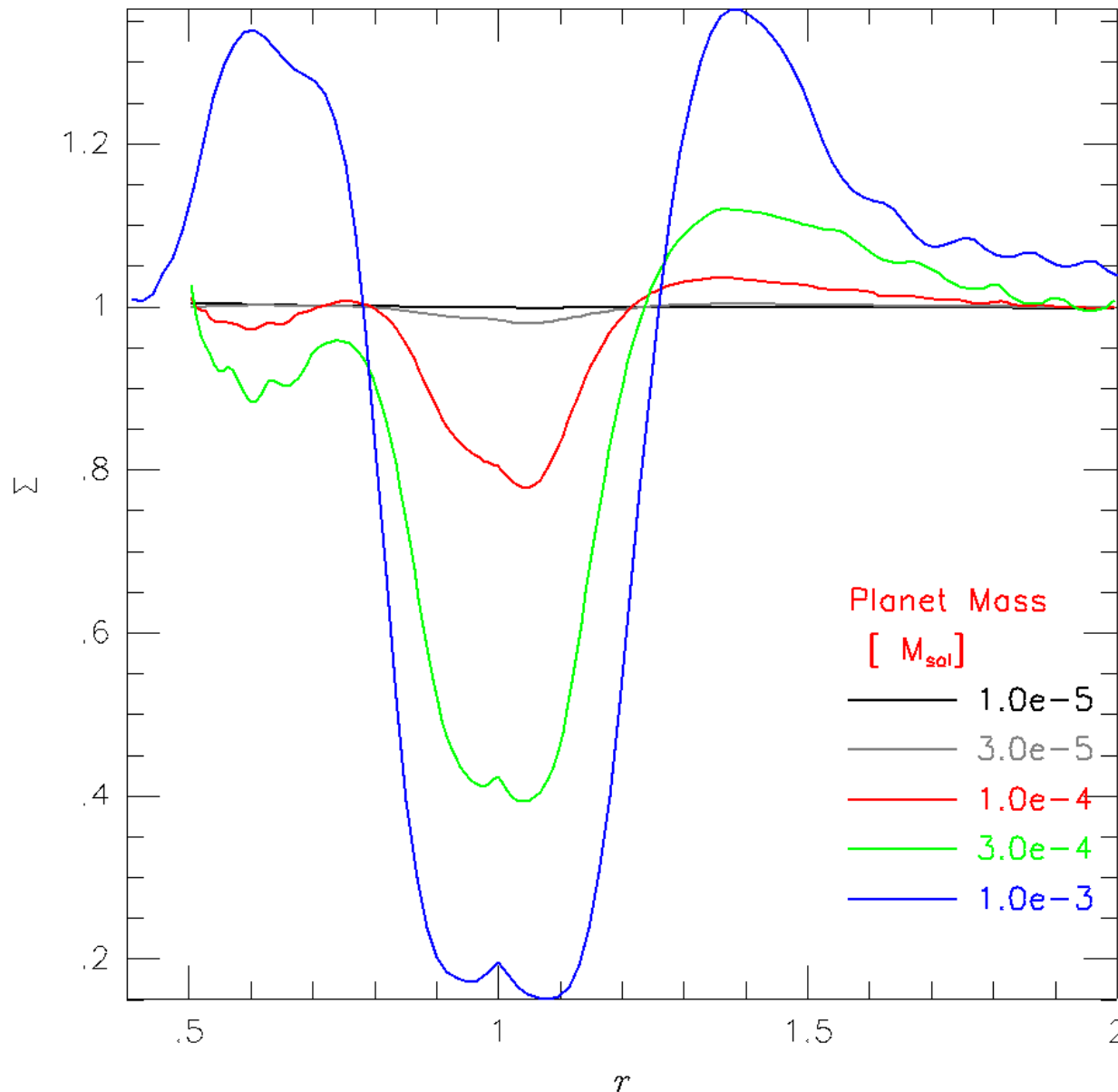
Total torque (spirals and corotation)

$$\Gamma = \Gamma_{\text{L}} + \Gamma_{\text{CR}} \quad (6)$$

Typically:  $|\Gamma_{\text{CR}}| < |\Gamma_{\text{L}}|$  Inward migration

NOTE: In MMSN  $\Sigma \propto r^{-3/2}$ , i.e.  $\alpha_{\Sigma} = 1.5 \implies \Gamma_{\text{CR}} = 0$

In **linear** theory: Migration inward and rapid !



$M_p = 0.01 M_{Jup}$

$M_p = 0.03 M_{Jup}$

$M_p = 0.1 M_{Jup}$

$M_p = 0.3 M_{Jup}$

$M_p = 1.0 M_{Jup}$

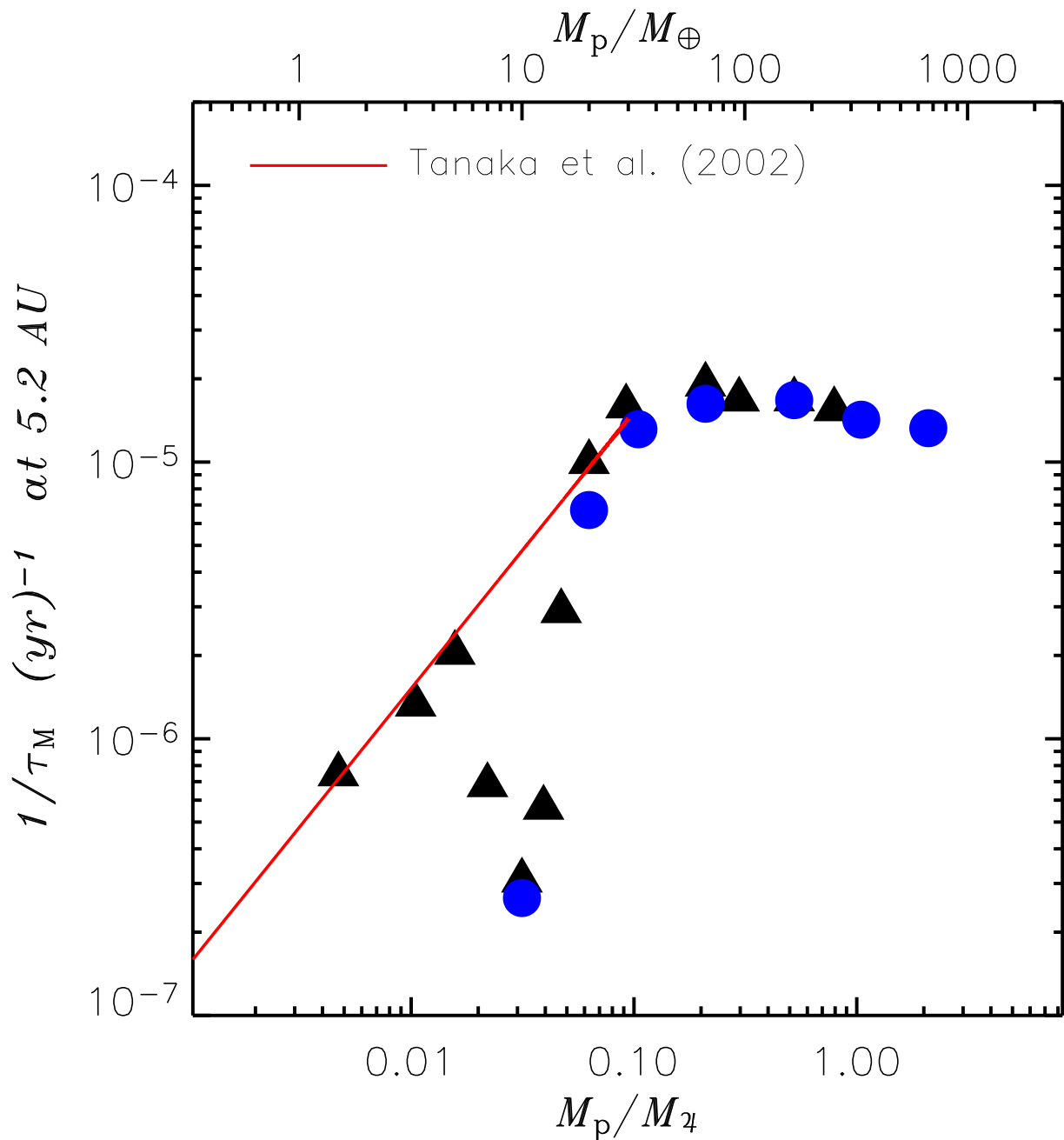
Depth depends on Planet Mass

Torques reduced:

Migration slows

Type I  $\Rightarrow$  Type II

linear  $\Rightarrow$  non-linear



$$\tau_M = \frac{a}{\dot{a}} \quad \text{VS} \quad M_P$$

Symbols:

Full Hydro

3D-Nested Grid

$$\Sigma(r) \propto r^{-1/2}$$

D'Angelo et al. (2003)

Red Line:

Linear Theory

Tanaka et al. (2002)

Dip:

Corotation Effects

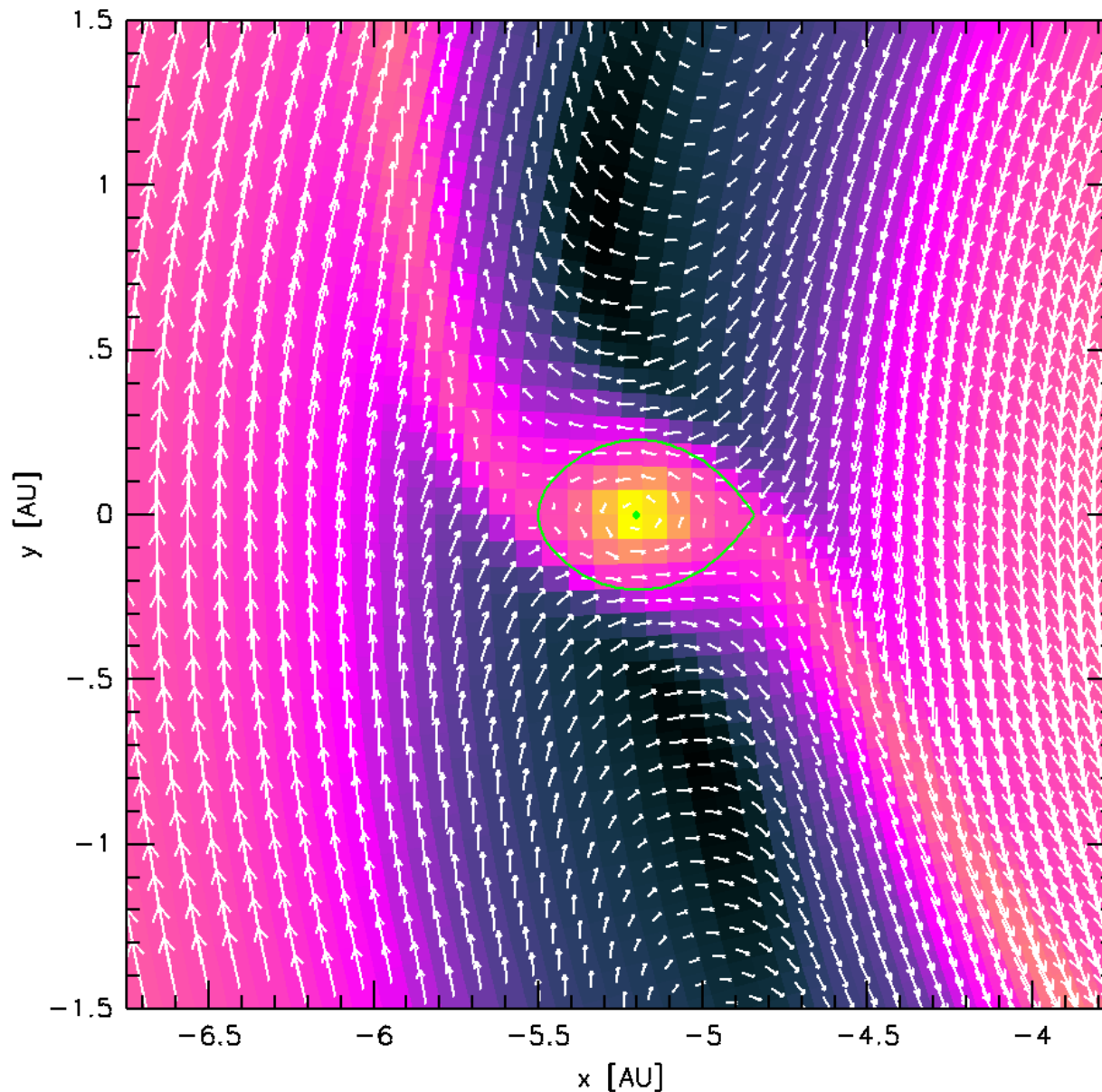
Non-linear for

$$M_p \approx 10M_{Earth}$$

Depend on  $\Sigma(r)$ ,  $H/r$

Analyzed by

(Masset et al. 2006)



Surface Density  
at 200 Orbits

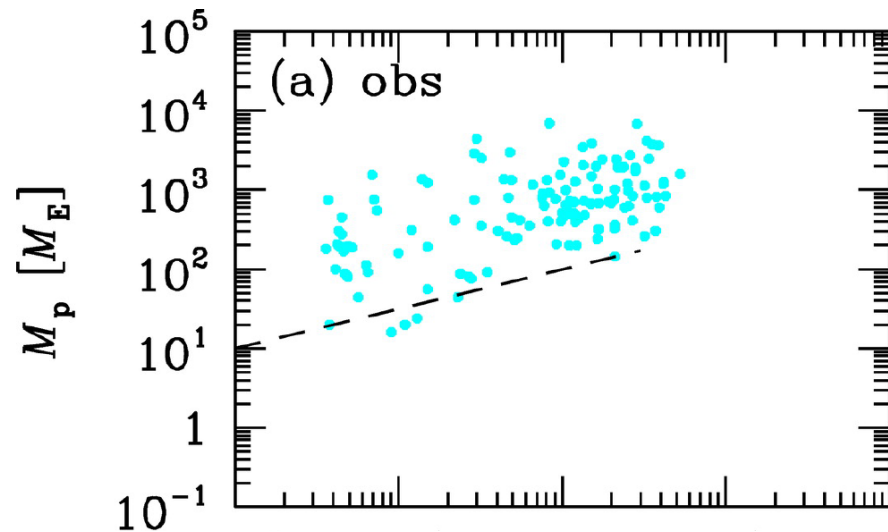
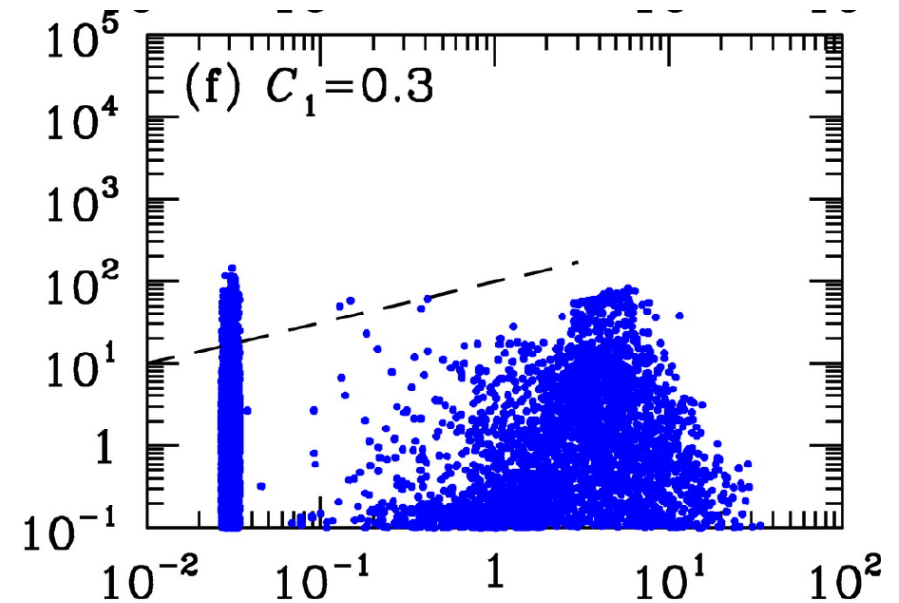
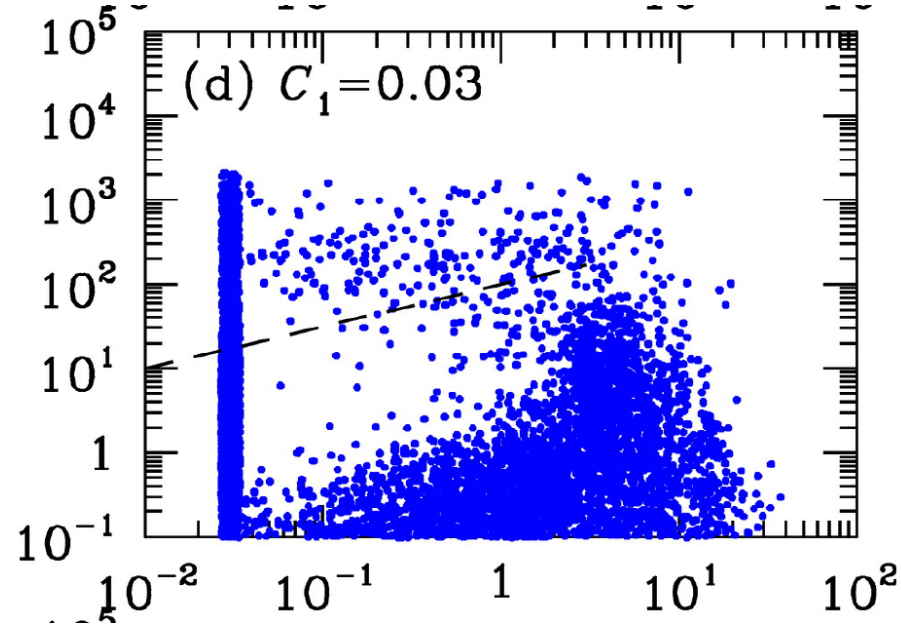
Green Dot: Planet  
Green Line:  
Roche-Lobe

$m_p = 1 M_{Jup}$   
 $a_p = 5.2 AE$

Flow-Field

→ Mass growth  
up to a few  $M_{Jup}$   
→ prograde rotation

(WK, 2000)



Migration too efficient!

Only strong reduction of Type I ( $C_1$ ) gives reasonable results

([Ida & Lin](#); [Mordasini, Alibert & Benz](#))

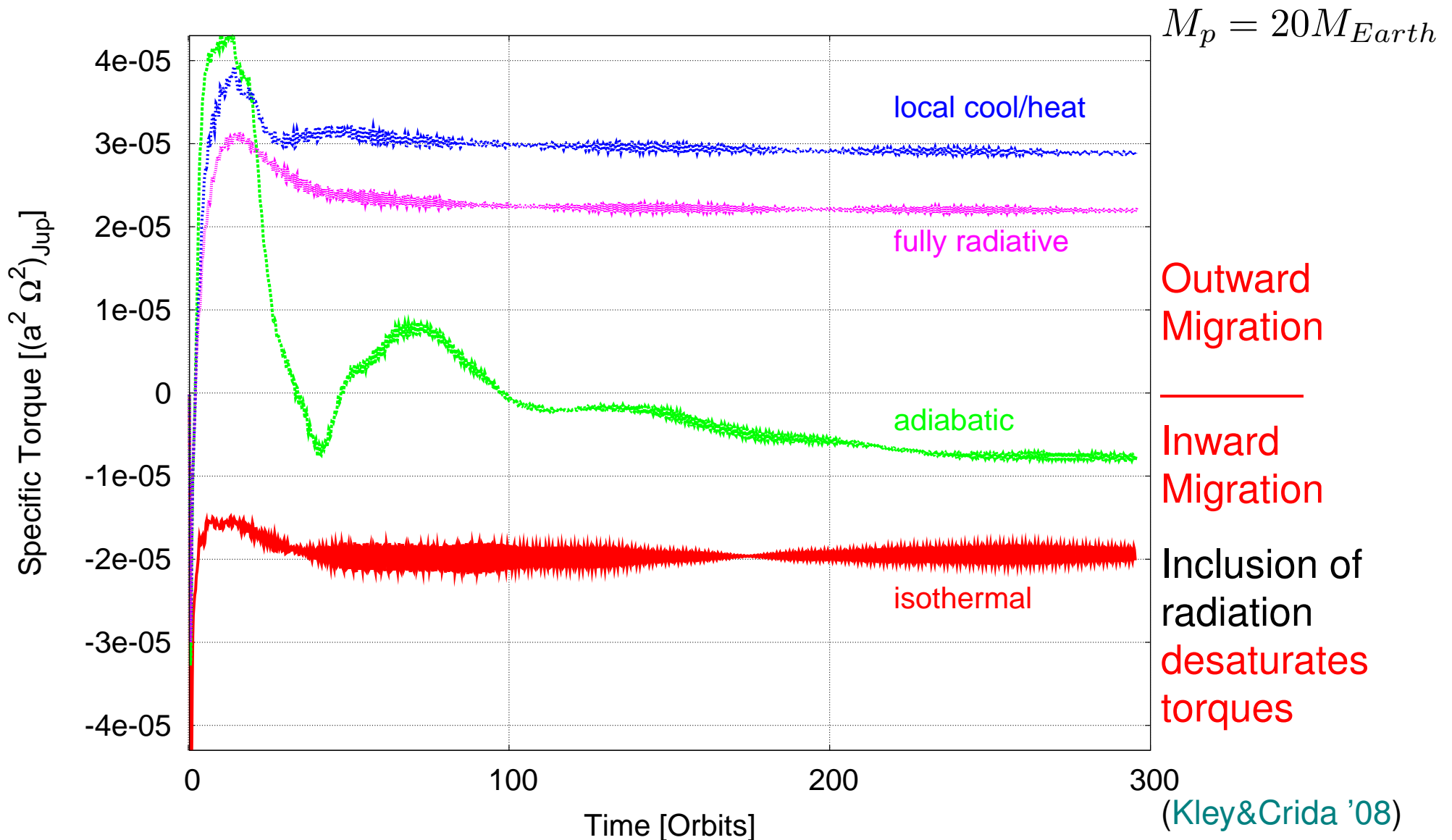
⇒ Need improvements:

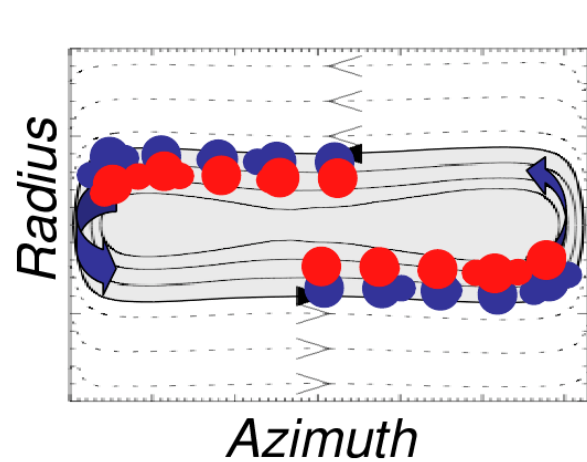
- stochastic migration
- inviscid, self-grav. discs
- here: **radiative disks**



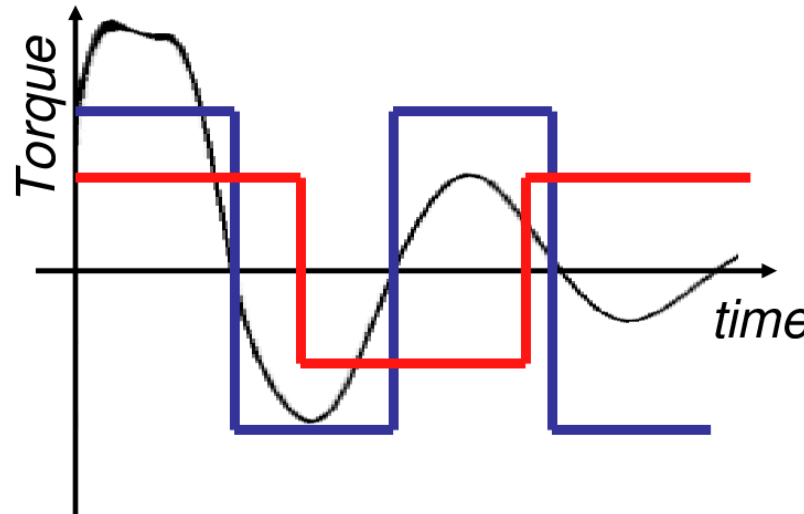


$$\frac{\partial \Sigma c_v T}{\partial t} + \nabla \cdot (\Sigma c_v T \mathbf{u}) = -p \nabla \cdot \mathbf{u} + D - Q - 2H \nabla \cdot \vec{F}$$



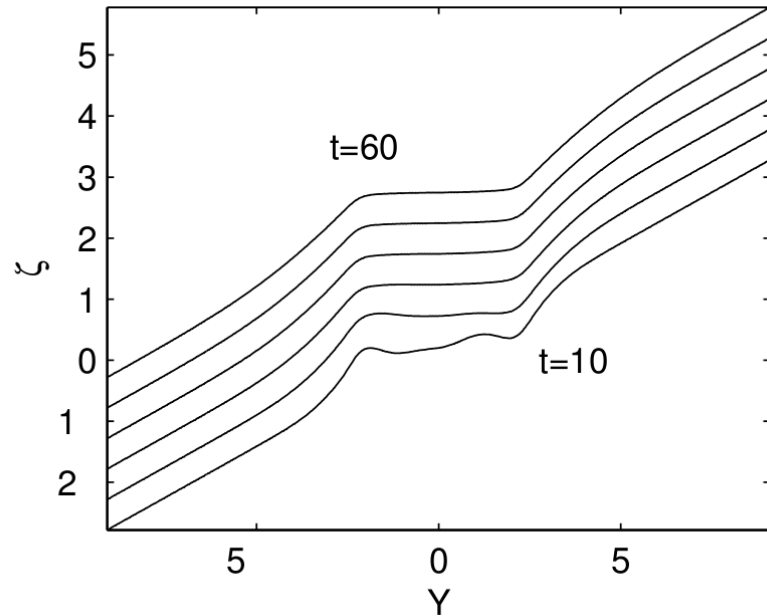


(F. Masset)

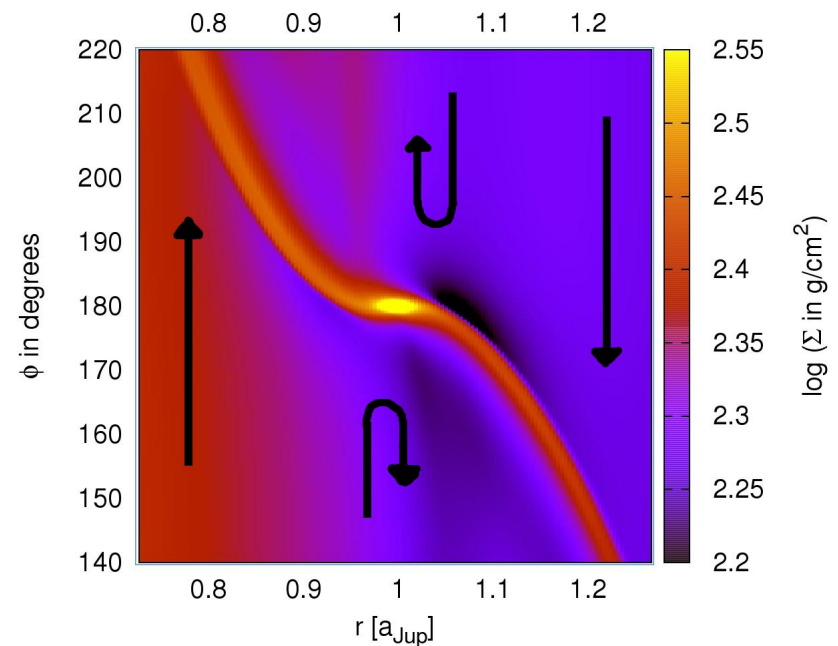


Red and Blue Orbits have different periods => Phase mixing

(a) Mean vorticity



(Balmforth & Korycanski)

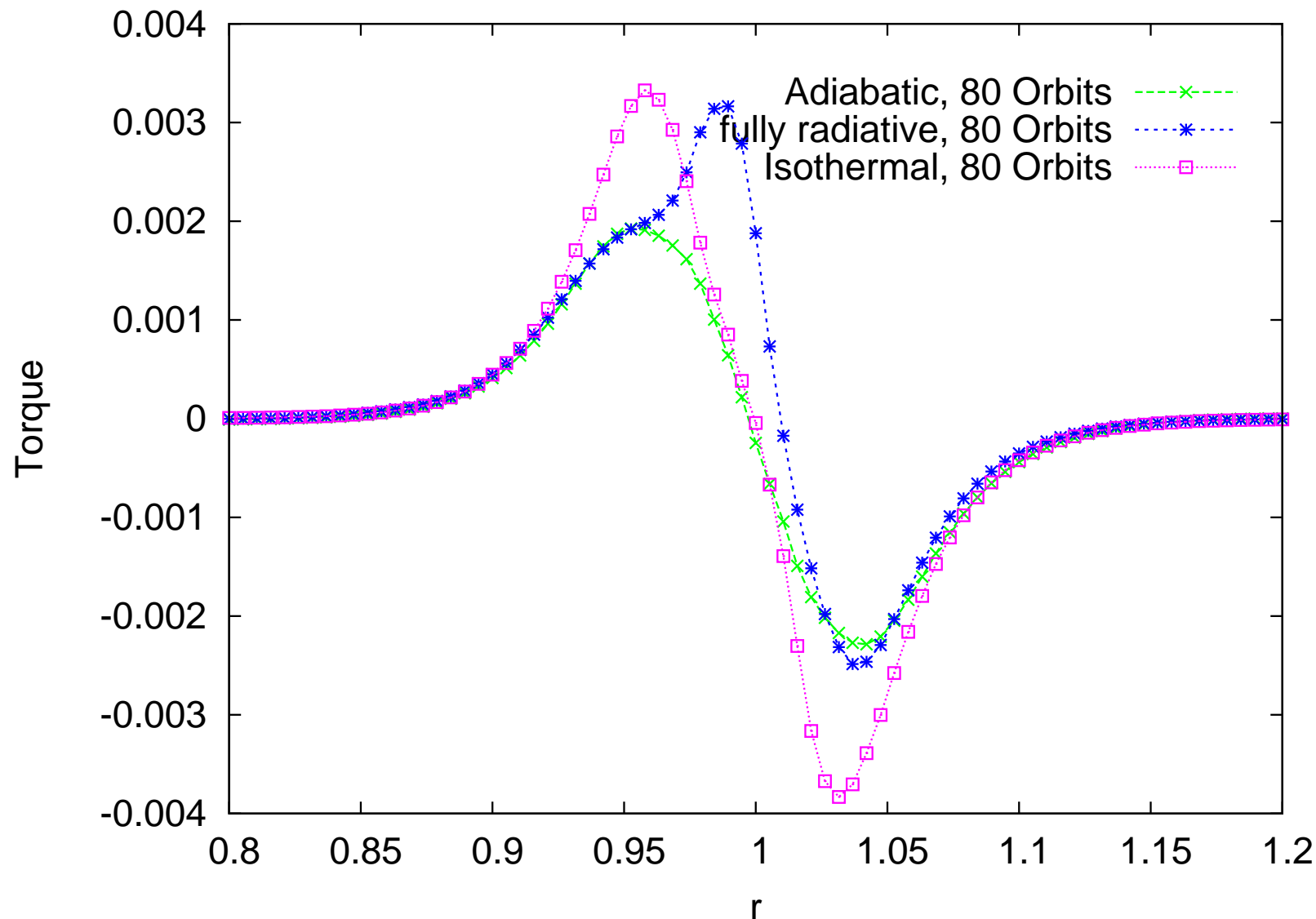


3D radiative disk (B. Bitsch)



3D-simulations, radiative diffusion,  $20 M_{Earth}$  planet (Kley, Bitsch & Klahr '09)

$\Gamma(r)$ , with  $\Gamma_{tot} = \int \Gamma(r) dr$  Radiative:  $\Rightarrow$  additional positive contrib.



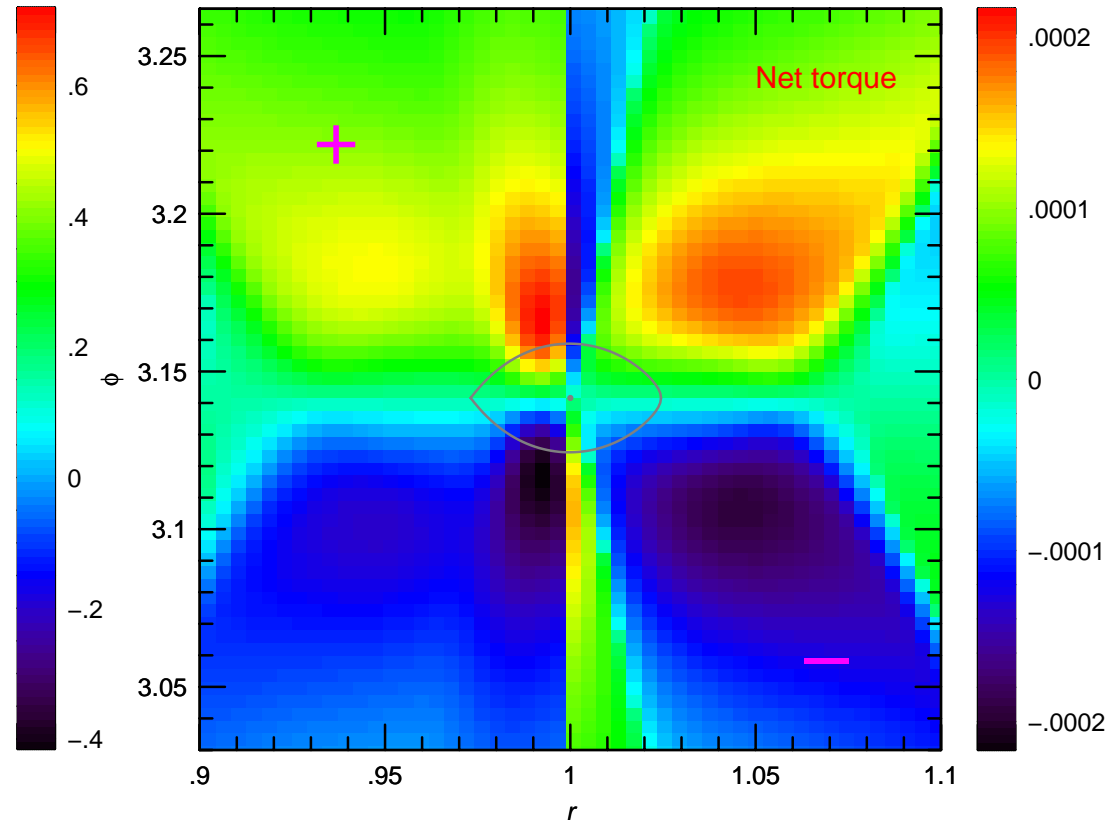
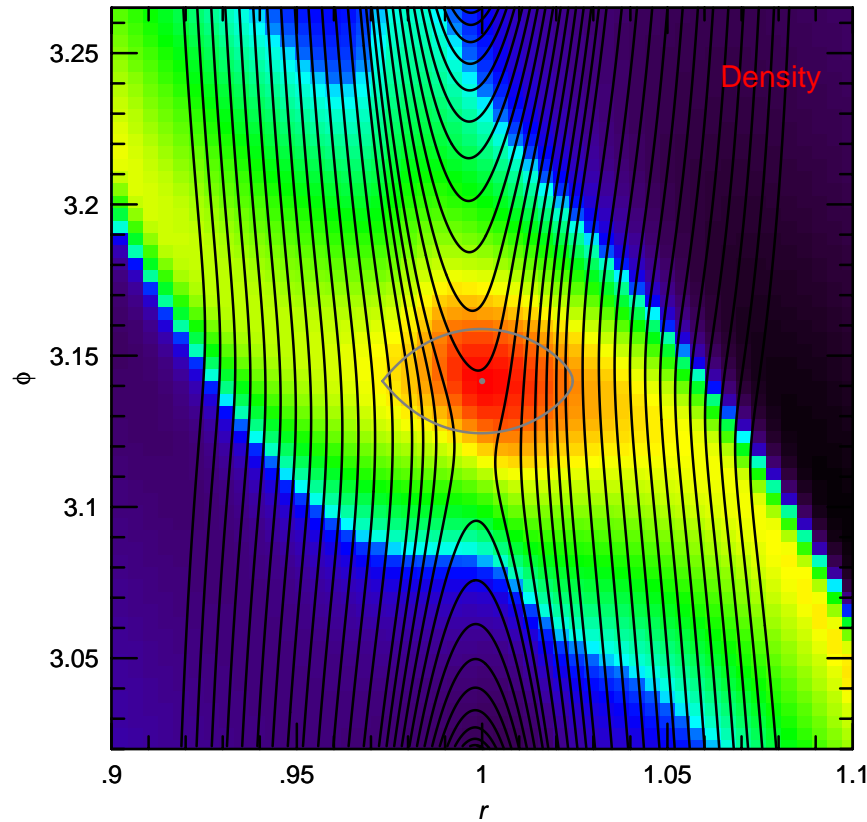


Perturbed Density

$$(\Sigma - \Sigma_0) / \Sigma_0$$

Net Torque contributions

$$\pm (\Gamma(r, \varphi_p + \varphi) - \Gamma(r, \varphi_p - \varphi))$$

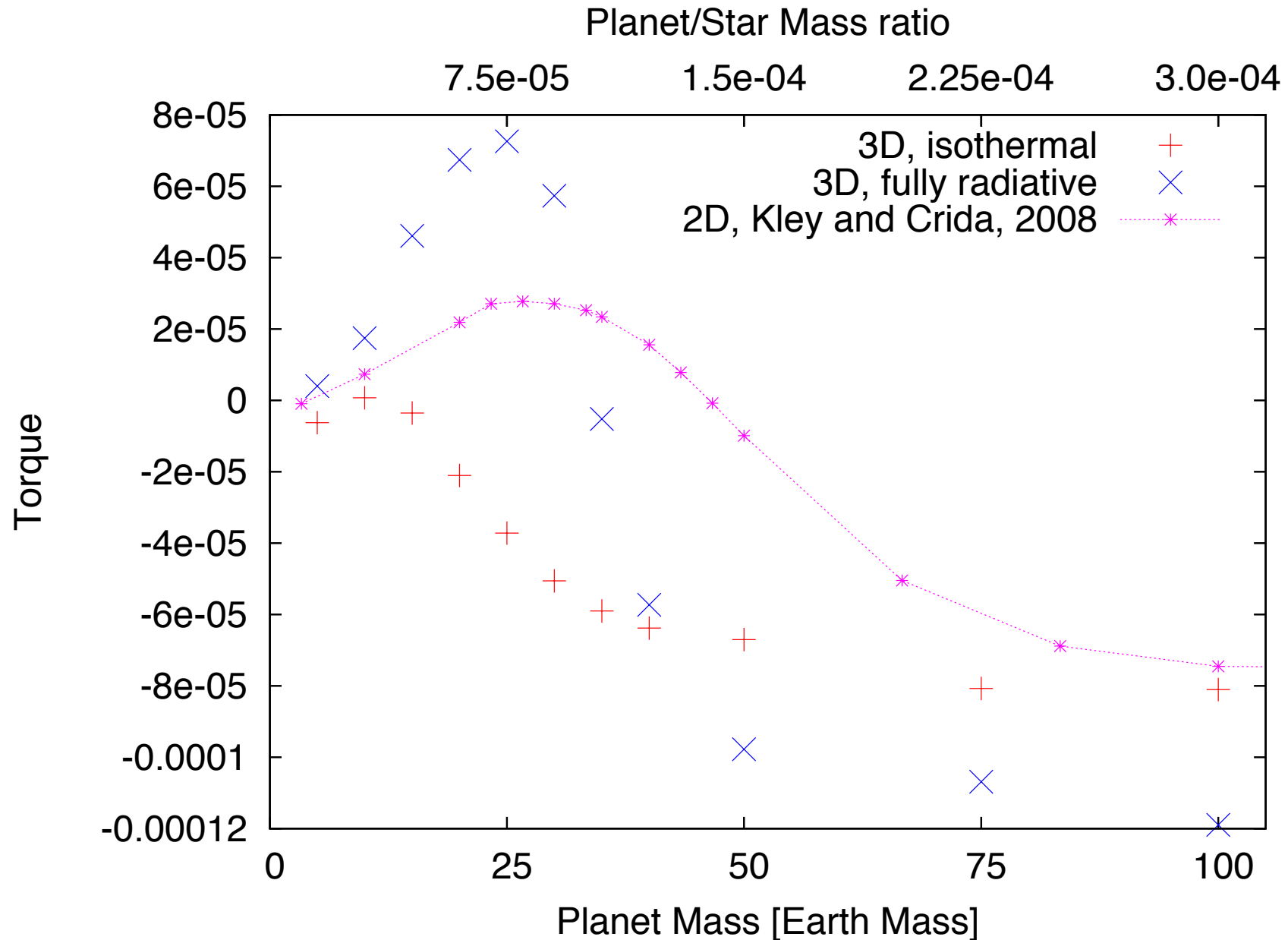


- asymmetric density and torques:      — Trailing      + Leading

⇒ positive torques      (Horseshoe drag: Ward, 1991)

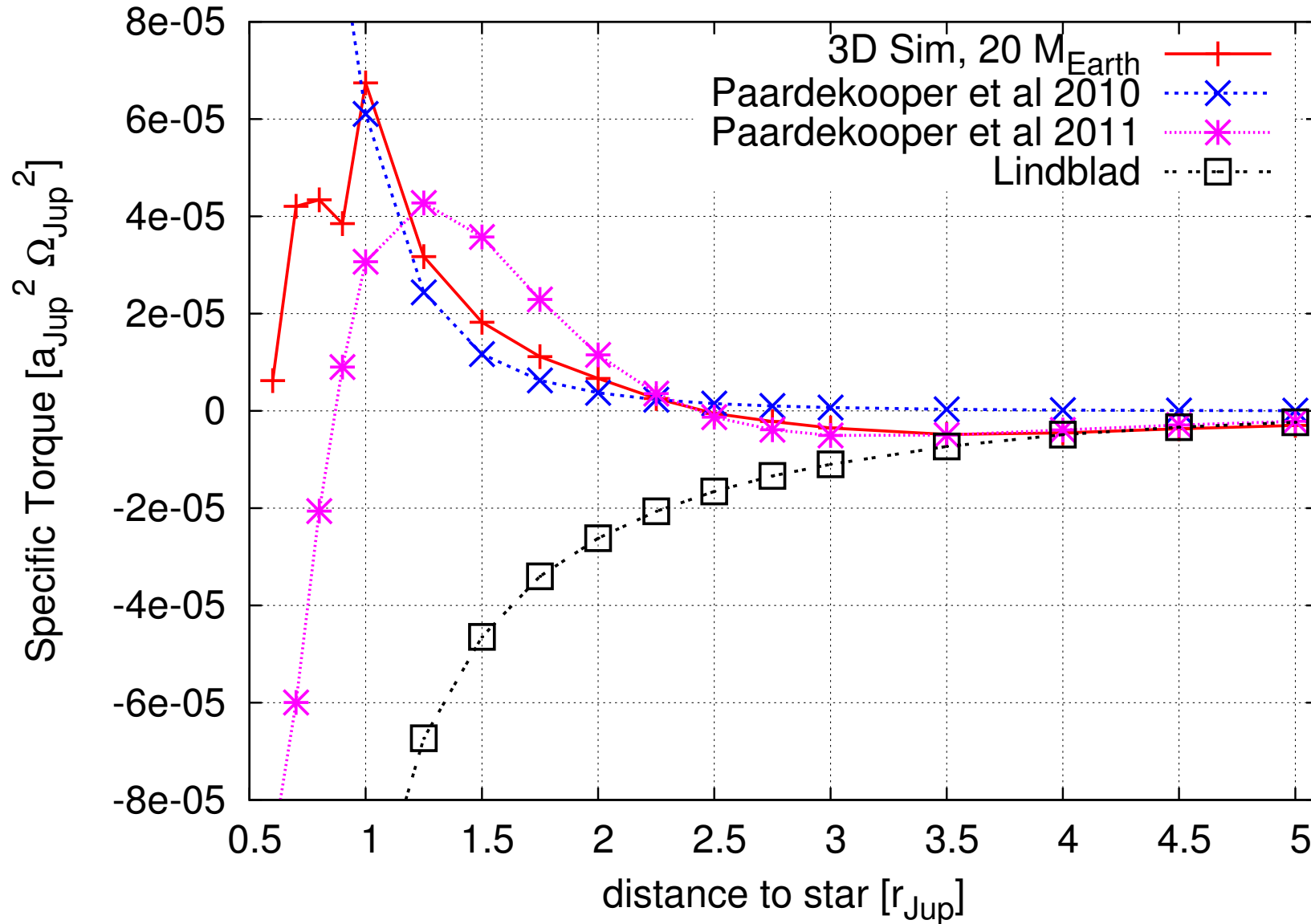


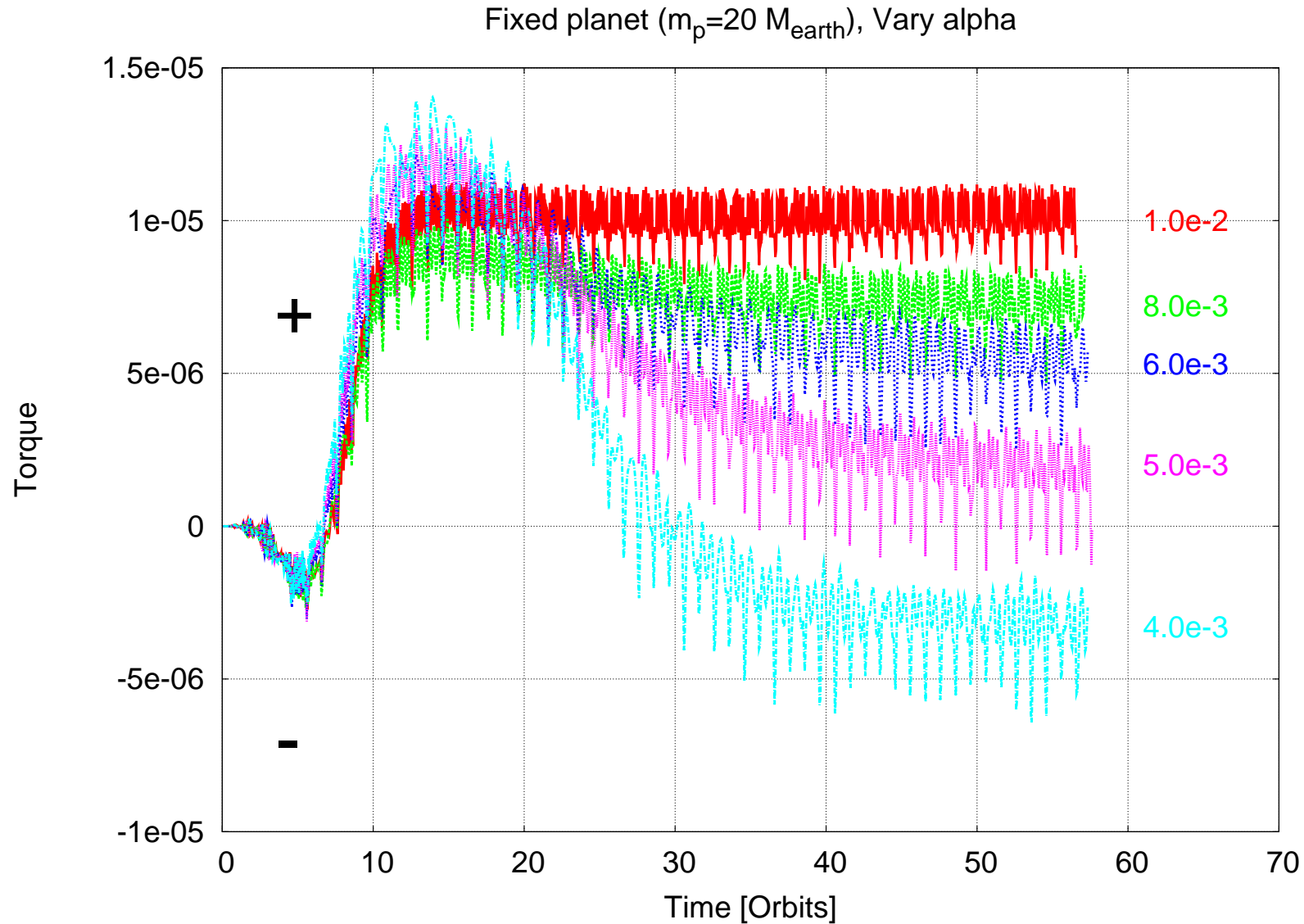
Isothermal and radiative models Outward migration for  $M_p \leq 35M_{Earth}$





Place planets at various distances (Bitsch & Kley, 2011)





Need viscosity (Turbulence) to maintain desaturation



Planet-disk interaction: Torques on Planet

Isothermal Migration is inward & rapid (lose planets)

But:  $\Gamma_{tot} = \Gamma_L + \Gamma_{HS,ent} + \Gamma_{HS,vort}$

Outward in radiative disks

Mass limit due to gap opening

Driven by:

Vortensity gradient

Entropy gradient

maintained by:

- rad. diffusion (or cooling)
- cooling time  $\approx$  libration time

Need viscosity

Approximate torque formula: [Masset, Casoli & Paardekooper ea 2010](#)

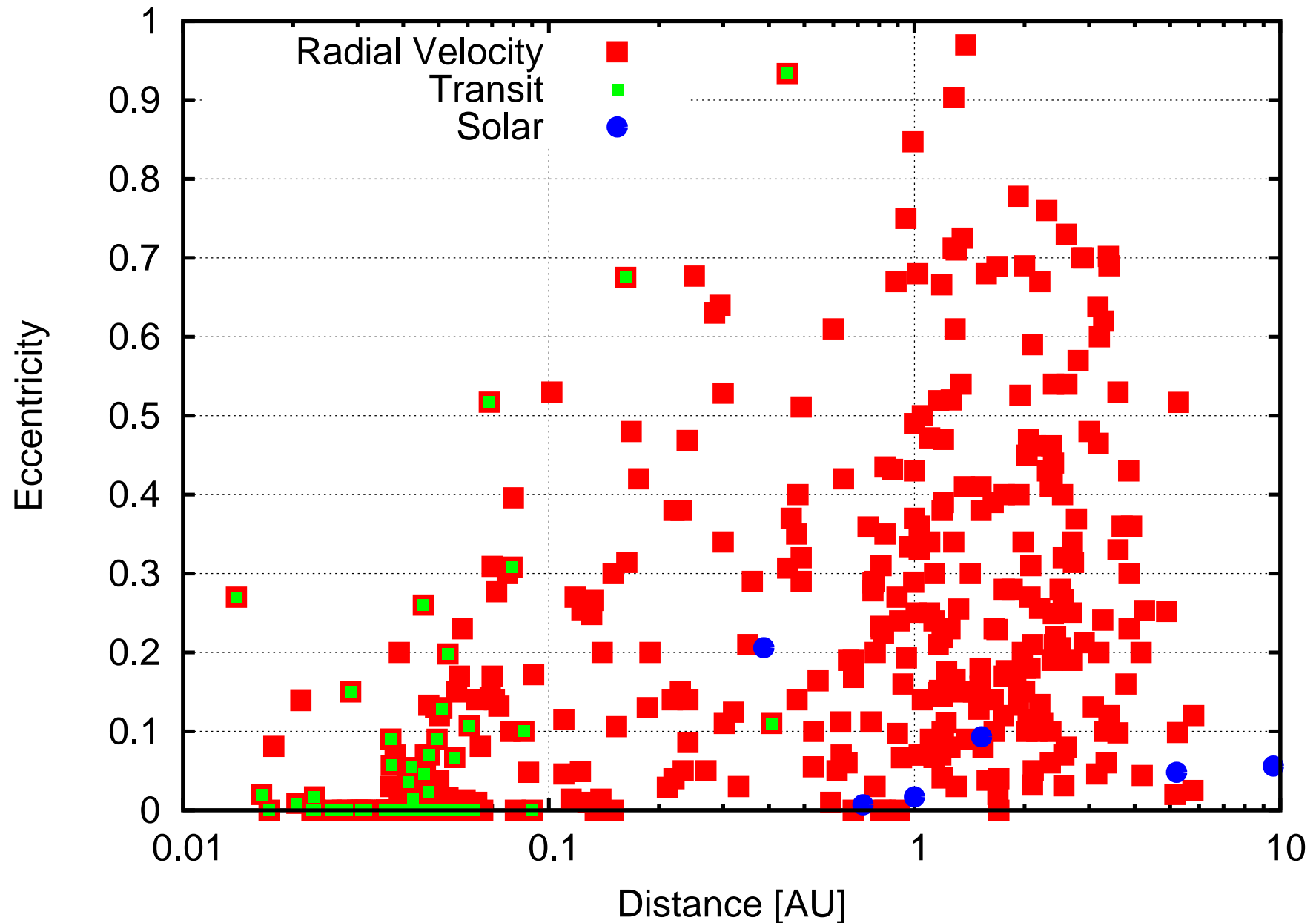
Helps to prevent loss of planets see Talk: Y. Alibert





Large eccentricities (similar to binary stars)

(Data: exoplanet.eu)



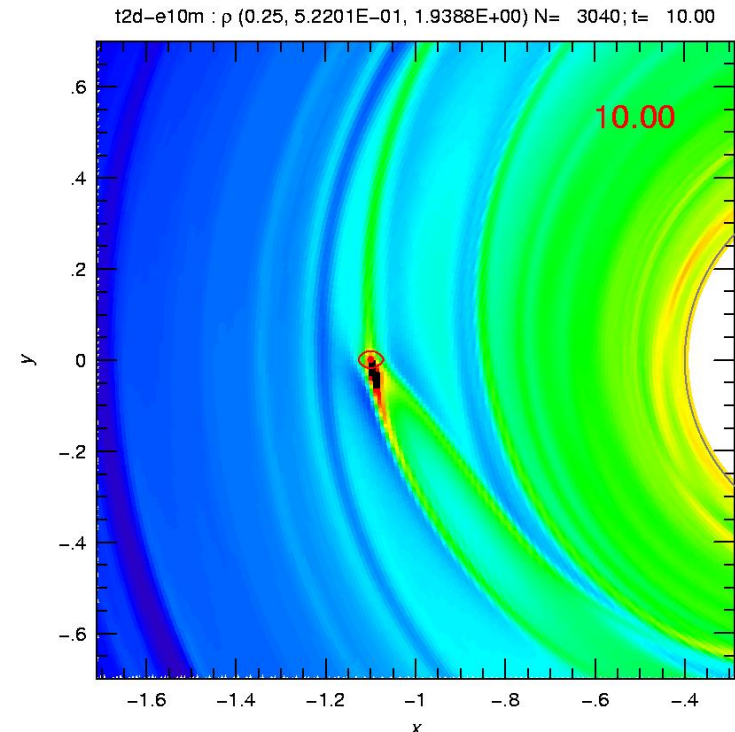
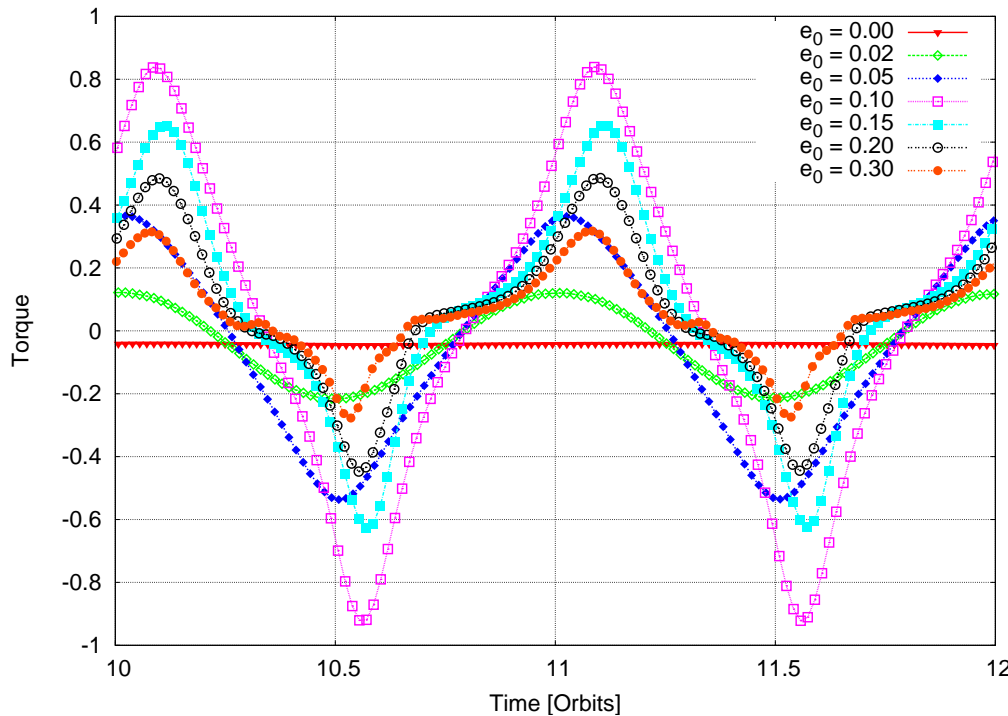


**Torque** on planet due to disk

**Power:** Energy loss of planet

$$\Gamma_{\text{disk}} = \int_{\text{disk}} (\vec{r}_P \times \vec{F}) \Big|_z df$$

$$P_{\text{disk}} = \int_{\text{disk}} \dot{\vec{r}}_P \cdot \vec{F} df$$



$$L_p = m_p \sqrt{GM_* a} \sqrt{1 - e^2}$$

$$\frac{\dot{L}_p}{L_p} = \frac{1 \dot{a}}{2a} - \frac{e^2 \dot{e}}{1 - e^2 e} = \frac{\Gamma_{\text{disk}}}{L_p}$$

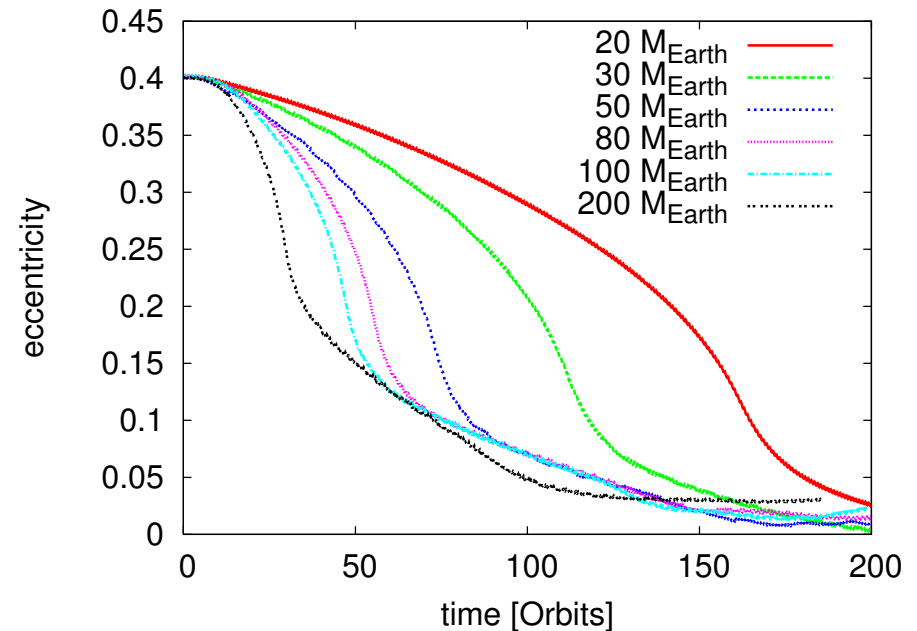
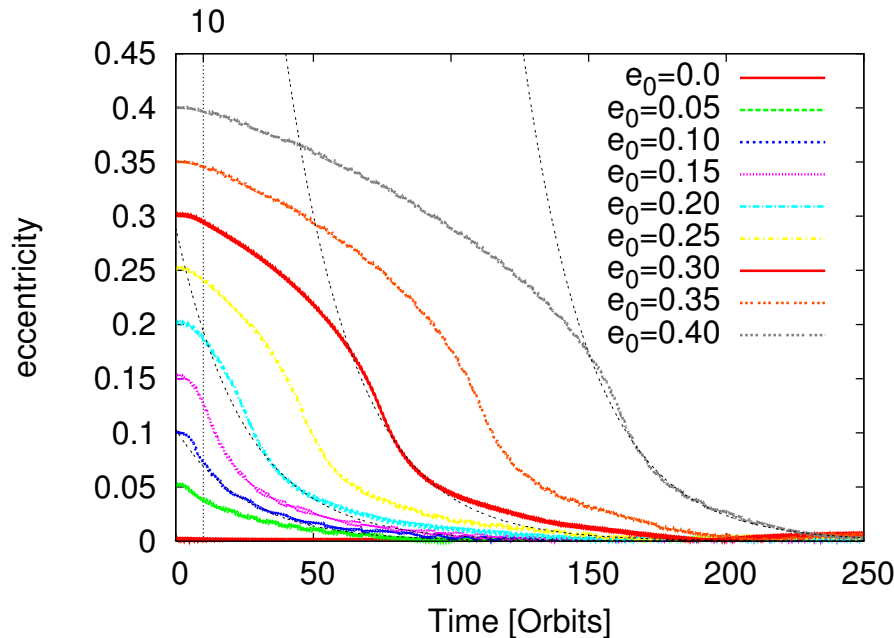
$$E_p = -\frac{1}{2} \frac{GM_* m_p}{a}$$

$$\frac{\dot{E}_p}{E_p} = \frac{\dot{a}}{a} = \frac{P_{\text{disk}}}{E_p}$$



Fix planet mass  $M_p = 20M_{Earth}$   
 - Vary initial Eccentricity

Vary Planet Mass 10 – 200  $M_{Earth}$   
 - Same  $e_0 = 0.40$

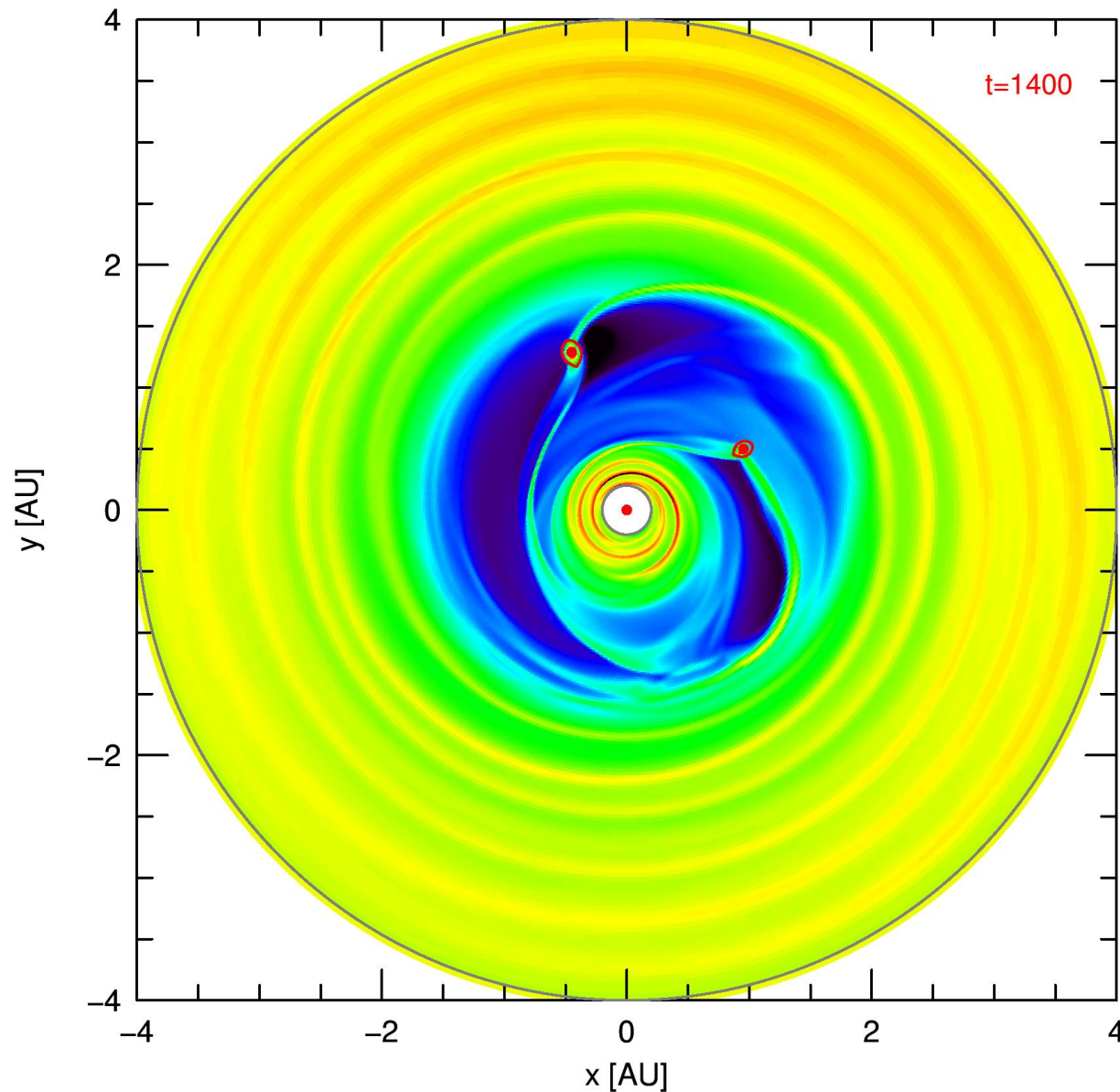


(Bitsch&Kley '10)

- **$e$ -damping for all planet masses.** ( $\implies$  Poster Bertram Bitsch)  
 Small  $e$ : exponential damping, **large  $e$ :  $\dot{e} \propto e^{-2}$**
  - Need  $e < 0.01 - 0.02$  for outward migration to work (radiative disks)
- $\implies$  Need multiple objects ! (and Scattering)



## 2 massive Planets in disk



Two planets:  
joint, large gap

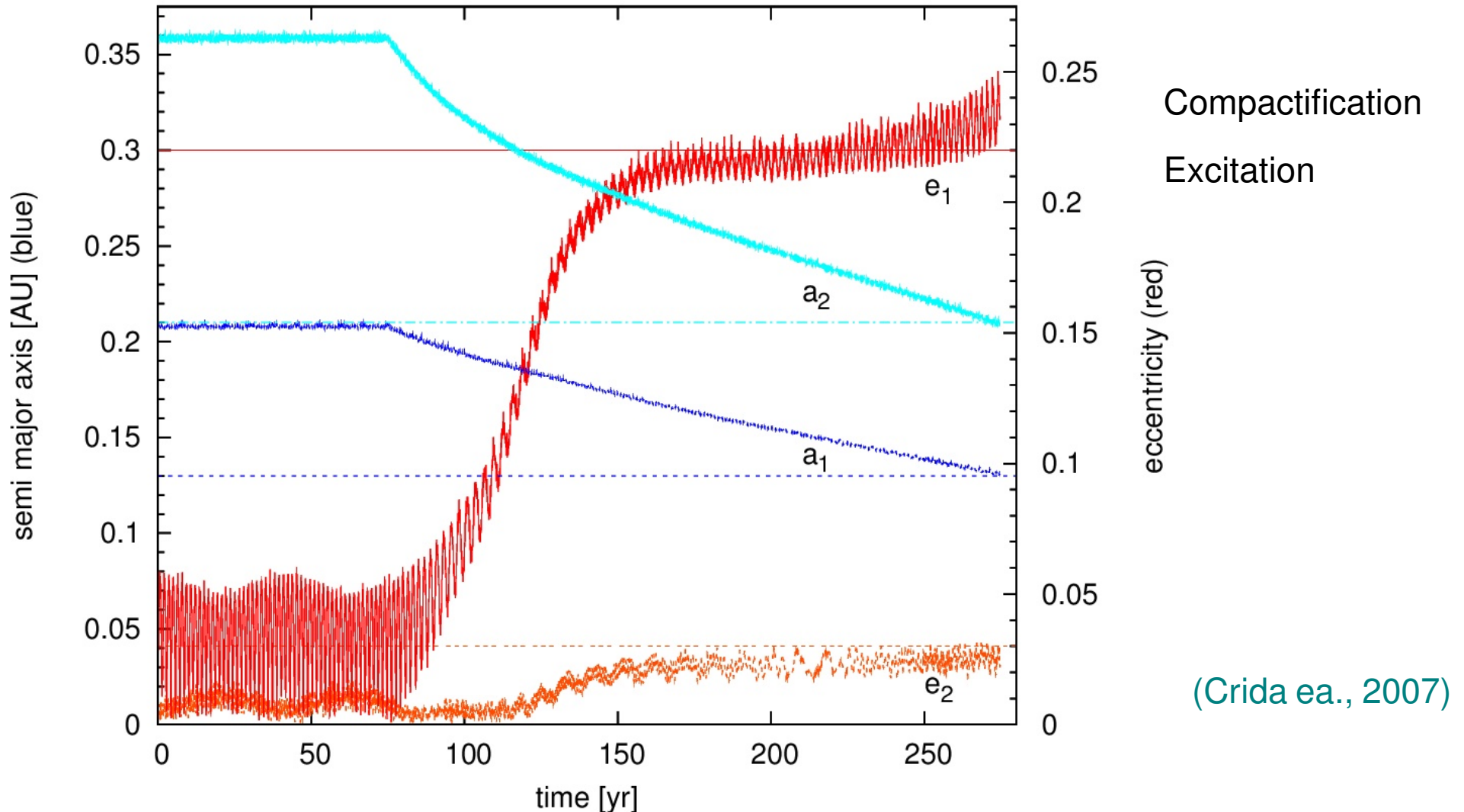
Outer planet :  
Pushed inward

Inner planet :  
Pushed outward

Separation reduction:  
Resonant capture



Here: **System-parameter of GJ 876** (2 planets in 2:1 resonance, 60:30 days)



System ends in: **apsidal corotation**, with **correct eccentricities**

Less disk damping:  $\Rightarrow$  much higher  $e \Rightarrow$  Instability



- Radial Velocity technique
    - HD 45364: system in 3:2 resonance
    - HD 60532: system in 3:1 resonance
    - GJ 876: additional outer planet, 4:2:1 Laplacian resonance ?  
(System with clearest sign of 2:1 resonance)
  - Transit timing
    - Kepler: 5 new multiple planet systems (tbc)  
(3 near resonance,  $2 \times 2:1$ ,  $1 \times 5:2$ )
    - WASP-3b: need outer perturber; near 2:1 or 5:2
    - NN Ser: eclipsing post-common-envelope binary,  $P_{orb} = 3.12\text{hrs}$   
WD & M4 dwarf, 2 planets in 2:1 resonance
  - Direct Imaging
    - HR 8799: 3 planet system at large distances)  
(massive planets: 7, 10, 10  $M_{Jup}$ )  
(at 24, 38, 68 AU; (stable only if in: 4:2:1 resonance )
- About 30% of multi-planet system close to MMR ([Wright ea. 2011](#))

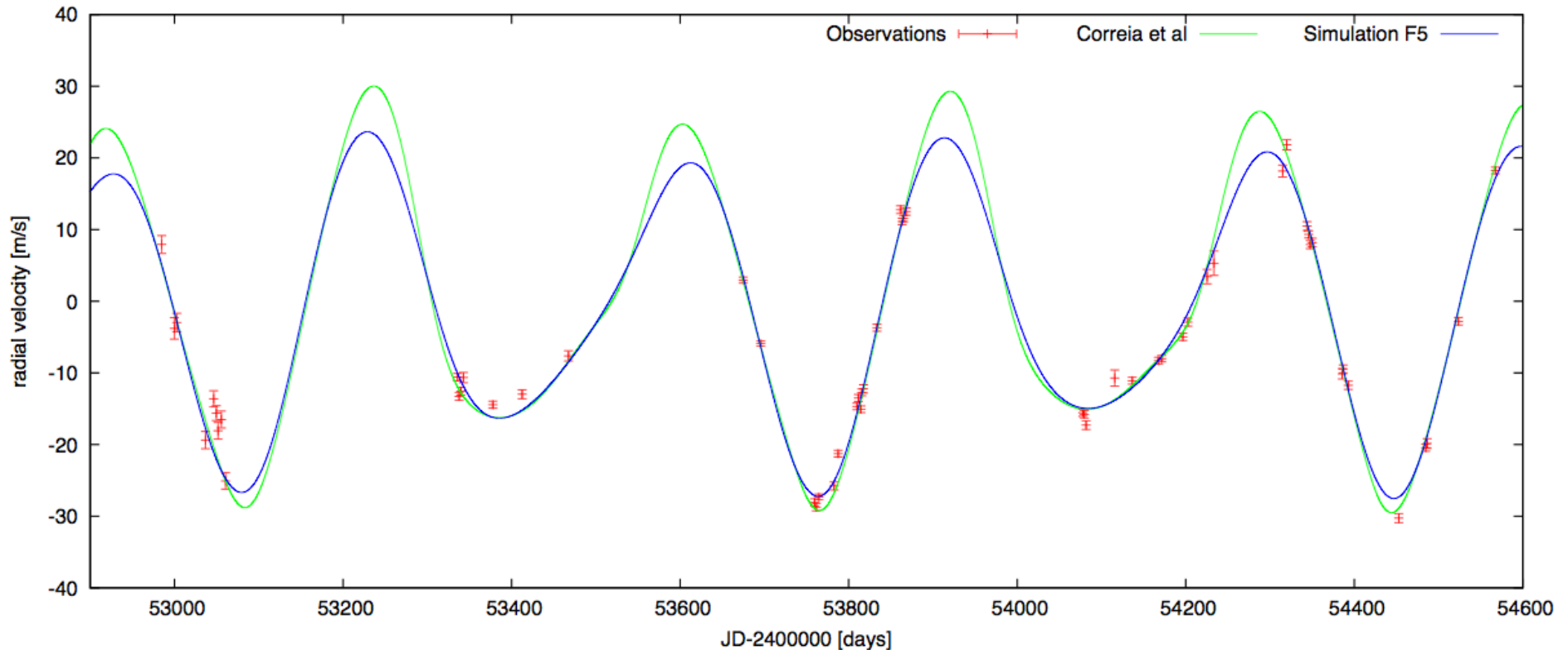




Announced by: [Correia et al. 2009](#)

3:2 Resonance,  $m_1 = 0.19, m_2 = 0.69M_J$ , at 0.68, 0.89 AU

System formation through full 2D hydrodyn. simulations!



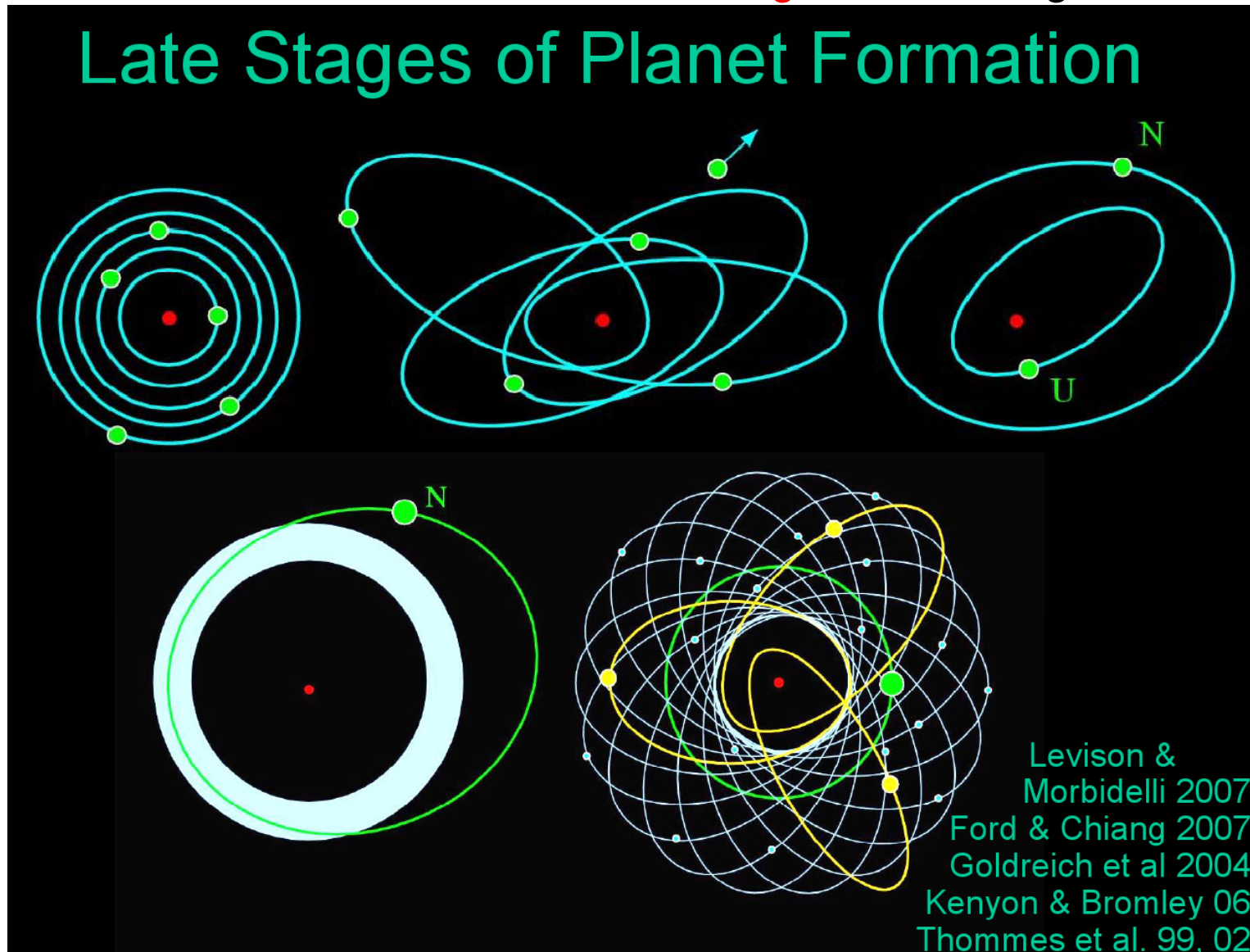
Observed  $e$ : 0.17, 0.097, Simulation: 0.036, 0.017, same  $\chi^2$

(Rein, Papaloizou, Kley, 2010)

For clarification: [More observations](#) !



Gravitational interaction after disk has gone!  $\Rightarrow$  High eccentricities



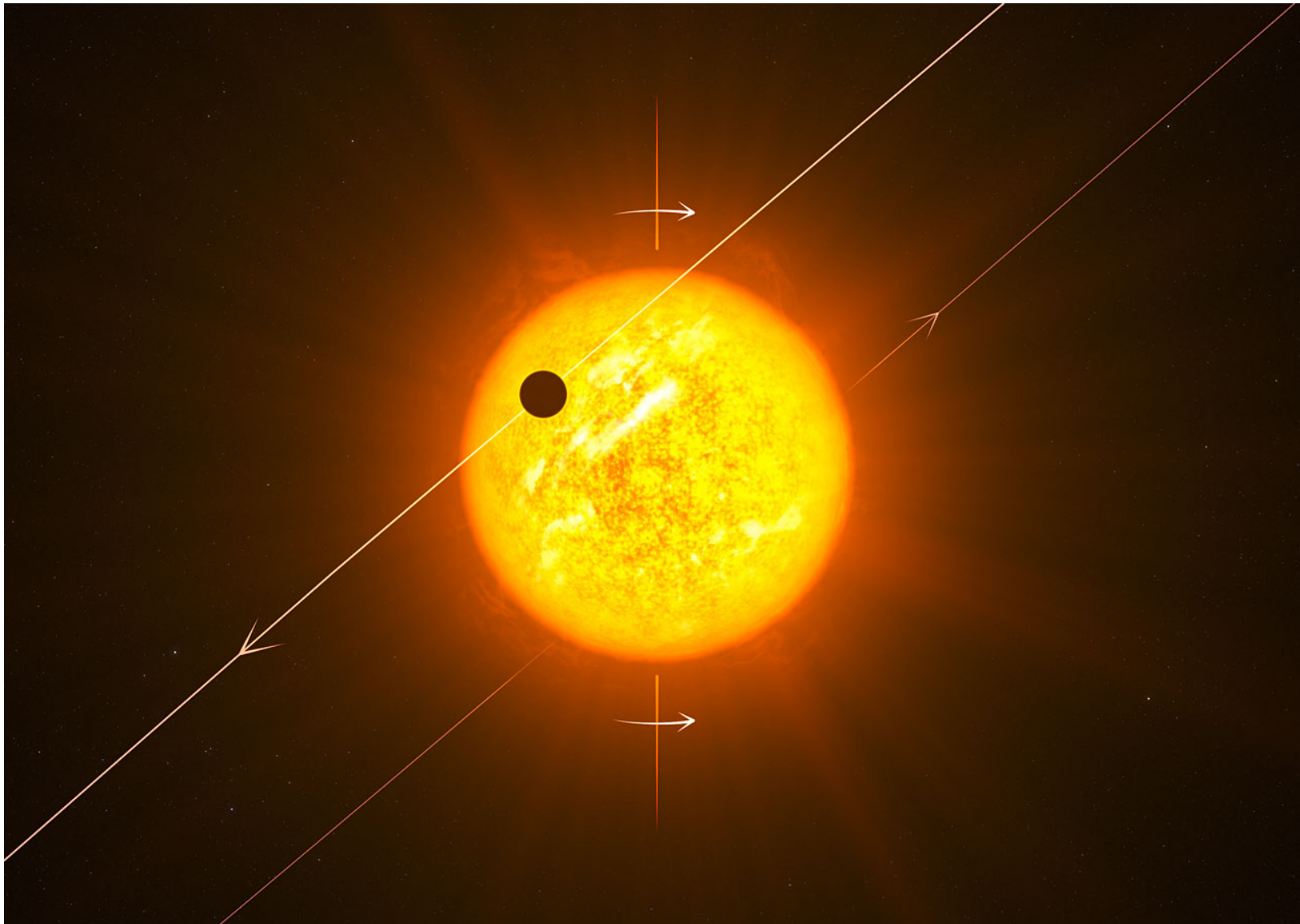
(Slide from Eric Ford)





ESO press release 13. April, 2010

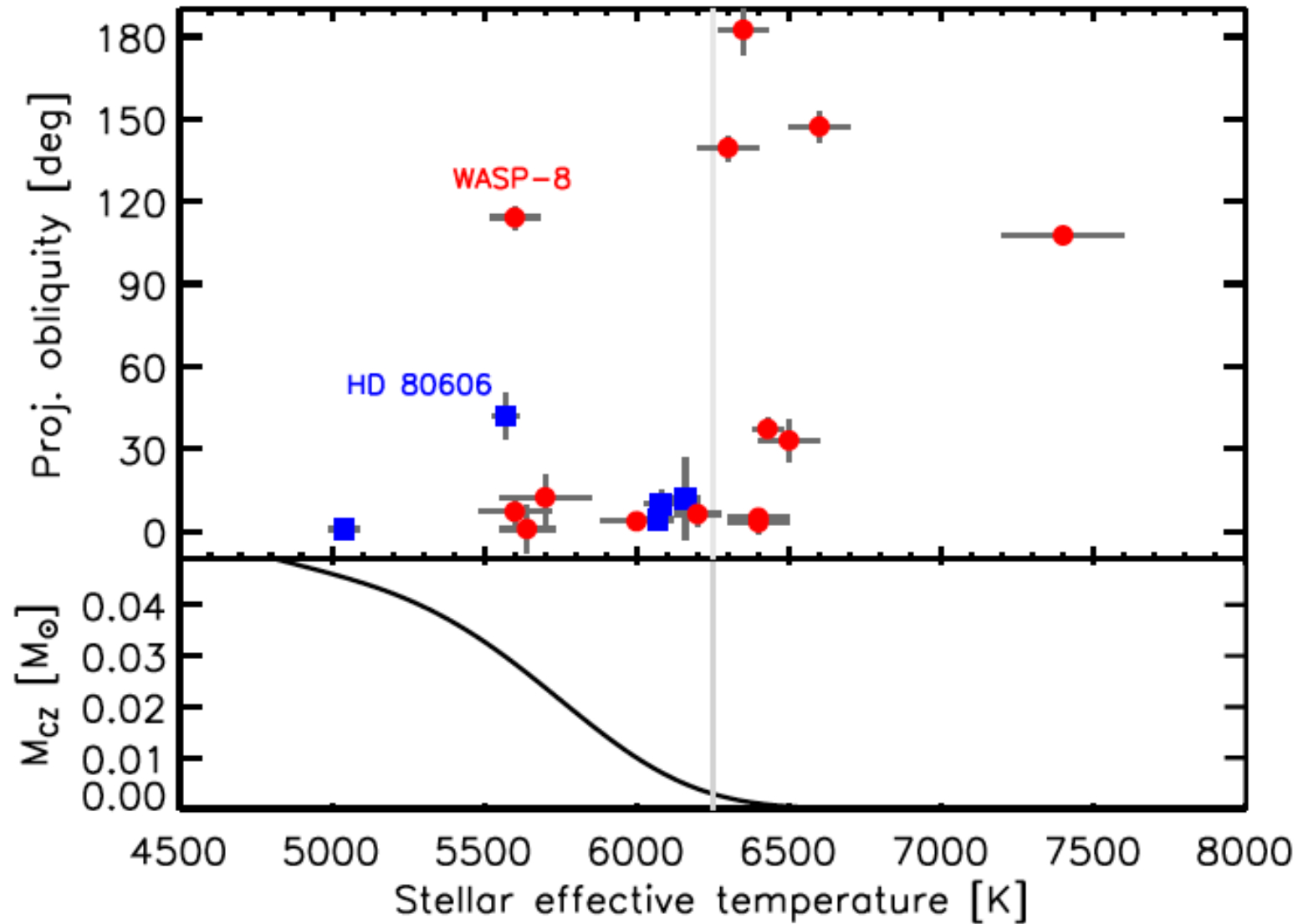
“Misalignment of planetary orbit and stellar rotation” (Triaud et al. 2010)





Sky projected angle

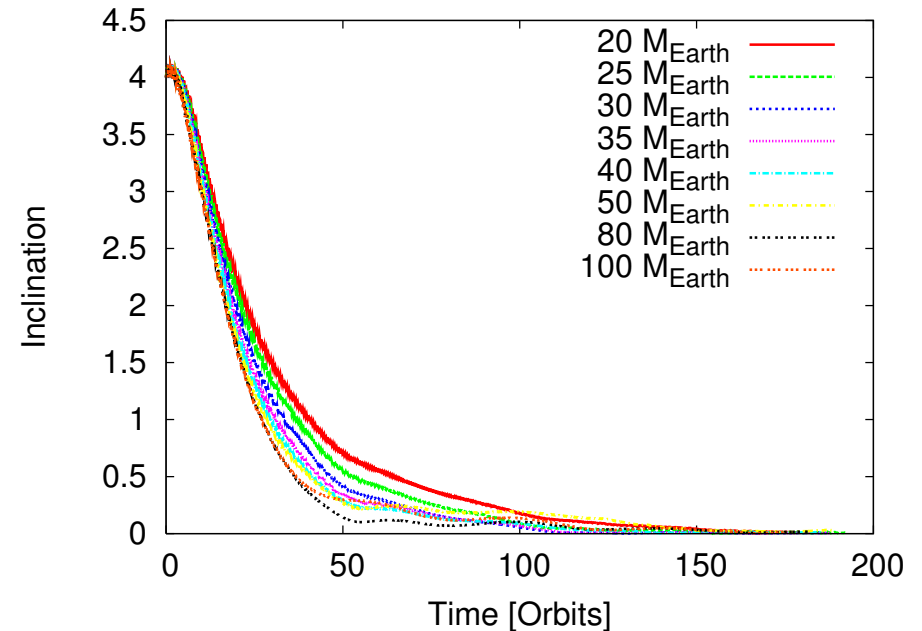
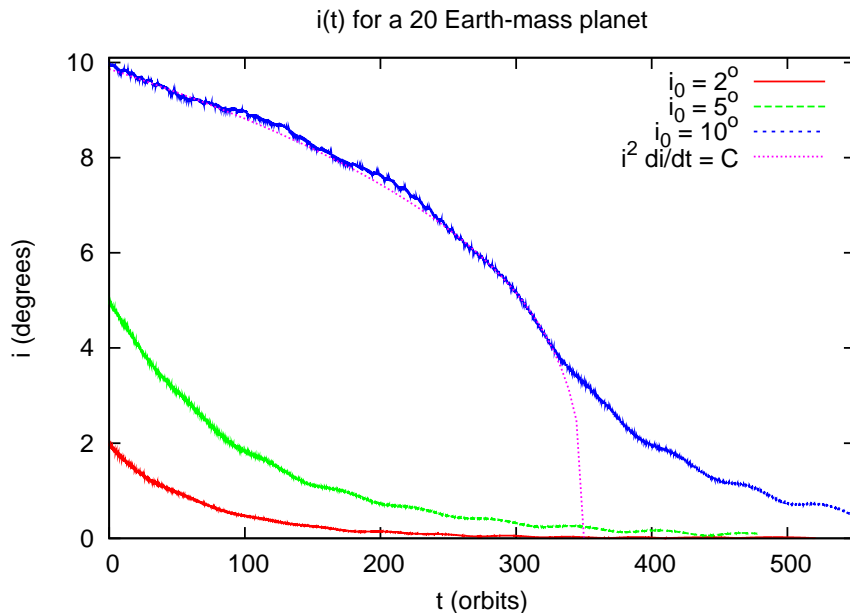
(Winn et al., 2010)





Fix planet mass  $M_p = 20M_{Earth}$   
 - Vary initial Inclination

Vary Planet Mass 20 – 100  $M_{Earth}$   
 - Same  $i_0 = 4deg$



(Cresswell et al 2007; Bitsch&Kley 2011)

- **$i$ -damping for all planet masses.** ( $\implies$  Poster Bertram Bitsch)  
 Small  $i$ : exponential damping, **large  $i$ :  $i \propto i^{-2}$**
- Migration still outward upto  $i \approx 4^\circ$   
 $\implies$  Need multiple objects ! (Scattering)



- 
- Planet-disk interaction moves planets
    - Inward for isothermal disks
    - + possibly outward/slowed in **radiative disks**
    - for small planets, small eccentricities, opacities
    - + helps to avoid too rapid type I (see Pop.synthesis)
  - Eccentricity & Inclination damped by disk
  - Resonant migration
    - + explain resonant planets
    - + supplies initial conditions for scattering
  - Eccentric & inclined planets through scattering
    - Obliquity vs. stellar mass



Thank you for your attention !

(A. Crida)