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An improved mass-loss description for dust-driven superwinds and tip-AGB evolution models

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Abstract. We derive an improved description of dust-driven stellar mass-loss for the cool winds of carbon-rich tip-AGB stars. We use pulsating wind models in which the mass loss is driven by radiation pressure on dust grains, for C-rich chemistry. From a larger set of these models, selected for representative dynamical (pulsational velocity amplitude Δv, period P) and chemical (the $\epsilon_{\rm C}/\epsilon_{\rm O}$ abundance ratio) input parameters, an improved approximative mass-loss formula has been derived which depends only on the stellar parameters (effective temperature $T_{\rm eff}$, luminosity L and mass M). Due to the detailed consideration of the chemistry and the physics of the dust nucleation and growth processes, there is a particularly strong dependence of the mass-loss rate $\dot{M}$ (in $M_\odot$/yr) on $T_{\rm eff}$: $\log \dot{M} = 8.86 - 1.95 \cdot \log M/M_\odot - 6.81 \cdot \log T/{\rm K} + 2.47 \cdot \log L/L_\odot$. The dependence of the model mass-loss on the pulsational period has explicitly been accounted for in connection with the luminosity dependence, by applying an observed period–luminosity relation for C-rich Miras. We also apply the improved mass-loss description to our evolution models, and we revisit their tip-AGB mass-loss histories and the total masses lost, in comparison to our earlier work with a preliminary mass-loss description. While there is virtually no difference for the models in the lower mass range of consideration ($M_{\rm i} = 1.0$ to $\approx 1.3\,M_\odot$), we now find more realistic, larger superwind mass-loss rates for larger stellar masses: i.e., $\dot{M}$ between $\approx$0.4 and 1.0×10^{-4} $M_\odot$/yr for $M_{\rm i}$ between 1.85 and 2.65 $M_\odot$, removing between 0.6 and 1.2 $M_\odot$, respectively, during the final 30 000 yrs on the tip-AGB.

Key words. stars: carbon – stars: circumstellar matter – stars: evolution – stars: interiors – stars: late-type – stars: mass loss

1. Introduction

Strong stellar mass-loss by dust-driven "superwinds" (Renzini 1981) on the tip-AGB has long been identified as a major mechanism to recycle stellar material and to drive the chemical evolution of the galaxy. This applies to stars which do not undergo a supernova explosion ($M_* \lesssim 8\,M_\odot$) but, instead, lose several tenths to over one solar mass in a strong "cool" stellar wind during their final $\approx$30 000 years (Peimbert 1981; Kwok 1981) to form a planetary nebula (PN). During this brief phase of its evolution, the star becomes a very cool, strongly dust-enshrouded, long-period variable (LPV) object, and it exhibits mass-loss rates of the order of 10^{-4} $M_\odot$/yr, as derived from observations and models of LPVs (e.g., Knapp & Morris 1985; Winters et al. 2000). Because of the interpretational difficulties encountered with LPVs, there has long been a considerable interest in a theoretical description of tip-AGB mass-loss from both, an atmospheric and an evolutionary point of view.

Early descriptions of dust-driven mass-loss used for evolution models (Bowen 1988; Vassiliadis & Wood 1993; Blöcker 1995) did not take into account the highly temperature dependent chemistry of the dust-formation process, but considered the main energy input to be related to the pulsations of LPVs, rather then being dependent on the interaction of the radiation field with the freshly formed dust grains in the wind acceleration region. For this reason, the derived dependence of the mass-loss rate on the actual stellar parameters, especially on the effective temperature $T_{\rm eff}$, was in need of considerable improvement. Nevertheless, the magnitude of the total mass lost on the tip-AGB by those models did already match the

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evolutionary context, i.e., the final-mass/initial-mass relation as derived by Weidemann (1987, 1997), as that was just a matter of adopting the right scaling of the mass-loss description (Groenewegen et al. 1995).

A much more sophisticated approach to dust-driven mass-loss has been developed over the past 10 years, in terms of pulsating, hydrodynamic and self-consistent numerical models, which describe in detail the complex physics of a dust-driven wind and the dust-formation process around a cool AGB star (Fleischer et al. 1992; Sedlmayr 1994; Sedlmayr & Winters 1997), for a given set of stellar parameters and carbon-rich chemistry. Mass-loss rates are then obtained by averaging the outflow of the time-dependent models.

Since individual wind models are very computer time intensive, a first attempt was made to describe the model mass-loss in terms of the stellar and other input parameters by Arndt et al. (1997), using a multi-dimensional, maximum-likelihood fitting procedure on the set of C-rich model mass-loss rates available at that time. By averaging over the existing range of non-stellar (dynamical and chemical) input parameters, a mass-loss description was obtained which depends on the principle stellar parameters, $T_{\rm eff}$, L and M, only. That description differs remarkably from other approaches in that it is very sensitive to the effective temperature of the star. In fact, the characteristics of the macroscopic system is strongly influenced by the temperature-sensitivity of the microscopic dust-formation process.

In using the mass-loss description derived by Arndt et al. (1997) with tip-AGB evolution models, Schröder et al. (1999) found good agreement with observed properties of "superwinds", and even with some observed short bursts of strong mass-loss, suggesting that the gross picture was about right. Meanwhile, however, there has been progress on several points which has created a need for revising the mass-loss description of Arndt et al. (1997):

- many more wind models have been computed in the meantime;
- minimum requirements for the radiative acceleration have been identified (Schröder et al. 1999; Winters et al. 2000) and under-critical wind models should not be considered;
- the non-stellar parameters have either been brought into a physical context, or have been selected for those values which produce models that match observed objects best (see below).

With consideration of the above points, we here provide an improved mass-loss formula for dust-driven, C-rich winds. We then apply it to tip-AGB evolution models to investigate the significance of the new formula to superwind properties and total mass-loss yields by comparison with our previous work (Schröder et al. 1999; Schröder & Sedlmayr 2001).

Table 1. Range of the parameters covered by the selected model grid.

M	$[M_\odot]$	0.8–1.2
T	[K]	2200–3000
L	$[L_\odot]$	3500–15 000
P	[d]	104–1000

2. Deriving the improved mass-loss formula

Our description of the mass loss during the tip-AGB is based on an substantially updated set of self-consistent, dynamical wind models for dust-forming, carbon-rich atmospheres. The range of model parameters covered matches the observed range and is given by Table 1. These models include time-dependent hydrodynamics, (equilibrium) chemistry, (stationary, grey) radiative transfer and carbon dust formation, growth and evaporation processes. A recent description of the wind model code has been given by Winters et al. (2000).

The hydrodynamical wind models used to calculate the mass loss rates treat in detail the circumstellar environment around pulsating long-period variable stars and therefore require the prescription not only of the three fundamental stellar parameters (i.e. stellar mass M, effective temperature $T_{\rm eff}$, luminosity L) plus the abundance ratio $\epsilon_{\rm C}/\epsilon_{\rm O}$ of carbon over oxygen. In addition, two parameters are required for the prescription of the inner boundary condition which simulates a pulsating stellar surface as a means of mechanical energy input. This inner boundary condition is provided by a sinusoidal variation of the innermost gridpoint in the hydro-model, parameterized by a velocity amplitude Δv and a pulsation period P ("piston approximation").

From the output of the time-dependent models, a mass-loss rate is obtained by averaging the outflow between $r = 30$ and 60 R_*, and over a time span of 20 pulsation periods. So far, the models consider only carbon-rich winds, i.e. $\epsilon_{\rm C}/\epsilon_{\rm O} > 1$. This ratio determines the amount of carbon available for dust formation, making the reasonable approximation that all oxygen and an equal amount of carbon is locked up in CO molecules.

To obtain a physically relevant representation of the theoretical mass-loss rates given by the large set of computed wind models, we applied the multidimensional maximum-likelihood method (as described by Arndt et al. 1997). The input parameters for each wind model are completely independent of each other. Hence, the data set to be described consists of a maximum of 6 independent (M, $T_{\rm eff}$, L, $\epsilon_{\rm C}/\epsilon_{\rm O}$, P and Δv) and one dependent variable ($\dot{M}$) for each model. The mass-loss formula should be preferably simple and is therefore assumed to be linear: $\log \dot{M}_{\rm fit} = a_0 + a_1 \cdot \log x_1 + \cdots + a_n \cdot \log x_n$.

In order to also give a mass-loss approximation described only by the three fundamental stellar parameters M, L and $T_{\rm eff}$, Arndt et al. (1997) simply averaged over the mass-loss rates from models with otherwise differing input parameters. Consequently, their simplified formula,

which we used for our evolution models in earlier work (Schröder et al. 1999; Schröder & Sedlmayr 2001), neglects any possible dependence of the mass-loss on $\epsilon_{\rm C}/\epsilon_{\rm O}$, P or Δv.

To explore the dependence on any of these parameters and its significance, we first fitted the full set of models available up to date for all 6 parameters to obtain the power of each parameter in the updated representation. We then used the same multidimensional maximum-likelihood method on appropriately reduced parameter sets to compare the quality of the fits, i.e., taking the correlation coefficient of each mass-loss rate approximation $\log \dot{M}_{\rm fit}$ and the set of the model mass-loss rates $\log \dot{M}$. If, by dropping a certain parameter, the correlation coefficient decreased significantly, and if a parameter enters the mass-loss formula with a comparatively large power, its significance would need further investigation. In this process, we have given special consideration to the following points:

– *Exclusion of under-critical wind models*:
Schröder et al. (1999) found that a minimum luminosity, equivalent to a minimum radiative acceleration (Winters et al. 2000), is required in order to maintain a stable dust-driven wind. Under-critical wind models yield mass-loss deficiencies which differ by at least one order of magnitude from the characteristics of stable mass-loss. This, however, had not yet been considered by Arndt et al. (1997). From all currently available wind models, we now selected only the solutions with sufficient radiative acceleration for a stable wind, yielding the high mass-loss rates typical for the tip of the AGB. Precisely, we considered the ratio α of the radiative acceleration $a^{\rm rad}$ to the gravitational acceleration $a^{\rm grav}$ and required that its time averaged value $\langle\alpha\rangle_{\rm t}$ is bigger than 1.

– *Significance of the carbon-to-oxygen ratio* $\epsilon_{\rm C}/\epsilon_{\rm O}$:
By including this abundance ratio in the fitting procedure, it turned out that for the models under consideration there is only a slight dependence of $\dot{M} \propto 0.32 \cdot \log(\epsilon_{\rm C}/\epsilon_{\rm O})$. Compared to the other parameters the carbon-to-oxygen ratio thus has the least influence on the mass-loss rate. Also, there is virtually no difference in fit quality between the correlation coefficients with $\epsilon_{\rm C}/\epsilon_{\rm O}$ resolved (0.966), or it been dropped (0.965).
For the purposes of this paper, we therefore considered it a reasonable simplification to neglect any such dependence.

– *The right choice of the piston amplitude* Δv:
The mass-loss characteristics of wind models with otherwise identical parameters show a dependence on the chosen piston amplitude. But only models with physically appropriate values give a realistic description of the mass-loss rate. In fact, an appropriate value of Δv can be found, and is constrained sensitively, by matching the amplitudes of observed infrared lightcurves of pulsating AGB stars, which vary almost linearly with the choice of Δv.
Table 2 summarizes the amplitudes of the K- and M lightcurves of a sample of C-stars monitored by Le Bertre (1992), as well as the mass loss rates derived from phase-dependent spectral modelling by Le Bertre (1997). It can be seen from this table that those objects of the sample, which have a mass loss rate in excess of 10^{-6} $M_\odot$/yr, show K amplitudes ranging between 0.8 and 2 mag, and M amplitudes between 0.70 and 1.6 mag. Best matches of these amplitudes are achieved by models with piston amplitudes which all fall in the range of 3 to 6 $\mathrm{km\,s^{-1}}$. For deriving the mass-loss formula, we therefore restricted the set of available models to a choice of the piston amplitude of $\Delta v = 5$ $\mathrm{km\,s^{-1}}$.

Table 2. Mean K- and M-amplitudes and mass-loss rates of a set of C-stars.

Name	$\langle\Delta K\rangle^{(1)}$	$\langle\Delta M\rangle^{(1)}$	$\dot{M}^{(2)}$
R Scl	0.29	0.35	6.6e-8
R Lep	0.45	0.48	2.3e-7
R For	0.84	0.76	7.8e-7
AFGL 799	0.83	0.80	1.7e-6
AFGL 1062	0.91	0.71	2.4e-6
AFGL 2392	1.06	1.04	1.8e-6
AFGL 1235	1.07	0.94	4.7e-6
IRSV 1519-5115	1.16	0.84	9.2e-6
AFGL 1085	1.30	1.16	5.7e-6
IRC +10401	1.32	0.93	6.3e-6
AFGL 935	1.40	1.02	2.8e-6
IRC -10502	1.41	1.11	7.0e-6
AFGL 971	1.48	1.17	5.9e-6
AFGL 865	1.73	1.57	1.7e-5
AFGL 3099	1.78	1.21	8.0e-6
AFGL 3068	1.90	1.14	4.8e-5
IRC +10216	2.03	1.47	1.2e-5

(1) Le Bertre (1992), (2) Le Bertre (1997).

– *Dependence on the pulsation period* P:
In the initial 6 parameter representation of $\dot{M}$, the P term entered with a power of 0.67. Also, comparing the fit quality with and without including the pulsational period, there is a significant drop of the correlation coefficient from 0.965 to 0.88. Consequently, the impact of P on the mass-loss description is too significant to be neglected and must be accounted for.
Since a well observed relation exists for pulsating AGB stars between period and luminosity, it is possible to substitute the period term in the mass-loss approximation by an additional luminosity term. In this context, there are a number of models, which do not fall onto such a period–luminosity relation. However, these give a description of how the circumstellar shell model would respond to different lower boundary conditions,

by which we can properly account for those specific cases matched by real stars.

In the case of carbon-rich Miras, which are the most appropriate class of tip-AGB objects to compare with our models, the observed relation (Groenewegen & Whitelock 1996) is $\log P/\mathrm{d} = 0.965 \cdot \log L/L_\odot - 1.05$.

With the consideration of the above points, a set of 58 wind models (given in Table 2) has been selected. For applying the maximum-likelihood method on this final set, we now consider only 4 remaining independent input parameters, i.e., effective temperature $T_{\rm eff}$, luminosity L, actual mass M and pulsation period P.

For convenience, these parameters are normalized to physically representative reference values, i.e., 1 $M_\odot$ for the mass, 2600 K for $T_{\rm eff}$, 10^4 $L_\odot$ for luminosity, and 650 d for the period.

Based on that final set of models, the following mass-loss description is obtained:

$$\begin{aligned}\log \dot{M}_{\rm fit} \;=\; & -4.52 - 6.81 \cdot \log(T_{\rm eff}/2600\,\mathrm{K}) \\ & +1.54 \cdot \log(L/10^4\,L_\odot) - 1.95 \cdot \log(M/M_\odot) \\ & +0.959 \cdot \log(P/650\,\mathrm{d}). \qquad (1)\end{aligned}$$

It shows a very good correlation (with a coefficient of 0.965) with the set of model mass-loss rates. The typical deviation of $\dot{M}_{\rm fit}$ from any of the actual rates represented by it, is ±20%. This is demonstrated by Figs. 1a–1c where the logarithmic ratio of the mass loss rates is plotted against M, $T_{\rm eff}$ and L respectively.

In order to substitute the dependence on P, we now apply the above mentioned period-luminosity relation. Transformed to our reference values, it is virtually identical with $\log(P/650\,\mathrm{d}) = 0.965 \cdot \log(L/10^4\,L_\odot)$. We can therefore substitute the period term by an additional power of 0.93 of the luminosity. This finally reduces the approximative mass-loss formula to depend on the three fundamental stellar parameters M, L, and $T_{\rm eff}$, only, but without neglecting significant effects from a dependence of the mass-loss on any other parameters (P, in particular):

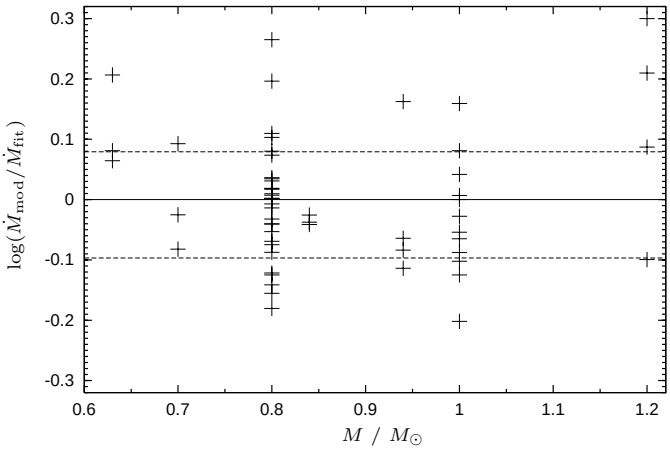
$$\begin{aligned}\log \dot{M}_{\rm fit} \;=\; & -4.52 - 6.81 \cdot \log(T_{\rm eff}/2600\,\mathrm{K}) \\ & +2.47 \cdot \log(L/10^4\,L_\odot) - 1.95 \cdot \log(M/M_\odot) \quad (2)\end{aligned}$$

where $\dot{M}_{\rm fit}$ is given in units of $M_\odot$/yr.

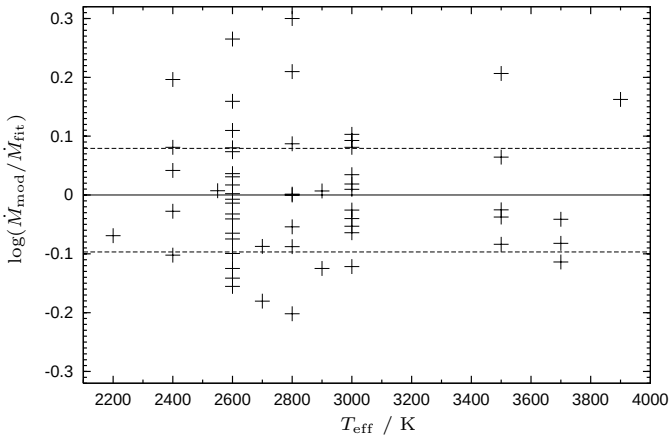
3. Application to tip-AGB evolution and revised mass-loss yields

In order to demonstrate the relevance and significance of the improved mass-loss formula for dust-driven, C-rich winds, we applied it to our evolution models in the exact same way as described by Schröder et al. (1999) and Schröder & Sedlmayr (2001). In particular, we use the same description of convection and overshooting, the critical luminosity for the onset of dust-driven winds, and of any prior mass-loss.

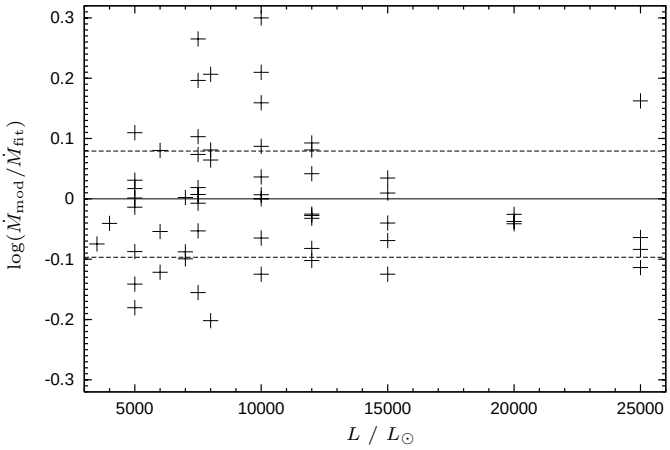
By comparison, the resulting tip-AGB evolution models and their mass-loss histories are virtually the same in the lower range of initial stellar masses considered here (i.e., $M_{\rm i} = 1.0$ to ≈1.3 $M_\odot$, see Figs. 2, 3). A proper,



(a) Logarithmic mass loss ratio over initial mass



(b) ...over effective temperature



(c) ...and over luminosity

Fig. 1. Deviation of the mass-loss rate given by the fit formula from the model mass-loss rate represented by it. The dashed lines mark a deviation of 20%.

Table 3. Listing of the 58 wind models, selected by the criteria explained in the text, to realistically represent dust-driven, carbon-rich mass-loss of real tip-AGB objects.

No.	$M_*/M_\odot$	$T_{\rm eff}$/K	$L_\star/L_\odot$	P/d	$\langle\dot{M}\rangle$	$\langle\alpha\rangle_{\rm t}$	W-No.	$\epsilon_{\rm C}/\epsilon_{\rm O}$
1	0.63	3000	8000	820	3.0e-5	2.50	w155	1.30
2	0.63	3500	8000	460	5.8e-6	2.76	w172	1.30
3	0.63	3500	8000	820	1.4e-5	5.30	w159	1.30
4	0.70	3000	12 000	1100	6.2e-5	6.61	w156	1.30
5	0.70	3500	12 000	650	1.0e-5	4.15	w174	1.30
6	0.70	3700	12 000	650	6.0e-6	1.59	w176	1.30
7	0.80	2200	15 000	300	1.1e-4	5.42	w38	1.30
8	0.80	2400	7500	104	1.4e-5	3.94	w125/2	1.50
9	0.80	2550	7500	104	6.0e-6	3.16	w130/1	1.50
10	0.80	2600	3500	400	4.9e-6	1.30	w135	1.30
11	0.80	2600	4000	400	6.5e-6	1.38	w46	1.30
12	0.80	2600	5000	300	7.4e-6	1.34	w49/14	1.30
13	0.80	2600	5000	350	6.4e-6	1.74	w110	1.30
14	0.80	2600	5000	400	1.3e-5	1.82	w44	1.30
15	0.80	2600	5000	500	1.3e-5	1.88	w111	1.30
16	0.80	2600	5000	600	1.6e-5	1.96	w112	1.30
17	0.80	2600	6000	400	1.6e-5	2.28	w51	1.30
18	0.80	2600	7000	450	1.9e-5	2.61	w60	1.30
19	0.80	2600	7500	300	1.0e-5	2.38	w113	1.30
20	0.80	2600	7500	450	2.5e-5	2.61	w48	1.30
21	0.80	2600	7500	600	5.1e-5	1.96	w114	1.30
22	0.80	2600	7500	800	3.6e-5	2.28	w141	1.30
23	0.80	2600	10 000	640	5.0e-5	2.02	w63	1.30
24	0.80	2600	12 000	800	7.0e-5	4.65	w61	1.30
25	0.80	2600	15 000	1000	9.9e-5	3.84	w62	1.30
26	0.80	2700	5000	300	3.9e-6	1.47	w167/2	1.30
27	0.80	2700	5000	350	5.6e-6	1.76	w168/4	1.30
28	0.80	2800	5000	400	6.1e-6	1.26	w52	1.30
29	0.80	3000	6000	400	3.8e-6	1.85	w47	1.30
30	0.80	3000	7500	400	9.0e-6	2.78	w124/2	1.50
31	0.80	3000	7500	450	8.3e-6	8.49	w32	1.80
32	0.80	3000	7500	650	1.0e-5	4.86	w31	1.80
33	0.80	3000	15 000	300	1.7e-5	3.70	w30	1.50
34	0.80	3000	15 000	650	3.0e-5	8.90	w28	1.50
35	0.80	3000	15 000	800	4.1e-5	9.32	w29	1.50
36	0.84	3000	20 000	1200	7.9e-5	10.07	w157	1.30
37	0.84	3500	20 000	880	2.0e-5	5.47	w177	1.30
38	0.84	3700	20 000	710	1.1e-5	4.07	w178	1.30
39	0.94	3000	25 000	1300	8.8e-5	11.30	w158	1.30
40	0.94	3500	25 000	1000	2.3e-5	6.77	w180	1.30
41	0.94	3700	25 000	810	1.2e-5	2.85	w181	1.30
42	0.94	3900	25 000	1300	2.5e-5	11.90	w183	1.30
43	1.00	2400	12 000	300	2.6e-5	3.15	w122/1	1.30
44	1.00	2400	12 000	500	5.9e-5	4.04	w144	1.30
45	1.00	2400	12 000	600	7.7e-5	4.09	w120	1.30
46	1.00	2400	12 000	800	7.9e-5	3.71	w145	1.30
47	1.00	2600	10 000	640	4.3e-5	2.64	w64	1.30
48	1.00	2600	10 000	650	2.6e-5	8.78	w13	1.80
49	1.00	2800	6000	400	4.6e-6	1.41	w169/5	1.35
50	1.00	2800	7000	400	5.4e-6	1.21	w78	1.30
51	1.00	2800	8000	400	5.1e-6	1.68	w97/1	1.30
52	1.00	2800	10 000	640	1.8e-5	2.92	w69	1.30
53	1.00	2900	10 000	578	1.3e-5	3.09	w71	1.30
54	1.00	2900	10 000	578	9.6e-6	1.98	w72	1.25
55	1.20	2600	7000	400	6.1e-6	1.05	w53	1.30
56	1.20	2800	10 000	400	1.3e-5	2.90	w109	1.80
57	1.20	2800	10 000	400	1.6e-5	3.37	w108/6	1.50
58	1.20	2800	10 000	400	9.8e-6	2.06	w126	1.40

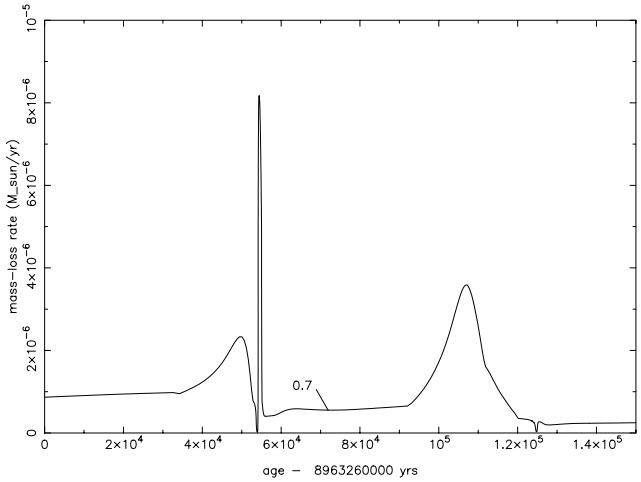


Fig. 2. Tip-AGB and final 10^5 yrs of mass-loss history for a C-rich evolution model with $M_\mathrm{i} = 1.10\ M_\odot$ and the improved mass-loss description – note the short burst of mass-loss as already discussed by Schröder et al. (1998, 1999). The label denotes the actual mass (in $M_\odot$) at that time.

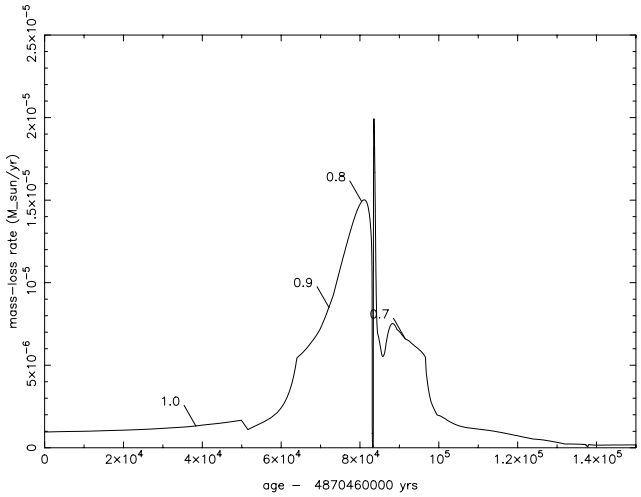


Fig. 3. As in Fig. 2, but for $M_\mathrm{i} = 1.30\ M_\odot$, with the improved mass-loss description. This is the minimum initial mass required to obtain a regular superwind.

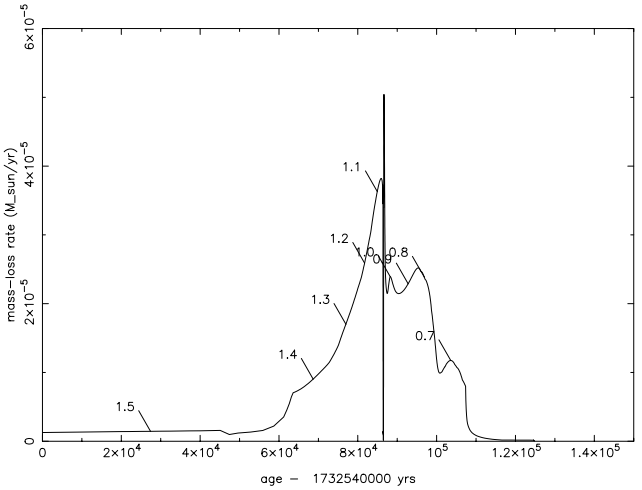


Fig. 4. Tip-AGB and final 10^5 yrs of mass-loss history for a C-rich evolution model with $M_\mathrm{i} = 1.85\ M_\odot$ and the improved mass-loss description. Otherwise as Fig. 2.

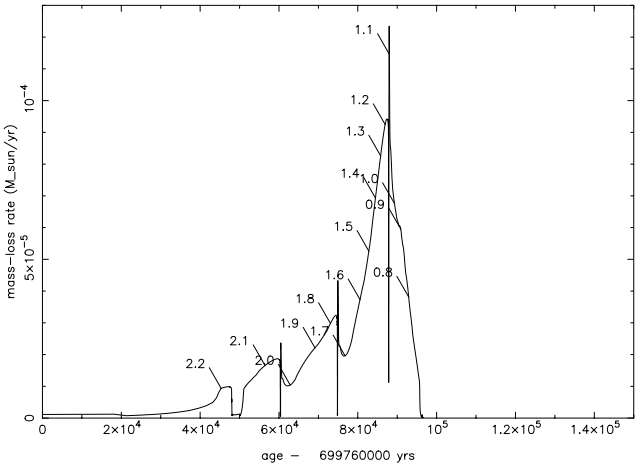


Fig. 5. As in Fig. 4, but for $M_\mathrm{i} = 2.65\ M_\odot$. Note the larger peak mass-loss rate for larger initial stellar mass.

continuous superwind sets in only for initial masses of $\approx$1.3 $M_\odot$ and larger (as found before), while short bursts of strong mass-loss are obtained for less massive tip-AGB models. Our new model with $M_\mathrm{i} = 1.1\ M_\odot$ produces the same short ($\lesssim$800 yrs) burst of mass-loss (nearly $10^{-5}\ M_\odot$/yr), with the same properties as found by us before (Schröder et al. 1999), and removes obout 0.007 $M_\odot$ in that event. This is in excellent agreement with the latest observations of stunningly thin, spherical CO shells, especially the one of TT Cyg observed by Olofsson et al. (2000), who derive a total shell mass of 0.007 $M_\odot$. In addition, their observed outflow velocity (12.6 $\mathrm{km\,s^{-1}}$) of the shell is in very good agreement with the outflow velocity of our respective wind model.

Towards larger initial masses (see Figs. 4, 5), we now obtain higher superwind peak mass-loss rates and larger fractions of stellar mass lost by the superwind altogether: The $M_\mathrm{i} = 1.85\ M_\odot$ model yields $4 \times 10^{-5}\ M_\odot$/yr and a total loss of 0.62 $M_\odot$ during the final 30 000 yrs (before: 0.50 $M_\odot$). The $M_\mathrm{i} = 2.65\ M_\odot$ model even develops a superwind mass-loss peak of about $1 \times 10^{-4}\ M_\odot$/yr, removing 1.22 $M_\odot$ (before: 0.80 $M_\odot$) in that phase.

The new results agree very well with the characteristics of PNs, and we believe that they now match even better than before. Basically, the new mass-loss description yields larger rates for stars with slightly higher T_eff, M and for larger L. These stellar parameters are the ones reached by more massive models, which have their tip-AGB at higher luminosities and slightly larger T_eff. This means, more massive stars produce denser circumstellar shells and, eventually, more massive PNs.

In total, we re-computed 27 C-rich evolution models and their mass-loss histories, for initial stellar masses in the range of 1.0 to 2.8 $M_\odot$. The basic characteristics of these models are listed in Table 4. This table compares

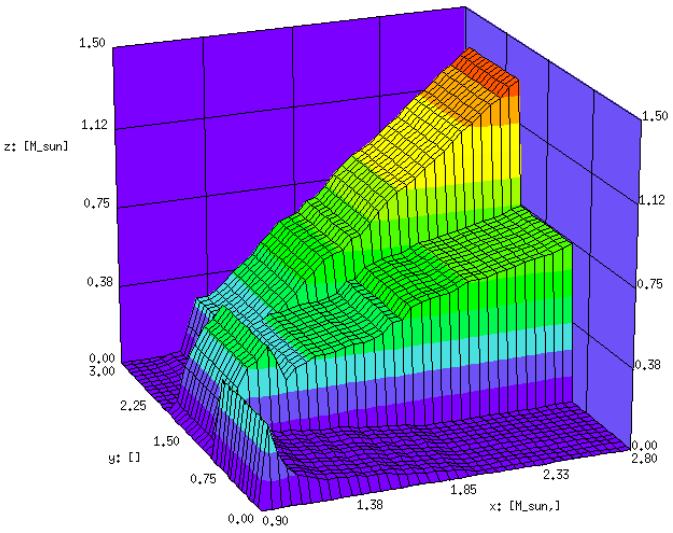


Fig. 6. Mass lost by stars of different initial stellar mass (x-axis) during the RGB (front), the AGB (middle, excl. superwind) and the superwind (back, final 30 000 yrs) phases.

exactly to Table 2 of Schröder & Sedlmayr (2001). The now stronger mass-loss rates of more massive models lead to significantly larger yields during the superwind phase, which now exceed the AGB mass-losses prior to the superwind, giving more weight to the C-rich superwind yields for their contribution to the galactic mass re-injection rate. The same can be seen from Fig. 6, which compares to Fig. 8 of Schröder & Sedlmayr (2001), and which shows the total mass lost during each evolution phase relevant to stellar mass-loss, i.e., the Red Gant Branch (RGB), the Asymptotic Giant Branch (AGB, excl. superwind) and the superwind (final 30 000 yrs).

For the same reason, slightly smaller final masses are now obtained, approximately $M_\mathrm{f}/M_\odot = 0.55 \cdot (M_\mathrm{i}/M_\odot)^{0.27}$, which is in even better agreement with the observed initial-mass, final-mass relation of Weidemann (1997) than before: for $M_\mathrm{i} = 2.5\ M_\odot$, Weidemann (1997) derived $M_\mathrm{f} = 0.68\ M_\odot$, while we obtain 0.70 $M_\odot$ (instead of 0.72 $M_\odot$ by Schröder & Sedlmayr 2001).

Looking at a complete stellar sample of stars, distributed in the range of $M_\mathrm{i} = 1.0\ M_\odot$ to 2.5 $M_\odot$ with a logarithmic IMF representative of the solar neighbourhood stars ($\propto M_\mathrm{i}^{-1.8}$, see Schröder & Sedlmayr 2001), we find a re-injection of 57% of the total stellar mass involved, which is the same fraction as derived before by us (Schröder & Sedlmayr 2001). Again, however, the difference lies in the mass fraction lost during the C-rich superwind, which we now find to reach nearly 22% (was 17%), while the fraction lost during the prior AGB phases is nearly 29% (was 33%). Again, this enhances the importance of the C-rich winds on the tip-AGB relative to the cool but still O-rich winds prior to dust-driven mass-loss, in terms of their contribution to the galactic mass re-injection.

Table 4. Listing of the revised evolution models, giving the initial stellar mass M_i, the masses lost during the RGB, AGB (excl. superwind) and the superwind (final 30 000 yrs) phases, and the final mass M_f of the remnant WD.

M_i	$\int \dot{M}_\mathrm{RGB}$	$\int \dot{M}_\mathrm{AGB}$	$\int \dot{M}_\mathrm{SW}$	M_f	
1.00	0.24	0.20	—	0.55	
1.05	0.16	0.30	—	0.56	
1.10	0.12	0.38	0.01	0.56	[1]
1.15	0.11	0.41	0.03	0.57	[1]
1.20	0.09	0.47	0.03	0.58	[1]
1.25	0.08	0.40	0.15	0.59	[1]
1.30	0.08	0.30	0.28	0.60	
1.35	0.07	0.35	0.28	0.60	
1.40	0.07	0.37	0.31	0.61	
1.45	0.06	0.39	0.34	0.61	
1.50	0.06	0.39	0.38	0.62	
1.55	0.06	0.40	0.42	0.62	
1.60	0.05	0.41	0.46	0.63	[2]
1.65	0.04	0.42	0.51	0.63	
1.70	0.04	0.42	0.55	0.63	
1.75	0.03	0.43	0.60	0.64	
1.80	0.03	0.45	0.62	0.64	
1.85	0.02	0.50	0.63	0.64	
1.90	0.02	0.50	0.68	0.65	
1.95	0.01	0.55	0.69	0.65	[3]
2.05	0.001	0.58	0.79	0.66	
2.15	—	0.59	0.87	0.67	
2.25	—	0.63	0.92	0.68	
2.35	—	0.64	1.00	0.69	
2.50	—	0.67	1.11	0.70	
2.65	—	0.70	1.22	0.71	
2.80	—	0.74	1.32	0.72	
Sample:	6.3%	28.7%	21.7%	43.3%	[4]

[1] Only brief superwind burst(s).
[2] Onset of core overshooting on MS at $M_\mathrm{i} \approx 1.6\ M_\odot$.
[3] RGB evolution ends with He flash for $M_\mathrm{i} \leq 1.95\ M_\odot$.
[4] Fractions of mass lost by a stellar sample (1.0 to 2.5 $M_\odot$, IMF $\propto M_*^{-1.8}$).

4. Discussion

Despite significant improvement, there are a few points which will require some more work in the future. In particular, we believe that the most critical of the remaining simplifications concerns the description of the mechanical energy input, Δv in particular, for which a fixed value may not be applicable to the whole range of tip-AGB models. Hopefully, a larger set of models, in combination with a better knowledge of stellar parameters of individual pulsating objects, will reveal any kind of relationship between the best choice of Δv with any fundamental stellar parameter, i.e., with L. Then, any Δv term in the mass-loss

description could be considered in a similar way as we treated the P term.

We would like to emphasize that the here presented mass-loss formula applies only to *carbon-rich* mass-loss, and for strong dust-driven winds. The chemistry of dust-formation in oxygen-rich winds is substantially different and requires its own modelling. Such work is on the way (Jeong et al. 1999) and a specific oxygen-rich mass-loss description is in preparation.

The application of the improved mass-loss approximation to the computation of individual evolution models, as well as to collective mass-loss yields, demonstrates the far-reaching implications of any improvement to stellar and galactic astrophysics.

Despite some remaining issues, we may conclude that we have derived a significantly improved and physically meaningful mass-loss description for carbon-rich, dust-driven winds of tip-AGB objects, based on a large set of theoretical wind models, selected to represent observed objects best. The application of the new formula to evolution models yields more massive superwinds and PNs towards the larger stellar masses as derived before with a preliminary mass-loss representation, and the results are in good agreement with observed properties of PNs.

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