The Interstellar Medium

IK-LECTURE

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Prologue

...it would be better for the true physics if there were no mathematicians on earth. Daniel Bernoulli

A THEORY that agrees with all the data *at any given time* is necessarily *wrong*, as at any given time *not all the data are correct.* Francis Crick

TELESCOPE, n. A device, having a relation to the eye similar to that of the *telephone* to the ear, enabling distant objects to plague us with a multitude of needless details.
Luckily it is unprovided with a *bell* ... Ambrose Bierce

For those who want some proof that physicists are human, the proof is in the idiocy of all the different units which they use for measuring energy. Richard P. Feynman

Overview

LECTURE 1: Non-thermal ISM Components

- Basic Plasma Physics (Intro)
  - Debye length, plasma frequency, plasma criteria
- Magnetic Fields
  - Basic MHD
  - Alfvén Waves
- Cosmic Rays (CRs)
  - CR spectrum, CR clocks, grammage
  - Interaction with ISM, propagation, acceleration

LECTURE 2: Dynamical ISM Processes

- Gas Dynamics & Applications
  - Shocks
  - HII Regions
  - Stellar Winds
  - Superbubbles
- Instabilities
  - Kelvin-Helmholtz Instability
  - Parker Instability

LECTURE 1
Non-Thermal ISM Components

I.1 Basic Plasma Physics

- Almost all baryonic matter in the universe is in the form of a plasma (> 99 %)
- Earth is an exception
- Terrestrial Phenomena: lightning, polar lights, neon & candle light
- Properties: ***collective behaviour***, wave propagation, dispersion, diagmagnetic behaviour, Faraday rotation

- Plasma generation:
  - Temperature increase

    However: no phase transition!

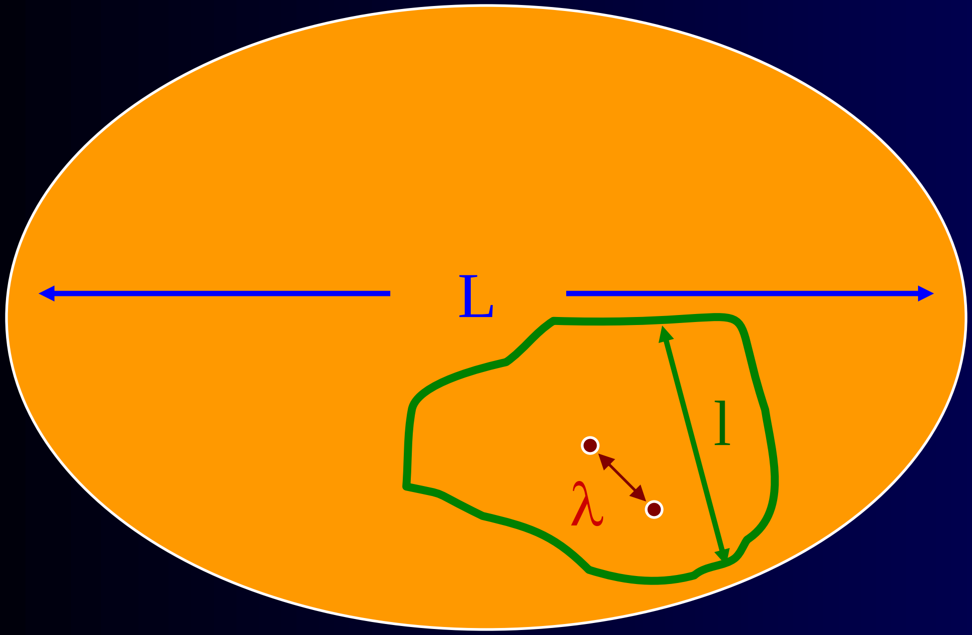
    Continuous increase of ionization (e.g. flame)
  - Photoionisation (e.g. ionosphere)
  - Electric Field („cold“ plasma, e.g. gas discharge)
- Plasma radiation:
  - Lines (emission + absorption)
  - Recombination (free-bound transition)
  - Bremsstrahlung (free-free transition)
  - Black Body radiation (thermodyn. equilibrium)
  - Cyclotron & Synchrotron emission

1. Definition:
   - System of electrically charged particles (electrons + ions) & neutrals
   - Collective behaviour $\Rightarrow$ long range Coulomb forces

2. Criteria & Parameters for Plasmas:

i. Macroscopic neutrality: Net Charge Q=0
   L >> λ_D (λ_D.. Debye length) for collective behaviour
ii. Many particles within Debye sphere $N_D \approx n_e \lambda_D^3 >> 1$
iii. Many plasma oscillations between ion-neutral damping collisions $\omega \tau_{en} >> 1$

Quasi-neutral Plasma



- Physical volume: L
- Test volume: l
- Mean free path: λ

- Requirement:

$$\lambda << l << L$$

Net Charge: Q = 0

- To violate charge neutrality within radius r requires electric potential:

$$q = \frac{4}{3}\pi r^3\left(n_i e + n_e(-e)\right) = \frac{4}{3}\pi r^3 e\left(n_i - n_e\right)$$

$$\Rightarrow \Phi = \frac{q}{r} = \frac{4}{3}\pi r^2 e\left(n_i - n_e\right)$$

For

$e = 4.803\times10^{-10}$ esu, 1 esu = 1 cm$^{3/2}$ g$^{1/2}$ s^{-1}

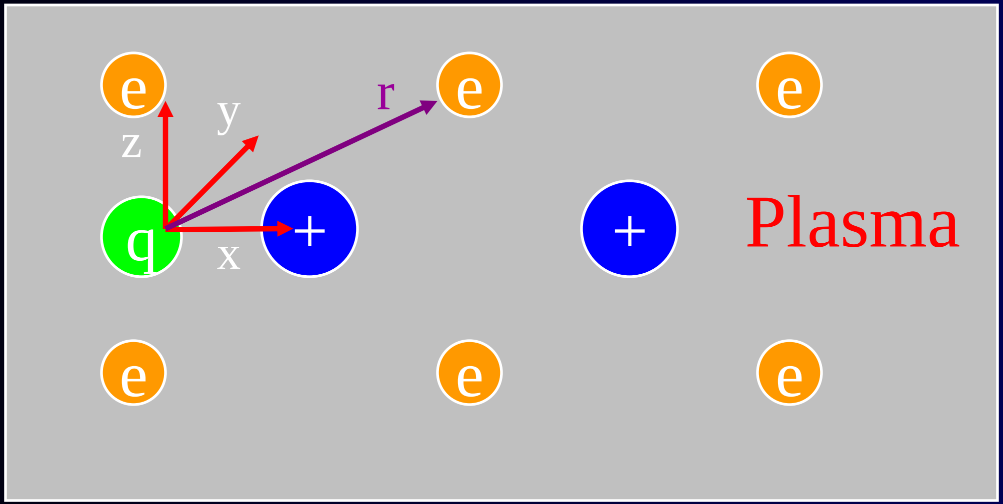
and $r = 1$ cm, $n = 10^{11}$ cm^{-3}, $\left|n_i - n_e\right| \approx 0.01\, n_i$

we need a voltage of

$\Phi = 6.03\times10^3$ V or $T \approx 7\times10^7$ K

Debye Shielding

- Violation of Q=0 only within Debye sphere



q ... positive test charge

Equilibrium is disturbed

Is there a new equil. state?

Look for steady state!

- Electrostatics: $\vec{\mathbf{E}} = -\vec{\nabla}\Phi(\vec{\mathbf{r}})$
- Equilibrium: Boltzmann distribution

$$n_e(\vec{\mathbf{r}}) = n_0 \exp\left[\frac{e\Phi(\vec{\mathbf{r}})}{k_B T}\right], \quad n_i(\vec{\mathbf{r}}) = n_0 \exp\left[\frac{-e\Phi(\vec{\mathbf{r}})}{k_B T}\right]$$

- Total charge density:

$$\rho(\vec{\mathbf{r}}) = -e(n_e(\vec{\mathbf{r}}) - n_i(\vec{\mathbf{r}})) + Q\delta(\vec{\mathbf{r}})$$

$$= -en_0\left\{\exp\left[\frac{e\Phi(\vec{\mathbf{r}})}{k_B T}\right] - \exp\left[\frac{-e\Phi(\vec{\mathbf{r}})}{k_B T}\right]\right\} + Q\delta(\vec{\mathbf{r}})$$

- Relation between charge distribution and charge density (Maxwell): $\vec{\nabla}\vec{\mathbf{E}} = 4\pi\rho(\vec{\mathbf{r}})$

- Disturbed potential thus given by diff.eq.:

$$\nabla^2\,\Phi(\vec{\mathbf{r}}) - 4\pi\,e\,n_0\left\{\exp\left[\frac{e\Phi(\vec{\mathbf{r}})}{k_B T}\right] - \exp\left[\frac{-e\Phi(\vec{\mathbf{r}})}{k_B T}\right]\right\} = -4\pi\,Q\delta(\vec{\mathbf{r}})$$

- Test charge with small electrostatic potential energy:

$$e\,\Phi(\vec{\mathbf{r}}) << k_B T \quad \Longrightarrow \quad \exp\left[\pm e\frac{\Phi(\vec{\mathrm{r}})}{k_B T}\right] \approx 1 \pm \frac{\Phi(\vec{\mathrm{r}})}{k_B T}$$

- Thus the transcendental equation can be approximated

$$\nabla^2\,\Phi(\vec{\mathbf{r}}) - 4\pi\, e\, n_0 \left\{ 2\,e\left[\frac{\Phi(\vec{\mathbf{r}})}{k_B T}\right]\right\} = -4\pi\, Q\,\delta(\vec{\mathbf{r}})$$

- Defining the Debye length:

$$\lambda_D = \sqrt{\frac{k_B T}{4\pi e^2 n_0}}$$

$$\Longrightarrow \quad \nabla^2\,\Phi(\vec{\mathbf{r}}) - \frac{2}{\lambda_D^2}\Phi(\vec{\mathbf{r}}) = -4\pi\, Q\,\delta(\vec{\mathbf{r}})$$

- Electrostatic forces are central forces: $\Phi(\vec{\mathbf{r}}) = \Phi(r)$

In spherical coordinates:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\Phi(r)\right) - \frac{2}{\lambda_D^2}\Phi(r) = 0$$

with boundary conditions :

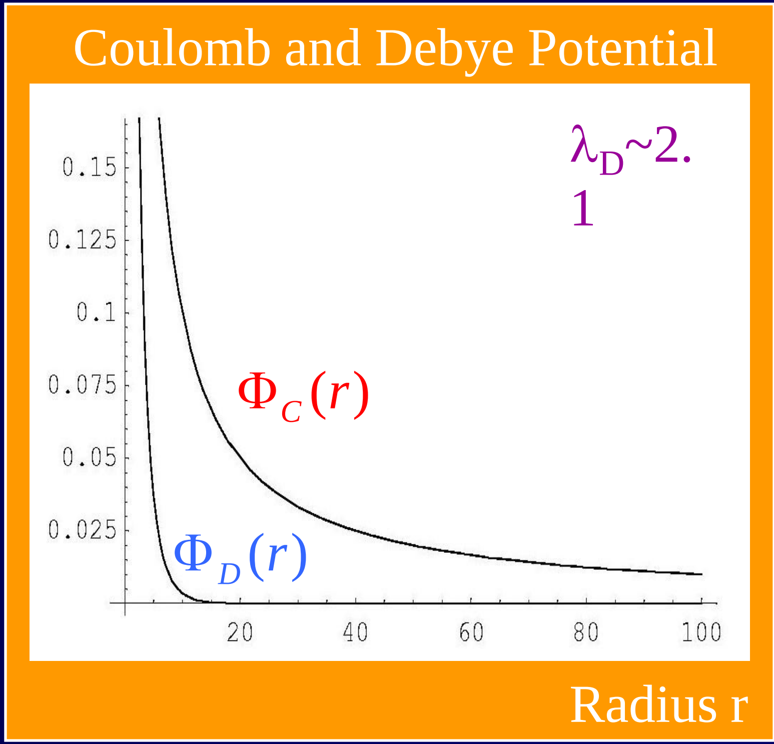
$$r \to 0:\ \ \Phi(r) \to \Phi_C(r) = \frac{Q}{r}$$

$$r \to \infty:\ \Phi(r) \to 0$$

- Result:

$$\boxed{\Phi_D(r) = \frac{Q}{r}\exp\left[\frac{-\sqrt{2}}{\lambda_D}r\right]}$$

Debye-Hückel Potential



- Is total charge neutrality fulfilled: $q_{tot} = 0$?

$$q_{tot} = \int_V \rho(\vec{r})\,\mathrm{d}^3 r = -\frac{q}{2\pi\lambda_D^2}\int_V \frac{1}{r}\exp\left(-\frac{\sqrt{2}}{\lambda_\mathrm{D}} r\right)\mathrm{d}^3 r + \int_V q\,\delta(\vec{r})\,\mathrm{d}^3 r$$

$$= -\frac{q}{2\pi\lambda_D^2}\int_0^\infty 4\pi\, r^2\,\frac{1}{r}\exp\left(-\frac{\sqrt{2}}{\lambda_\mathrm{D}} r\right)\mathrm{d}r + \underbrace{\int_0^\infty 4\pi\, r^2 q\,\delta(r)\,\mathrm{d}r}_{q}$$

$$= -\frac{q}{2\pi\lambda_D^2}\left[4\pi\, r\left(-\frac{\lambda_\mathrm{D}}{\sqrt{2}}\right)\exp\left(-\frac{\sqrt{2}}{\lambda_\mathrm{D}} r\right)\right]_0^\infty +$$

Partial Integration

$$+\frac{q}{2\pi\lambda_D^2}\int_0^\infty 4\pi\left(-\frac{\lambda_\mathrm{D}}{\sqrt{2}}\right)\exp\left(-\frac{\sqrt{2}}{\lambda_\mathrm{D}} r\right)\mathrm{d}r + q$$

$$= -\frac{q}{2\pi\lambda_D^2}\left[0-0\right] + \frac{q}{2\pi\lambda_D^2}\left[4\pi\,\frac{\lambda_D^2}{2}\exp\left(-\frac{\sqrt{2}}{\lambda_\mathrm{D}} r\right)\right]_0^\infty + q$$

$$= \frac{q}{2\pi\lambda_D^2}\left[0-2\pi\lambda_D^2\right] + q = q - q = 0 \quad \text{q.e.d.}$$

Importance of Debye Shielding

- Test charge q neutralized by neighbouring plasma charges $\longrightarrow$ within „Debye sphere“
- Charge neutrality guaranteed for $\boxed{r >> \lambda_D}$
- For $r \to 0$: $\Phi_D(r) \to \infty$ bec. $e\Phi << k_B T$ breaks down
- Factor „2“: due to non-equil. distr. of ions

If we neglect ion motions: $n_i = n_0$: $\boxed{\Phi_D(r) = \frac{Q}{r} \mathrm{e}^{-\frac{r}{\lambda_D}}}$

- Numbers: $\boxed{\lambda_D = 6.9 \sqrt{\frac{T[\mathrm{K}]}{n_e[\mathrm{cm}]^{-3}}}\ \mathrm{cm}}$

Ionosphere: T=1000 K, $n_e=10^6$ cm^{-3}, λ_D=0.2 cm

ISM: T=10^4 K, n_e~1 cm^{-3}, λ_D=6.9 m; L_{ISM}~3 10^{16} m

Discharge: T=10^4 K, $n_e=10^{10}$ cm^{-3}, λ_D=6.9 10^{-3} cm

- Number of particles in a Debye sphere:

$$N_D = \frac{4}{3}\pi\, n_e \lambda_D^3 \Rightarrow \text{Plasma parameter: } g = \left(n_e \lambda_D^3\right)^{-1}$$

- Charge neutrality can only be maintained for a sufficient number of particles in Debye sphere:

$$N_D >> 1 \Leftrightarrow g << 1$$

- Collective behaviour only for $r << \lambda_D$ for each particle: only here violation of $Q=0$ possible
- Debye shielding is due to collective behaviour
- $g<<1$: $\lambda_{mfp} << \lambda_D$

Plasma frequency

- Violation of Q=0: strong electrostatic restoring forces lead to *Langmuir oscillations* due to inertia of particles
- Electrons move, ions are immobile
- Averaged over a period: Q=0
- Longitudinal harmonic oscillations with plasma frequency: $$\boxed{\omega_p = \sqrt{\frac{4\pi e^2 n_e}{m_e}}}$$

Oscillations damped by collisions between electrons and neutral particles

$\tau_{en} \sim 1/\nu_{en}$ ... mean collision time

To restore charge neutrality we need:

$\omega >> \nu_{en} \Leftrightarrow \omega\tau_{en} >> 1$

Plasma Criteria

i. L >> λ_D

ii. $N_D \approx n_e \lambda_D^3 >> 1$

iii. $\omega\tau_{en} >> 1$

Exercise:

Check the validity of plasma criteria: ISM – DIG:
T_e ~ 8000 K, n_e ~ 1 cm^{-3}

I.2 Magnetic Fields

- Magnetic Fields (MFs) are ubiquitous in universe
- Observational evidence in ISM:
  - Polarization of star light –> dust –> gives $B_{\perp}$
  - Zeeman effect –> HI –> gives $B_{//}$
  - Synchrotron radiation –> relativistic e$^-$ –> gives $B_{\perp}$
  - Faraday rotation –> thermal e$^-$ –> gives $B_{//}$
- Sources of MFs are electric currents $\vec{j}$
- In ISM conductivity σ is high, thus large scale electric fields are negligible and $\vec{j} = \frac{c}{4\pi}\left(\vec{\nabla} \times \vec{B}\right)$

Basic Magnetohydrodynamics (MHD)

- Plasma is ensemble of charged (electrons + ions) and neutral particles –> characterized by distribution function in phase space $f_i(\vec{x}, \vec{p}, t)$ –> evol. by Boltzmann Eq.
- MHD is macroscopic theory marrying Maxwell's Eqs. with fluid dynamic eqs.: „magnetic fluid dynamics"
- Moving charges produce currents which interact with MF –> backreaction on fluid motion
- To see basic MHD effects, a single fluid MHD is treated (mass density in ions, high inertia compared to e^-)
- For simplicity gas treated as perfect fluid (eq. of state)
- Neglect dissipative processes: molecular viscosity, thermal conductivity, resistivity

Basic MHD assumptions

1. Low frequency limit: $\omega << \nu_c$
   - Consider large scale (λ big, ω small) gas motions
   - Consider volume V of extension L: $\omega \sim \frac{1}{\tau} \sim \frac{v_{th}}{L} << \nu_c \sim \frac{v_{th}}{\lambda_{mfp}}$

     (hydrodynamic limit) $\Leftrightarrow \frac{\lambda_{mfp}}{L} \equiv Kn << 1$
   - If $\omega << \nu_{ei}$ then $P_e \approx P_i$ as assumed
   - Note that also $\omega << \omega_g$ (for el. + ions) holds
   - Ampére's law: $\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$

     simplifies to $\vec{j} = \frac{c}{4\pi}\left(\vec{\nabla} \times \vec{B}\right)$ since $\left|\frac{1}{c}\frac{\partial \vec{E}}{\partial t}\right| \Big/ \left|\vec{\nabla} \times \vec{B}\right| \sim \frac{{}^{1}\!/_{c}\,\frac{E}{\tau}}{{}^{B}\!/_{L}} = \frac{L}{\tau}\frac{v}{c^2} << 1$

     using (see 2.) $E \sim \frac{v}{c} B$

2. Non-relativistic limit : v/c << 1

– Electric + magnetic field in plasma rest frame are then:

$$\vec{E}' = \vec{E} + \frac{1}{c}\left(\vec{v} \times \vec{B}\right)$$

$$\vec{B}' = \vec{B} - \frac{1}{c}\left(\vec{v} \times \vec{E}\right)$$

– For v<<c, e⁻ due to high mobility prevent large scale E-fields, i.e.

$$\vec{\nabla}\Phi \to 0 \Leftrightarrow \vec{E}' \to 0$$

$$\Rightarrow \vec{E} = -\frac{1}{c}\left[\vec{v} \times \vec{B}\right],\ \vec{B}' = \vec{B} + 1/c^2\left[\vec{v} \times \left(\vec{v} \times \vec{B}\right)\right] \approx \vec{B} \Rightarrow \vec{j}' = \vec{j}$$

– Current density

$$\vec{j} = Zen_i\vec{v}_i - en_e\vec{v}_e = -en_e\vec{u}_e$$

with $\vec{u}_e = \vec{v}_e - \vec{v}_i$ (drift speed), and $\rho_e = Zen_i - en_e \equiv 0$

Negligible drift speed between electrons and ions:

$$u_e \sim \frac{j}{en_e} \sim \frac{c}{4\pi}\frac{B}{en_e L}$$

Consider ISM with B-field: B~ 3 μG, L~1 pc, n_e~1 cm^{-3}

$u_e \sim 4.8\ 10^{-11}$ km/s as compared to $c_s \sim 1$ km/s

→ motion of ions and electrons coupled via collisions; small drift speed for keeping up B-field extremely low!

3. High electric conductivity $\sigma \to \infty$: ideal MHD

– In principle u_e is needed to calculate current density however Ohm‘s law can be used instead:

$$\text{Since } \vec{B}' = \vec{B}, \text{ we have } \quad \vec{j}' = \vec{j} = \sigma \vec{E}' = \sigma\left(\vec{E} + \frac{1}{c}\left(\vec{v} \times \vec{B}\right)\right)$$

$$\Rightarrow \vec{E} = -\frac{1}{c}\left(\vec{v} \times \vec{B}\right), \text{ for } \sigma \to \infty$$

Thus Faraday's law is given by:

$$\frac{\partial \vec{B}}{\partial t} = -\frac{1}{c}\left[\vec{\nabla} \times \vec{E}\right] = -\frac{1}{c}\left(\vec{\nabla} \times \left[\vec{v} \times \vec{B}\right]\right)$$

Magnetic pressure and tension

- The equation of motion (including Lorentz force term):

$$\rho \frac{d\vec{v}}{dt} = \rho \frac{\partial \vec{v}}{\partial t} + \rho\left(\vec{v}\vec{\nabla}\right)\vec{v} = -\vec{\nabla}P + \vec{F}_{mag} + \vec{F}_{ext}$$

with $\vec{F}_{mag} = \rho \frac{1}{c}\left[\vec{v} \times \vec{B}\right] = \frac{1}{c}\left[\vec{j} \times \vec{B}\right] = \frac{1}{4\pi}\left[\vec{\nabla} \times \vec{B}\right] \times \vec{B}$

Note: if $\frac{1}{4\pi}\left[\vec{\nabla} \times \vec{B}\right] \times \vec{B} = 0 \Rightarrow \vec{B} = -\vec{\nabla}\psi$ field is force free

Potential field!

- Magnetic pressure and tension:
  - Aside: killing vector cross products use ε-tensor ε_{ijk}

    and write $\vec{a}\times\vec{b} = a_i b_j \varepsilon_{ijk}$ with summation convention

    and the identity: $\varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

Thus:

$$\left[\vec{\nabla}\times\vec{B}\right]\times\vec{B} = -\vec{B}\times\left[\vec{\nabla}\times\vec{B}\right]$$

$$\rightarrow -B_i\left(\partial_j B_k \varepsilon_{jkl}\right)\varepsilon_{ilm} = -B_i\left(\partial_j B_k\right)\varepsilon_{jkl}\varepsilon_{ilm}$$

$$= B_i\left(\partial_j B_k\right)\varepsilon_{jkl}\varepsilon_{lim} = B_i\left(\partial_j B_k\right)\left[\delta_{ji}\delta_{km} - \delta_{jm}\delta_{ki}\right]$$

$$= B_i\left(\partial_i B_m\right) - B_i\left(\partial_m B_i\right)$$

$$\rightarrow \left(\vec{B}\vec{\nabla}\right)\vec{B} - \frac{1}{2}\left(\vec{\nabla}B^2\right)$$

- Thus $\vec{F}_{mag} = \frac{1}{4\pi}\left[\vec{\nabla}\times\vec{B}\right]\times\vec{B} = \frac{\left(\vec{B}\vec{\nabla}\right)\vec{B}}{4\pi} - \frac{B^2}{8\pi}$

Magnetic tension   Magnetic pressure

  - If field lines are parallel $\left(\vec{B}\vec{\nabla}\right)\vec{B} = 0$ i.e. no magnetic tension, but magnetic pressure will act on fluid
  - If field lines are bent, magnetic tension straightens them
  - Magnetic tension acts along the field lines

(Example: tension keeps refrigerator door closed)

Ideal MHD Equations

- Note that $\vec{\nabla}\vec{B}=0$ is included in Faraday's law as *initial condition!*

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}(\rho\,\vec{v})=0$$

$$\rho\frac{\partial\vec{v}}{\partial t}+\rho(\vec{v}\vec{\nabla})\vec{v}=-\vec{\nabla}P+\frac{1}{4\pi}(\vec{\nabla}\times\vec{B})\times\vec{B}+\vec{F}_{ext}$$

$$\frac{\partial\vec{B}}{\partial t}=\vec{\nabla}\times(\vec{v}\times\vec{B})$$

$$\frac{d}{dt}\left(\frac{P}{\rho^{\gamma}}\right)=0$$

- *Here we used the simple adiabatic energy equation*

Magnetic Viscosity and Reynolds number

- For finite conductivity field lines can diffuse away
- Analyze induction equation:

$$\frac{\partial \vec{B}}{\partial t} = -c\left(\vec{\nabla} \times \vec{E}\right)$$

$$\vec{j} = \vec{j}' = \sigma\, \vec{E}' = \sigma\left(\vec{E} + \frac{1}{c}\left(\vec{v} \times \vec{B}\right)\right)$$

$$\Rightarrow \vec{E} = \frac{\vec{j}}{\sigma} - \frac{1}{c}\left(\vec{v} \times \vec{B}\right)$$ ... keep the term with σ!!!

$$\Rightarrow \frac{\partial \vec{B}}{\partial t} = -\frac{c}{\sigma}\left(\vec{\nabla} \times \vec{j}\right) + \vec{\nabla} \times \left(\vec{v} \times \vec{B}\right)$$

$$= \vec{\nabla} \times \left(\vec{v} \times \vec{B}\right) - \frac{c^2}{4\pi\sigma}\left[\vec{\nabla}\left(\vec{\nabla}\vec{B}\right) - \nabla^2 \vec{B}\right]$$

$$= \vec{\nabla} \times \left(\vec{v} \times \vec{B}\right) + \eta_m \nabla^2 \vec{B}$$

$$\eta_m = \frac{c^2}{4\pi\sigma}$$

... magnetic viscosity

- First term is the kinematic MHD term, second is *diffusion term*
- Comparing both terms:

$$\left|\vec{\nabla} \times \left(\vec{v} \times \vec{B}\right)\right| \sim \frac{vB}{L}$$

$$\left|\eta_m \nabla^2 \vec{B}\right| \sim \eta_m \frac{B}{L^2}$$

- Magnetic Reynolds number:

$$R_m \equiv \frac{vB/L}{\eta_m B/L^2} = \frac{vL}{\eta_m}$$

- R_m >> 1: advection term dominates

  R_m << 1: diffusion term dominates

  cf. analogy to laminar and turbulent motions!

Magnetic field diffusion

- For R_m << 1 we get: $$\frac{\partial \vec{B}}{\partial t} = \eta_m \nabla^2 \vec{B}$$

- Deriving a magnetic diffusion time scale:

$$\frac{B}{\tau_D} \sim \frac{\eta_m B}{L^2} \Rightarrow \tau_D \sim \frac{L^2}{\eta_m} = \frac{4\pi\sigma L^2}{c^2}$$

Note: Form identical to heat conduction and particle diffusion

- Field diffusion decreases magnetic energy: field generating currents are dissipated due to finite conductivity -> Joule heating of plasma

Examples:

- Block of copper: L=10 cm, $\sigma=10^{18}$ s^{-1} $\Rightarrow \tau_D \approx 1.2$ s
- Sun: $R_\odot \sim L=7\ 10^{10}$ cm, $\sigma=10^{16}$ s^{-1}

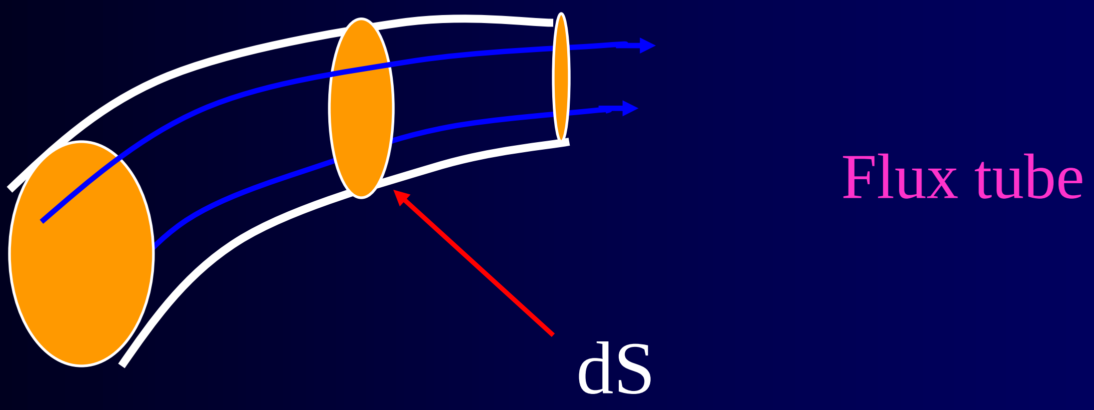
  $\Rightarrow \tau_D \approx 6\times10^{17}\,\text{s} \approx 2\times10^{10}$ yr!

  although conductivty is not so high, it is the ***large dimension*** L in the sun (as well as in ISM) that keep R_m high!
- Note that since $\tau_D \propto \sigma L^2$ turbulence decreases L and thus diffusion times

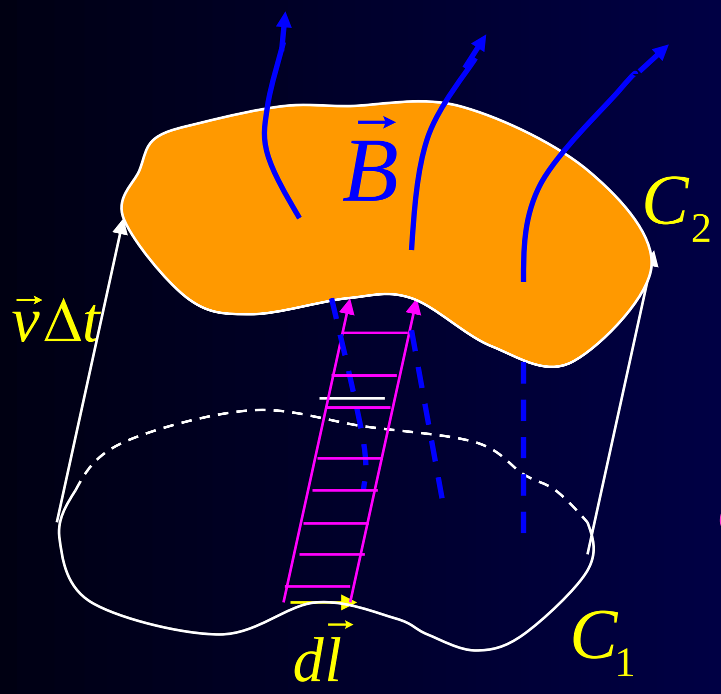
➡ thus fields in ISM have to be regenerated (dynamo?)

Concept of Flux freezing

- Flux freezing arises directly from the MHD kinematic (Faraday) equation ($R_m >> 1$): magnetic field lines are *advected* along with fluid, magnetic flux through any surface advected with fluid remains constant
- Theorem: Magnetic flux through bounded advected surface remains constant with time
- Proof: Consider flux tube



- Consider surface $\vec{S}_1 = \vec{S}(t)$ bounded by C_1 and $\vec{S}_2 = \vec{S}(t+\Delta t)$ by C_2



$$d\vec{S} = -(\vec{v}\Delta t)\times d\vec{l}$$

- Surface changes postion and shape with time
- Magnetic flux through surface at time t:

$$\Phi_B = \int_S \vec{B}(\vec{r},t)\, d\vec{S}$$

- Rate of change of flux through open surface:

$$\frac{d}{dt}\left[\int_S \vec{B}(\vec{r},t)\,d\vec{S}\right] = \lim_{\Delta t\to 0}\frac{1}{\Delta t}\left[\int_{S_2} \vec{B}(\vec{r},t+\Delta t)\,d\vec{S} - \int_{S_1} \vec{B}(\vec{r},t)\,d\vec{S}\right]$$

- Expand field in Taylor series

$$\vec{B}(\vec{r},t+\Delta t) = \vec{B}(\vec{r},t) + \frac{\partial \vec{B}(\vec{r},t)}{\partial t}\Delta t + \ldots$$

- So that

$$\frac{d}{dt}\left[\int_S \vec{B}(\vec{r},t)\,d\vec{S}\right] = \lim_{\Delta t\to 0}\left\{\int_{S_2}\frac{\partial \vec{B}(\vec{r},t)}{\partial t}\,d\vec{S} + \frac{1}{\Delta t}\left[\int_{S_2} \vec{B}(\vec{r},t)\,d\vec{S} - \int_{S_1} \vec{B}(\vec{r},t)\,d\vec{S}\right]\right\}$$

- Using Gauss law

$$\oint \vec{B}\,d\vec{S} = \int_V \vec{\nabla}\cdot\vec{B}\,d^3\vec{r} = 0$$

and applying to the closed surface consisting of $\vec{S}_1, \vec{S}_2$ and the cylindrical surface of length $\vec{v}\Delta t$

We obtain *bottom* *top* *mantle*

$$\oint \vec{B} d\vec{S} = -\int_{S_1} \vec{B}(\vec{r},t)\, d\vec{S} + \int_{S_2} \vec{B}(\vec{r},t)\, d\vec{S} - \oint_{C_1} \vec{B}(\vec{r},t)\left[(\vec{v}\Delta t)\times d\vec{l}\,\right] = 0$$

- Noting that in the limit $\Delta t \to 0,\ \vec{S}_2(t) = \vec{S}_2(t+\Delta t) \to \vec{S}_1(t) = \vec{S}(t)$

$$\frac{d}{dt}\left[\int_S \vec{B}(\vec{r},t)\, d\vec{S}\right] = \int_S \frac{\partial \vec{B}(\vec{r},t)}{\partial t} d\vec{S} + \oint_C \vec{B}(\vec{r},t)\cdot\left[\vec{v}\times d\vec{l}\,\right]$$

and using the vector identity $\vec{B}(\vec{r},t)\cdot\left(\vec{v}\times d\vec{l}\,\right) = -\left[\vec{v}\times\vec{B}(\vec{r},t)\right]\cdot d\vec{l}$

and Stokes‘ theorem $\oint_C \left[\vec{v}\times\vec{B}(\vec{r},t)\right]\cdot d\vec{l} = \int_S \vec{\nabla}\times\left[\vec{v}\times\vec{B}(\vec{r},t)\right]\cdot d\vec{S}$

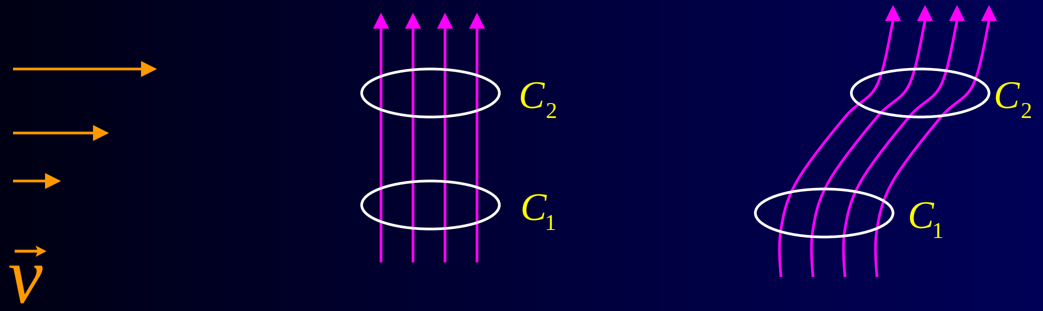
one gets

$$\frac{d}{dt}\left[\int_S \vec{B}(\vec{r},t)\, d\vec{S}\right] = \int_S \left\{\frac{\partial \vec{B}(\vec{r},t)}{\partial t} - \vec{\nabla}\times\left[\vec{v}\times\vec{B}(\vec{r},t)\right]\right\}\cdot d\vec{S}$$

- For a highly conducting fluid ($\sigma \to \infty$) and taking $\vec{v}$ as fluid velocity, field lines are linked to fluid motion and according to ideal MHD „flux freezing“ holds

$$\frac{d}{dt}\left[\int_S \vec{B}(\vec{r},t)\, d\vec{S}\right] = 0$$

- Note: motions parallel to field are not affected



- For finite conductivity field lines can diffuse out:

$$\frac{d}{dt}\left[\int_S \vec{B}(\vec{r},t)\, d\vec{S}\right] = \eta_m \int_S \vec{\nabla}^2 \vec{B}(\vec{r},t)\, d\vec{S}$$

MHD waves

- Perturbations are propagated with characteristic speeds
- For simplicity consider linear time-dependent perturbations in a static compressible ideal background fluid
- Ansatz:

$$\rho = \rho_0 + \delta\rho$$
$$P = P_0 + \delta P$$
$$\vec{v} = \vec{v}_0 + \delta\vec{v}$$
$$\vec{B} = \vec{B}_0 + \delta\vec{B}$$
$$\vec{E} = \vec{E}_0 + \delta\vec{E}$$
$$\vec{j} = \vec{j}_0 + \delta\vec{j}$$

where $\dfrac{\delta X}{X} << 1$

- Assume background medium at rest: $\vec{v}_0 = 0$

$$\frac{\partial \delta\rho}{\partial t} = -\vec{\nabla}\left(\rho_0\, \delta\vec{v}\right) = 0$$

$$\rho_0 \frac{\partial \delta\vec{v}}{\partial t} = -\vec{\nabla}\, \delta P + \frac{1}{c}\left(\delta\vec{j} \times \vec{B}_0\right)$$

$$\vec{\nabla} \times \delta\vec{B} = \frac{4\pi}{c}\, \delta\vec{j}$$

$$\vec{\nabla} \times \delta\vec{E} = -\frac{1}{c}\frac{\partial \delta\vec{B}}{\partial t}$$

$$\vec{\nabla}\, \delta\vec{B} = 0$$

$$\delta\vec{E} = -\frac{1}{c}\left(\delta\vec{v} \times \vec{B}_0\right)$$

$$\frac{\delta P}{P_0} = \gamma\, \frac{\delta\rho}{\rho_0}$$

Keep only first order terms!

Perturbed Equations

- Combining equations and eliminate all variables in favour of $\delta\vec{v}$
- Defining sound speed: $c_S^2 = \left(\frac{\partial P}{\partial \rho}\right)_S = \gamma \frac{P}{\rho}$

yields single perturbation equation

$$\frac{\partial^2 \delta\vec{v}}{\partial t^2} - c_s^2 \vec{\nabla}\left(\vec{\nabla}\,\delta\vec{v}\right) - \left\{\vec{\nabla}\times\left[\vec{\nabla}\times\left(\delta\vec{v}\times\frac{\vec{B}_0}{\sqrt{4\pi\rho_0}}\right)\right]\right\}\times\frac{\vec{B}_0}{\sqrt{4\pi\rho_0}} = 0$$

- Defining Alfvén speed:

$$\boxed{\vec{v}_A = \frac{\vec{B}_0}{\sqrt{4\pi\rho_0}}}$$

yields finally $$\frac{\partial^2 \delta\vec{v}}{\partial t^2} - c_s^2 \vec{\nabla}\left(\vec{\nabla}\delta\vec{v}\right) + \vec{v}_A \left\{\vec{\nabla} \times \left[\vec{\nabla} \times \left(\delta\vec{v} \times \vec{v}_A\right)\right]\right\} = 0$$

- We seek solutions for plane waves propagating *parallel* and *perpendicular* to B-field
- Wave ansatz:

$$\delta\vec{v}\left(\vec{x}, t\right) = A \exp\left[i\left(\vec{k}\vec{x} - \omega t\right)\right]$$

- Dispersion relation:

$$-\omega^2 \delta\vec{v} + \left(c_s^2 + v_A^2\right)\left(\vec{k}\delta\vec{v}\right)\vec{k} + \vec{v}_A \vec{k}\left[\left(\vec{v}_A \vec{k}\right)\delta\vec{v} - \left(\vec{v}_A \delta\vec{v}\right)\vec{k} - \left(\vec{k}\delta\vec{v}\right)\vec{v}_A\right] = 0$$

Case Study for different type of waves

- Case 1: $\vec{k} \perp \vec{v}_A$

  dispersion relation reads then

$$-\omega^2 \delta\vec{v} + \left(c_s^2 + v_A^2\right)\left(\vec{k}\,\delta\vec{v}\right)\vec{k} \qquad = 0$$

$\Rightarrow$ $\delta\vec{v} \,//\, \vec{k}$ *Longitudinal magnetosonic wave*

Phase velocity

$$v_{ph} = \frac{\omega}{k} = \sqrt{c_s^2 + v_A^2}$$

- Case 2: $\vec{k} \,//\, \vec{v}_A$, i.e. $\vec{k} \,//\, \vec{B}_0$

  dispersion relation reads then

$$\left(k^2 v_A^2 - \omega^2\right)\delta\vec{v} + \left(\frac{c_s^2}{v_A^2} - 1\right) k^2 \left(\vec{v}_A \cdot \delta\vec{v}\right)\vec{v}_A = 0$$

- Two different types of waves satisfy this DR:
  - Case A: $\vec{k} \,//\, \delta\vec{v} \Rightarrow \vec{v}_A \,//\, \delta\vec{v}$

    thus $(\vec{v}_A \cdot \delta\vec{v})\vec{v}_A = v_A \delta v \dfrac{v_A}{\delta v} \delta\vec{v} = v_A^2 \delta\vec{v}$

    and $\left(c_s^2 k^2 - \omega^2\right)\delta\vec{v} = 0$

    the solution is just an ordinary (longitudinal) sound wave
  - Case B: $\vec{k} \perp \delta\vec{v} \Rightarrow \vec{v}_A \perp \delta\vec{v} \Leftrightarrow \vec{v}_A \cdot \delta\vec{v} = 0$

    the DR reads in this case $\left(v_A^2 k^2 - \omega^2\right)\delta\vec{v} = 0$

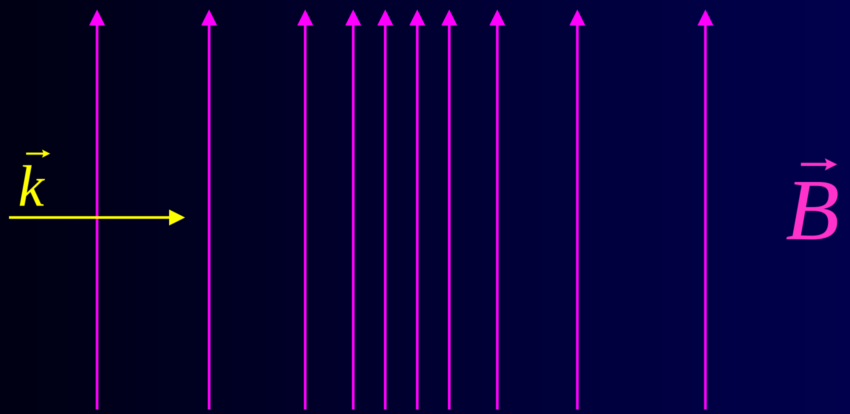
    thus $\dfrac{\omega}{k} = v_{ph} = v_A$

    the solution is a transverse Alfvén wave (pure MHD wave)

    driven by magnetic tension forces

    due to flux freezing gas (density ρ_0) must be set in motion!

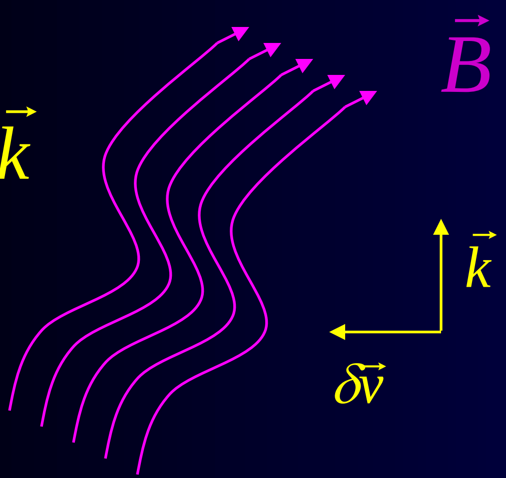
- Magnetosonic wave (longitudinal compression wave)



$\delta\vec{v} \,//\, \vec{k}$

- Transverse Alfvén wave

$\delta\vec{v} \perp \vec{k}$



Note 1: in both cases phase velocity independent of ω and k: dispersion free waves!

Note 2: in ISM density is low, Therefore Alfvén velocity high ~ km/s

I.3 Cosmic Rays

Cosmic Radiation

Includes -

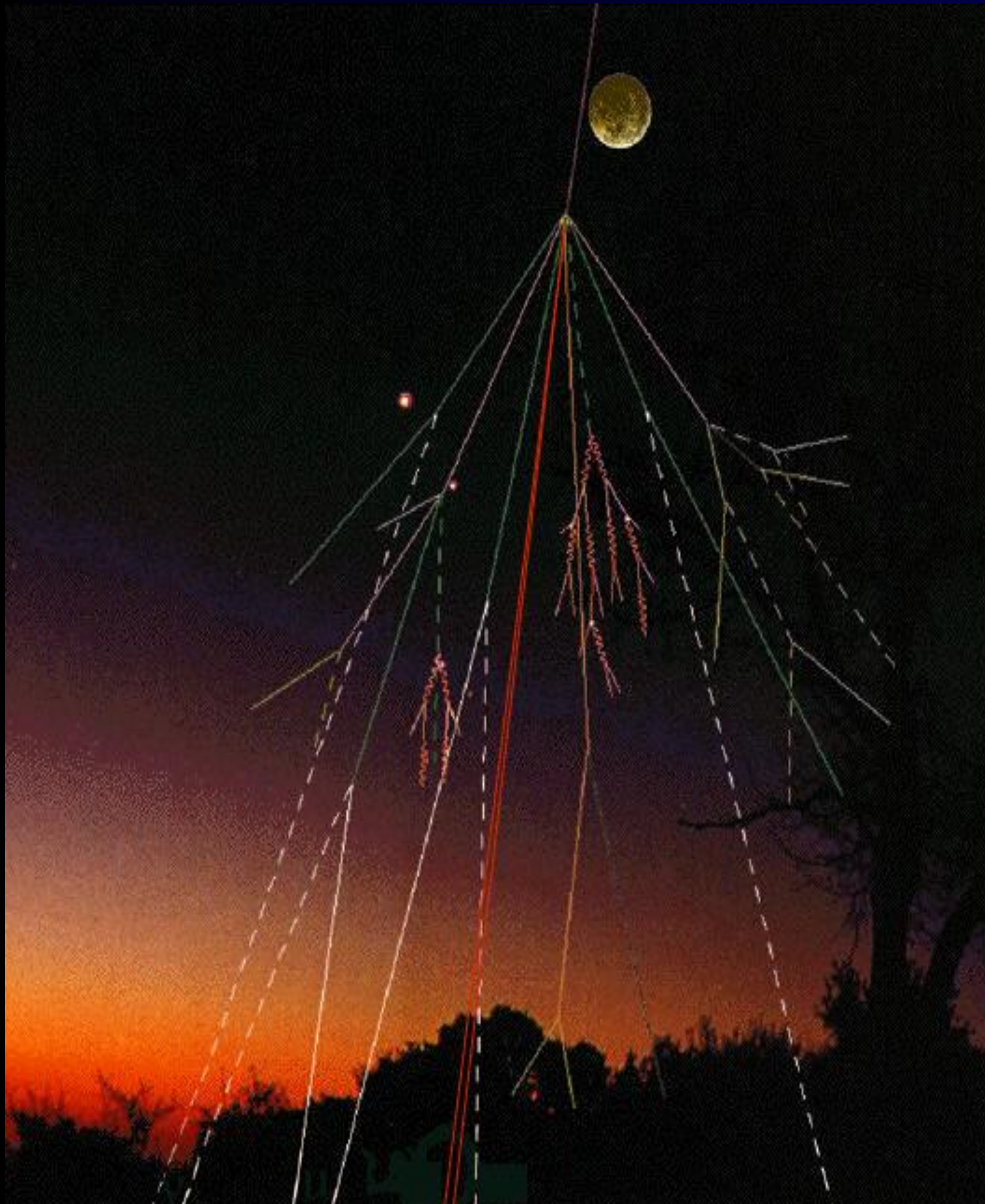
- **Particles** (2% electrons, 98% protons and atomic nuclei)
- **Photons**

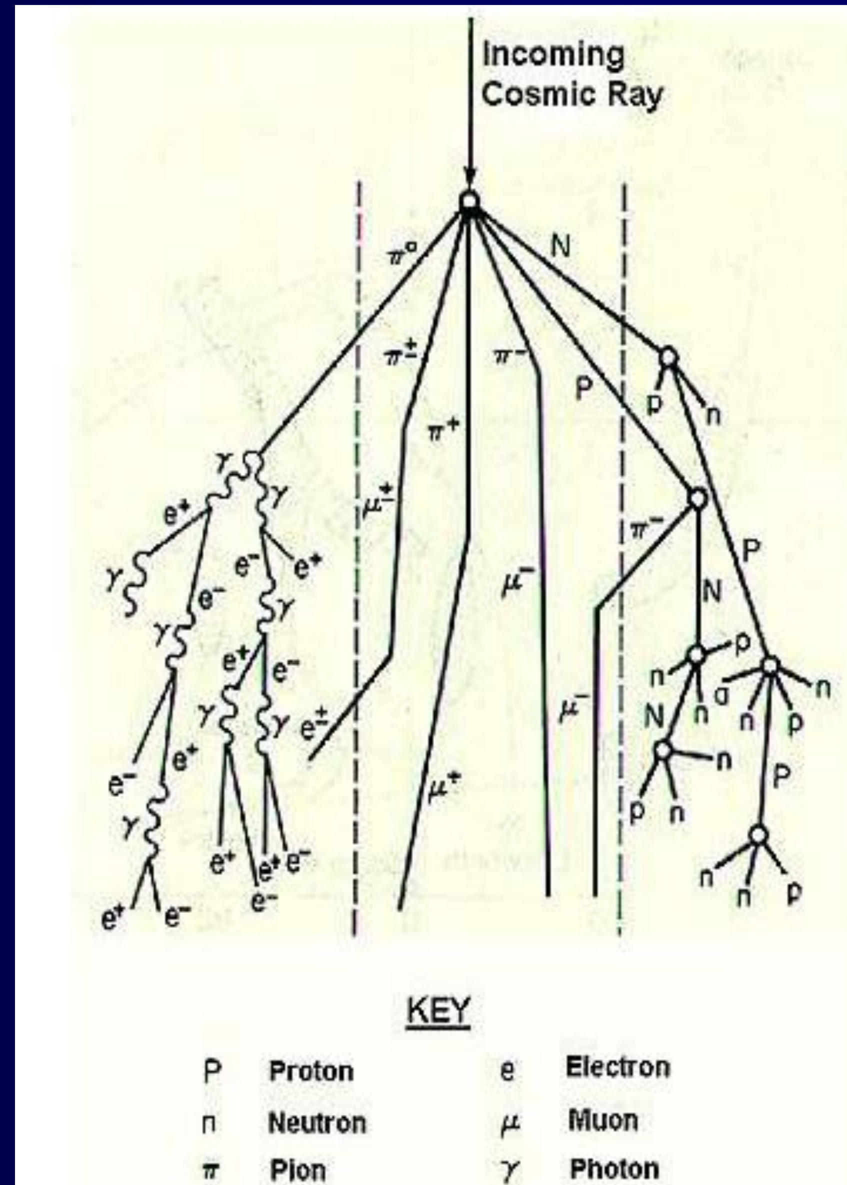
- Large energies ($10^9 eV \leq E \leq 10^{20} eV$)

□ γ-ray photons produced in collisions of high energy particles

Extraterrestrial Origin

- Increase of ionizing radiation with altitude
- 1912 Victor Hess‘ balloon flight up to 17500 ft. (without oxygen mask!)
- Used gold leaf electroscope



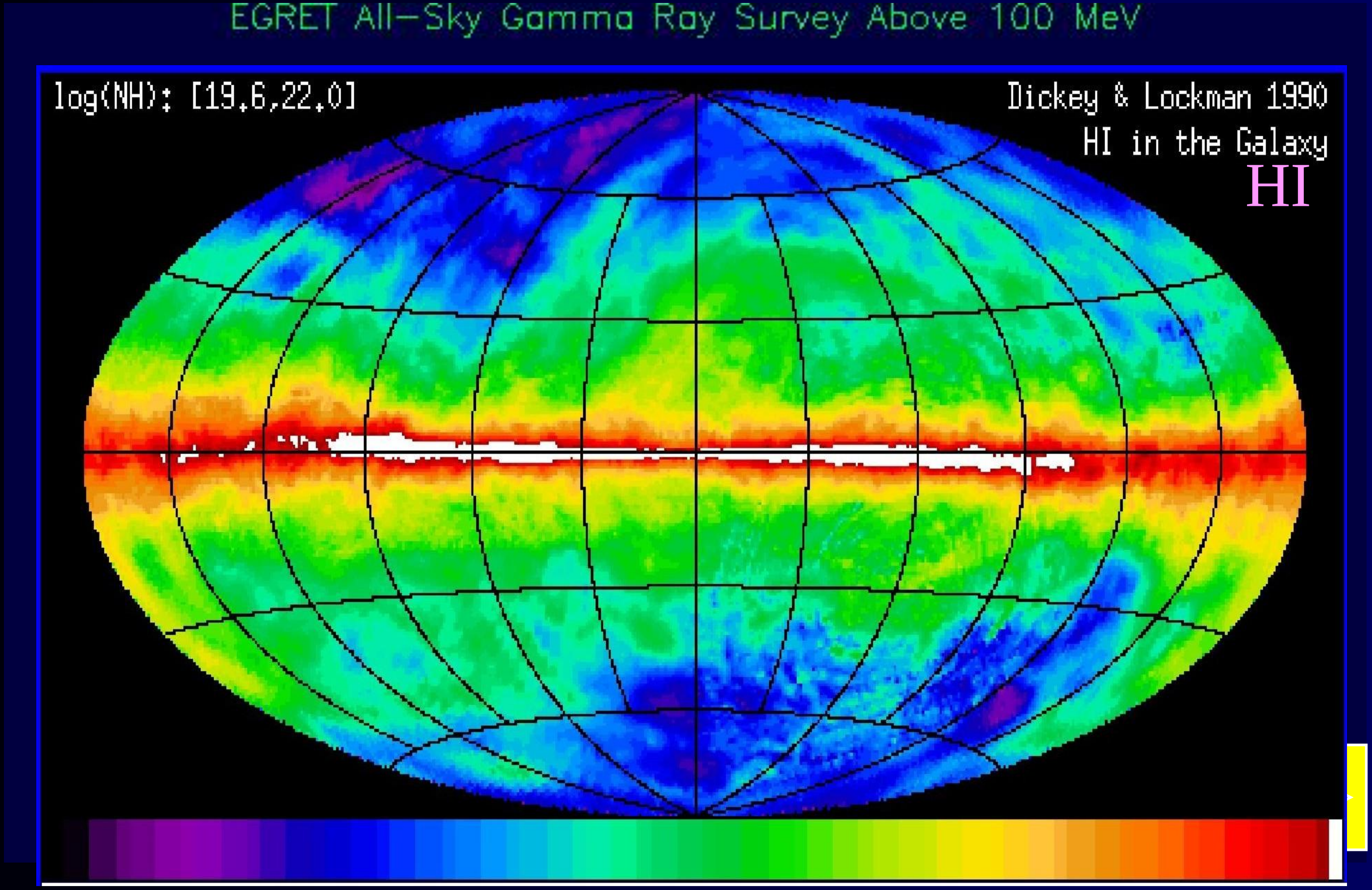

Incoming
Cosmic Ray
N
P
p
n
KEY
P Proton
n Neutron
π Pion
e Electron
μ Muon
γ Photon

Inelastic collision of CRs with ISM

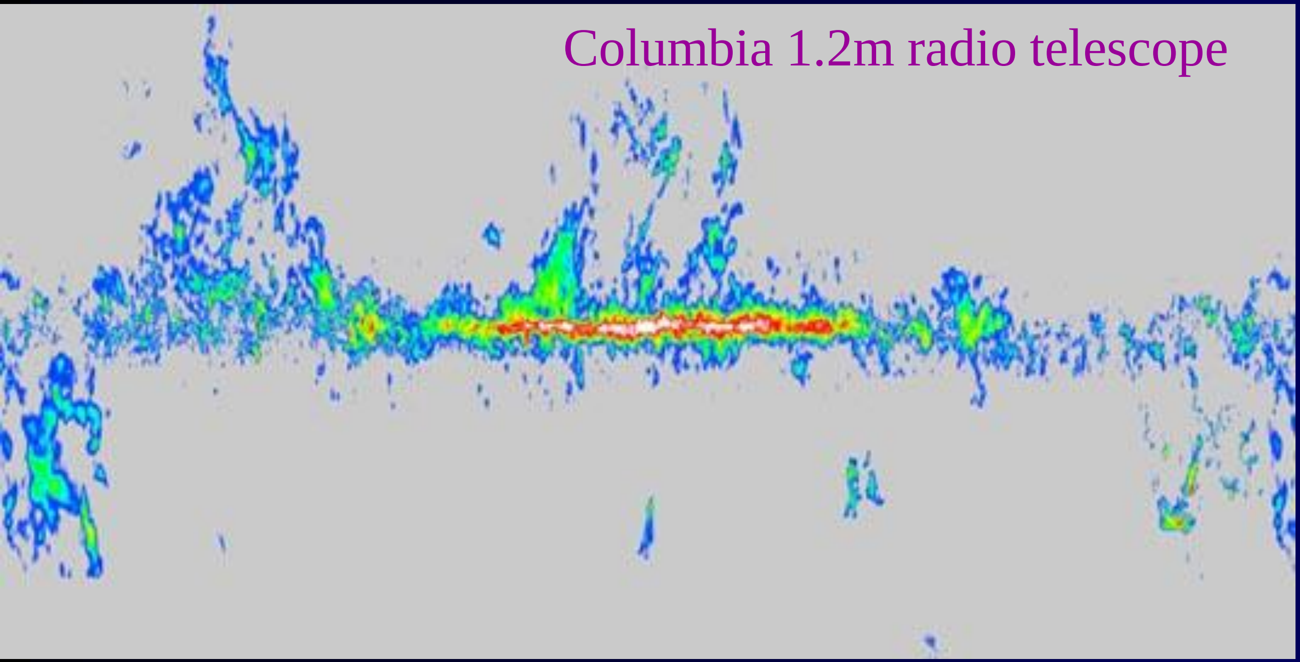
$$p + p \rightarrow p + p + \pi^0$$

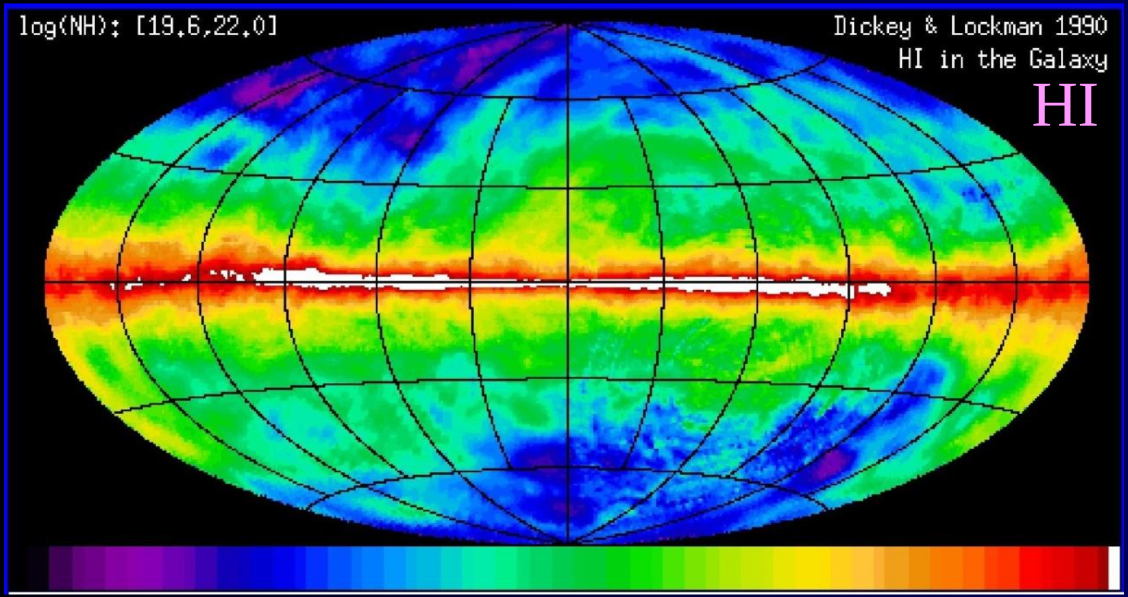
For E > 100 MeV dominant process

$$\pi^0 \rightarrow 2\gamma$$



- Diffuse γ-emission maps the Galactic HI distribution
- Discrete sources at high lat.: AGN (e.g. 3C279)





- Top: CO survey of Galaxy mapping molecular gas (Dame et al. 1997)
- Bottom: Galactic HI survey (Dickey & Lockman 1990)

γ-ray luminosity of Galaxy

- Probability that CR proton undergoes inelastic collision with ISM nucleus $P_{coll} = \sigma_{pp} n_H c, \quad \sigma_{pp} = 2.5 \times 10^{-26}\,\mathrm{cm}^2$
- 1/3 of pions are π^0 decaying with $< E_\gamma >\ \sim 180$ MeV
- If Galactic disk is uniformly filled with gas + CRs

  the total diffuse γ-ray luminosity is

$$L_\gamma = \frac{1}{3}\sigma_{pp} n_H c \sum n_{CR}(E) E = \frac{1}{3} P_{coll} \varepsilon_{CR} V_{gal}$$

- Galaxy with half thickness H=200 pc, $n_H \sim 1\ \mathrm{cm}^{-3}$, $\varepsilon_{CR} \sim 1\ \mathrm{eV/cm}^3$ ⟹ $V_{gal} \sim 2\ 10^{66}\ \mathrm{cm}^3$
- ⟹ $L_\gamma \approx 10^{39}$ erg/s in agreement with obs.!
- Thus γ-rays are tracer of Galactic CR proton distribution

Chemical composition

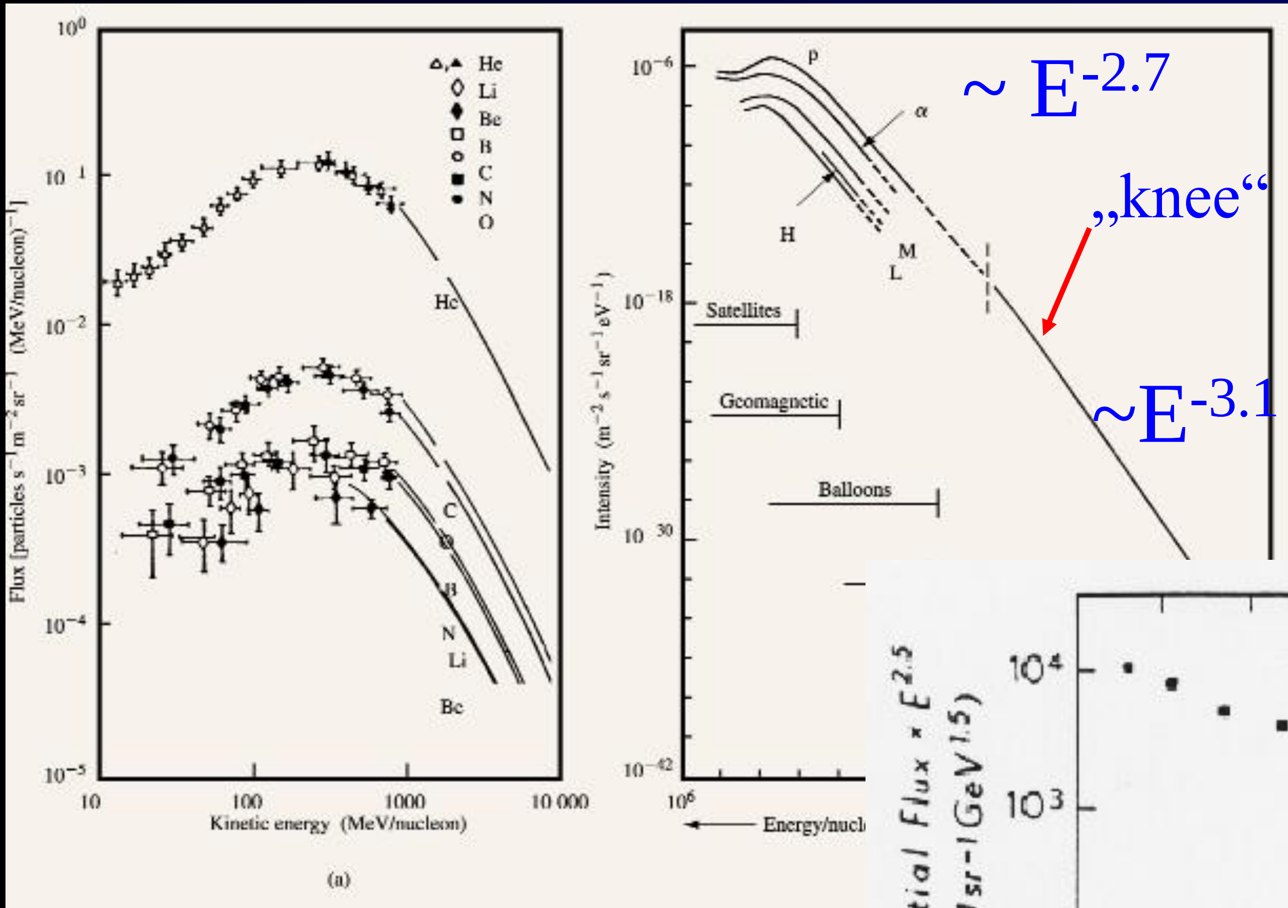
Groups of nuclei	Z	CR	Universe
Protons (H)	1	700	3000
α (He)	2	50	300
Light (Li, Be, B)	3-5	1	0.00001*
Medium (C,N,O,F)	6-9	3	3
Heavy (Ne->Ca)	10-19	0.7	1
V. Heavy	>20	0.3	0.06

Note: Overabundance of light elements → spallation!

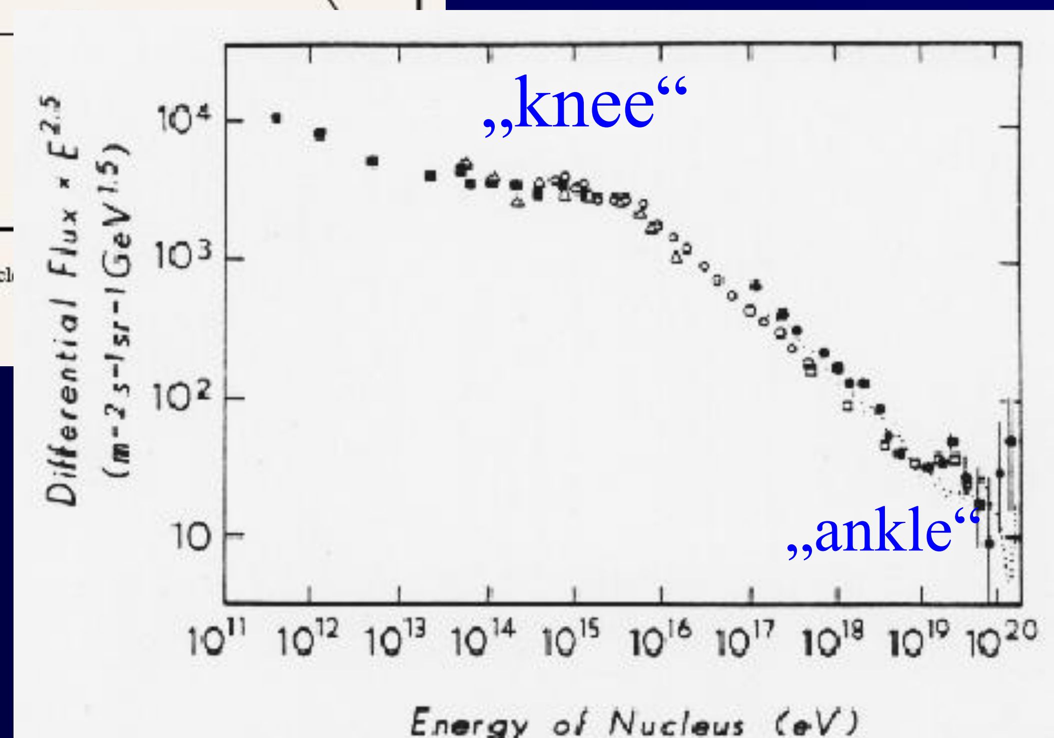
Origin of light elements

- Over-abundance of light elements caused by fragmentation of ISM particles in inelastic collision with CR primaries
- Use fragmentation probabilites and calculate transfer equations by taking into account all possible channels

Differential Energy Spectrum



- differential energy spectrum is power law for $10^{9} < E < 10^{15}$ eV $10^{16} < E < 10^{15}$ eV



- for $E > 10^{19}$ eV CRs are extragalactic ($r_g \sim 3$ kpc for protons)

Primary CR energy spectrum

- Power law spectrum for 10^9 eV < E < 10^{15} eV:

  $I_N(E) \propto E^{-\gamma}$ with $\gamma \approx 2.70$ or $N(E)dE = KE^{-\gamma}dE$

  $[I_N]$ = cm^{-2} s^{-1} sr^{-1} (GeV/nucleon)$^{-1}$
- Steepening for E > 10^{15} eV with γ = 3.08 („knee“)

  and becoming shallower for E > 10^{18} eV („ankle“)
- Below E ~ 10^9 eV CR intensity drops due to solar modulation (magnetic field inhibits particle streaming)

  gyroradius:

$$r_g = \frac{\gamma_L m_0 v \sin\vartheta}{ZeB} = \left(\frac{pc}{Ze}\right)\frac{\sin\vartheta}{Bc} = R\frac{\sin\vartheta}{Bc}$$

R ... rigidity, θ ... pitch angle

Example: CR with E=1 GeV has r_g ~ 10^{12} cm! For B ~ 1μG @ 10^{15} eV, r_g ~ 0.3 pc

Important CR facts:

- CR Isotropy:
  - Energies 10^{11} eV < E < 10^{15} eV: $\delta I / I \approx 6\times10^{-4}$ (anisotropy) consistent with CRs streaming away from Galaxy
  - Energies 10^{15} eV < E < 10^{19} eV:

    anisotropy increases -> particles escape more easily (energy dependent escape)

    Note: @ 10^{19} eV, $r_g \sim 3$ kpc
  - Energies E > 10^{19} eV: CRs from Local Supercluster?

    particles cannot be confined to Galactic disk
- CR clocks:
  - CR secondaries produced in spallation (from O and C) such as ^{10}Be have half life time $\tau_H \sim 1.6$ Myr -> β-decay into ^{10}B

– From amount of ^{10}Be relative to other Be isotopes and ^{10}B and τ_H the mean CR residence time can be estimated to be $\tau_{esc} \sim 2\ 10^7$ yr for a 1 GeV nucleon

CRs have to be constantly replenished!

What are the sources?

– Detailed quantitative analysis of amount of primaries and secondary spallation products yields a mean Galactic mass traversed („***grammage***" x) as a function of rigidity R:

$$x(R) = 6.9 \left(R / 20\,\mathrm{GV} \right)^{-\xi}\ \mathrm{g/cm^2},\ \xi = 0.6$$

for 1 GeV particle, x ~ 9 g/cm^2

– Mean measured CR energy density:

$$\boxed{\varepsilon_{CR} \sim \varepsilon_{mag} \sim \varepsilon_{th} \sim \varepsilon_{turb} \cong 1\,\mathrm{eV/cm^3}}$$

If *all* CRs were extragalactic, an extremely high energy production rate would be necessary (more than AGN and radio galaxies could produce) to sustain high CR background radiation

assuming energy equipartion between B-field and CRs radio continuum observations of starburst galaxy M82 give $\varepsilon_{CR}(M82) \sim 100\varepsilon_{CR}(Galaxy)$

CR production rate proportional to star formation rate

⇒ *no constant high background level!*

⇒ *CR interact strongly with B-field and thermal gas*

CR propagation:

- High energy nucleons are ultrarelativistic -> light travel time from sources $\tau_{lc} \sim L/c \approx 3\times10^4 \text{ yr} << \tau_{\text{esc}}$
- CRs as charged particles *strongly coupled to B-field*
- *B-field:* $\langle\vec{B}\rangle = \vec{B}_{reg} + \delta\vec{B}$ *with strong fluctuation component* $\delta\vec{B}$ *-> MHD (Alfvén) waves*
- Cross field *propagation by pitch angle scattering*

➡ random walk of particles!

➡ CRs **DIFFUSE** through Galaxy with mean speed

$$\langle v_{diff}\rangle \sim L/\tau_r \approx {}^{10\text{ kpc}}/_{2\times10^7\text{ yr}} = 490\text{ km/s} \sim 10^{-3}c$$

- Mean gas density traversed by particles

$$\langle\rho_h\rangle \approx x/c\tau_{esc} \sim 5\times10^{-25}\text{ g/cm}^3 \sim \frac{1}{4}\langle\rho_{ISM}\rangle$$

particles spend most time outside the Galactic disk in the Galactic *halo!* ⇒ *„confinement“ volume*

- CR „height“ ~ 4 times h_g (=250 pc) ~ 1 kpc
- CR diffusion coefficient:

$$\kappa \sim h_{CR} \times L/\tau_{esc} \approx 5\times 10^{28}\,\mathrm{cm}^2/\mathrm{s}$$

- Mean free path for CR propagation: $\lambda_{CR} \sim 3\kappa / c \sim 1\,\mathrm{pc}$

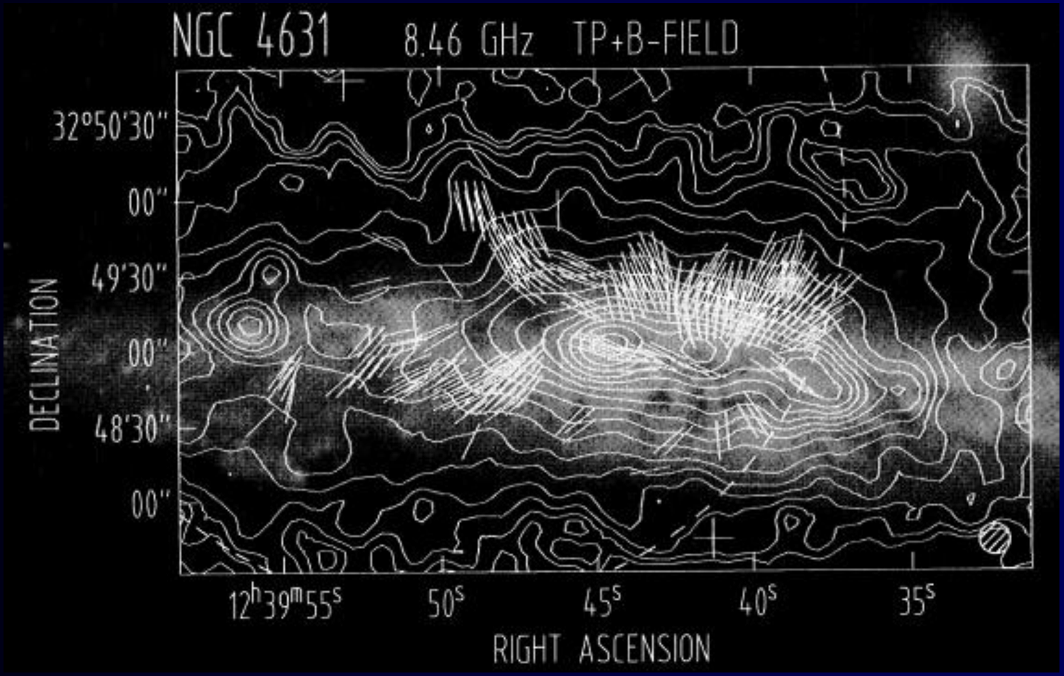
⇒ strong scattering off magnetic irregularities!

- Analysis of radioactice isotopes in meteorites: CR flux roughly constant over last 10^9 years

CR origin:

- CR electrons (~ 1% of CR paeticle density) must be of Galactic origin due to strong synchrotron losses in Galactic magnetic field and inverse Compton losses

Note: radio continuum observations of edge-on galaxies show strong halo field



- Estimate of total Galactic CR energy flux:

$$F_{CR} \sim \varepsilon_{CR} \, {}^{V_{conf}}\!\!\diagup_{\tau_{esc}} \approx 10^{41} \text{ erg/s}$$

Note: only ~ 1% radiated away in γ-rays!

- Enormous energy requirements leave as most realistic Galactic CR source supernova remnants (SNRs)
  - Available hydrodynamic energy:

    $$F_{SNR} \sim \nu_{SN} E_{SNR} \approx {}^{3}\!\diagup_{100 \text{ yr}} \cdot 10^{51} \text{erg} \approx 10^{42} \text{ erg/s}$$

    about 10% of total SNR energy has to be converted to CRs
  - Promising mechanism: *diffusive shock acceleration*
- *Ultrahigh energy CRs must be extragalactic*

  $$r_g \geq 100 \text{ kpc} > R_{gal} \quad (\text{for } E \sim 10^{20} \text{eV})$$

Diffusive shock acceleration:

- Problem: can mechanism explain power law spectrum?
- Diffusive shock acceleration (DSA) can also explain near cosmic abundances due to acceleration of ISM nuclei
- Acceleration in electric fields?

$$\frac{d}{dt}\left(\gamma_L m\vec{v}\right) = e\left(\vec{E} + \frac{1}{c}\left[\vec{v}\times\vec{B}\right]\right)$$

due to high conductivity in ISM, static fields of sufficient magnitude do not exist

thus strong induced E-fields from strongly time varying large scale B-fields could help -> no strong evidence!

Fermi mechanism:

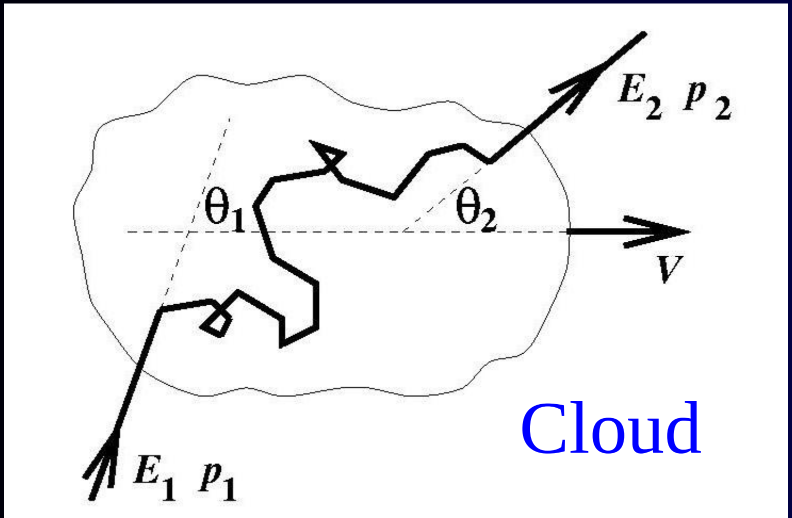
- Fermi (1949): randomly moving clouds reflect particles in converging frozen-in B-fields („magnetic mirrors“)

⇨ processes is 2nd order, because particles gain energy by head-on collisions and lose energy by following collisions (2nd order Fermi process)

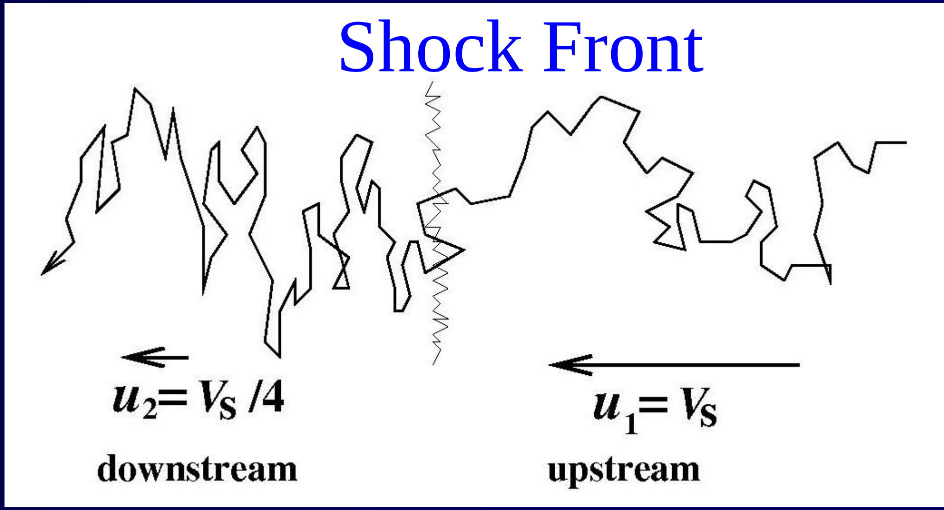
⇨ particles gain energy stochastically by collisions

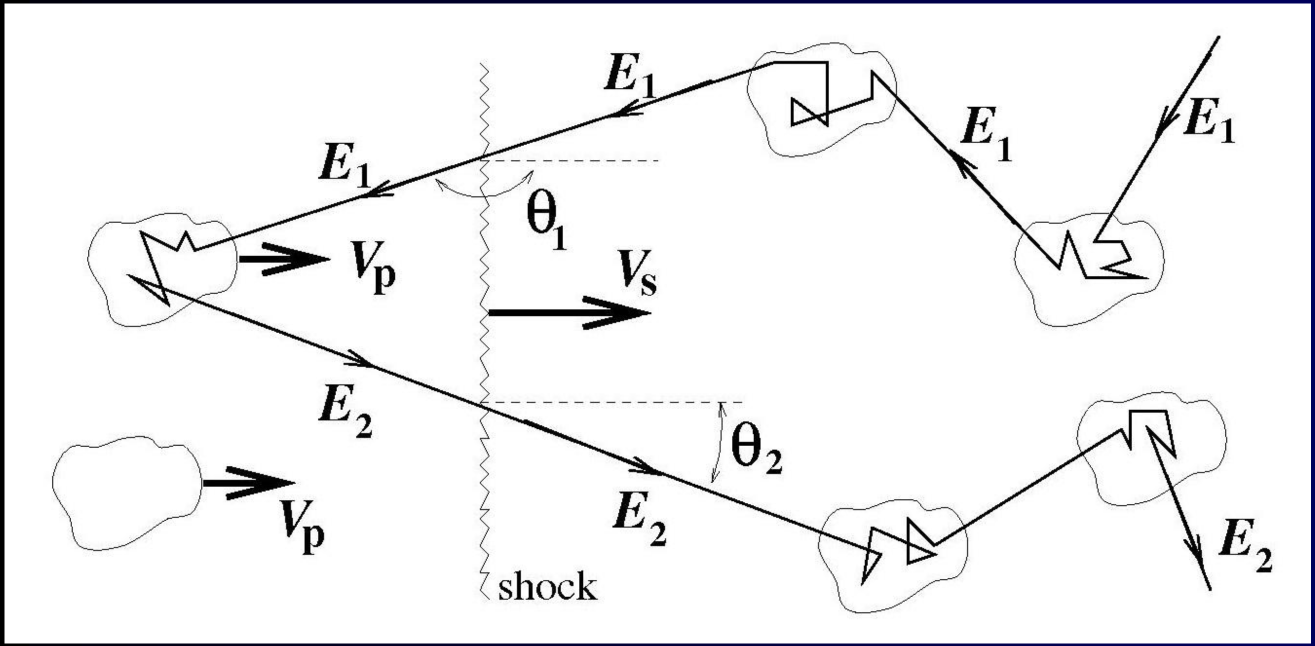
- 1st order Fermi process (more efficient):
  - Shock wave is a converging fluid
  - Particles are scattered (elastically) strongly by field irregularities (MHD waves) back and forth

2nd order Fermi



1st order Fermi





- Energy gain per crossing:

$$\frac{\Delta E}{E} \approx \frac{4}{3}\left(\frac{V}{c}\right)^2 \quad (2^{nd}\text{ order Fermi})$$

- Shock is converging fluid

$$\frac{\Delta E}{E} \approx \frac{4}{3}\frac{V_S}{c} \quad (1^{st}\text{ order Fermi})$$

- Energy gain per collision: ΔE/E ~ (Δv/c)
- Escape probability downstream increases with energy

$$P_{esc} \approx \frac{V_S}{v} \quad (v \ldots \text{particle speed}) \Longrightarrow P = 1 - P_{esc} \approx 1 - \frac{V_S}{v}$$

- Essence of statistical process:

  let $E = \beta\, E_0$ be average energy of particle after one collision

  and P be probability that particles remains in acceleration process -> after k collisions:

$$N(>E) = N_0 P^k \text{ particles with energies } E = E_0 \beta^k$$

$$\Rightarrow \frac{\ln(N/N_0)}{\ln(E/E_0)} = \frac{\ln P}{\ln \beta}$$

$$\boxed{\beta \equiv \frac{E}{E_0} = 1 + \frac{\Delta E}{E} \approx 1 + \frac{V_S}{c}}$$

$$\Rightarrow \frac{N}{N_0} = \left(\frac{E}{E_0}\right)^{\ln P/\ln \beta}$$

$$\boxed{\frac{\ln \beta}{\ln P} = \frac{\ln\left(1 + \frac{V_S}{c}\right)}{\ln\left(1 - \frac{V_S}{v}\right)} \approx \frac{1 + \frac{V_S}{c}}{-\frac{v}{c}\left(1 - \frac{V_S}{v}\right)} \approx -\frac{1}{v/c} \approx -1, \quad (\text{for } V_S << v \le c)}$$

– Note: that N=N(>E), since a fraction of particles is accelerated to higher energies

therefore differential spectrum given by

$$N(E)dE = const. \times E^{(\ln P/\ln \beta)-1} dE$$

– Note: statistical process leads *naturally* to *power law* spectrum!

– Detailed calculation yields: $\frac{\ln P}{\ln \beta} \approx -1$ e.g. Bell (1978)

– Hence the spectrum is:

$$N(E)dE = const. \times E^{-2} dE$$

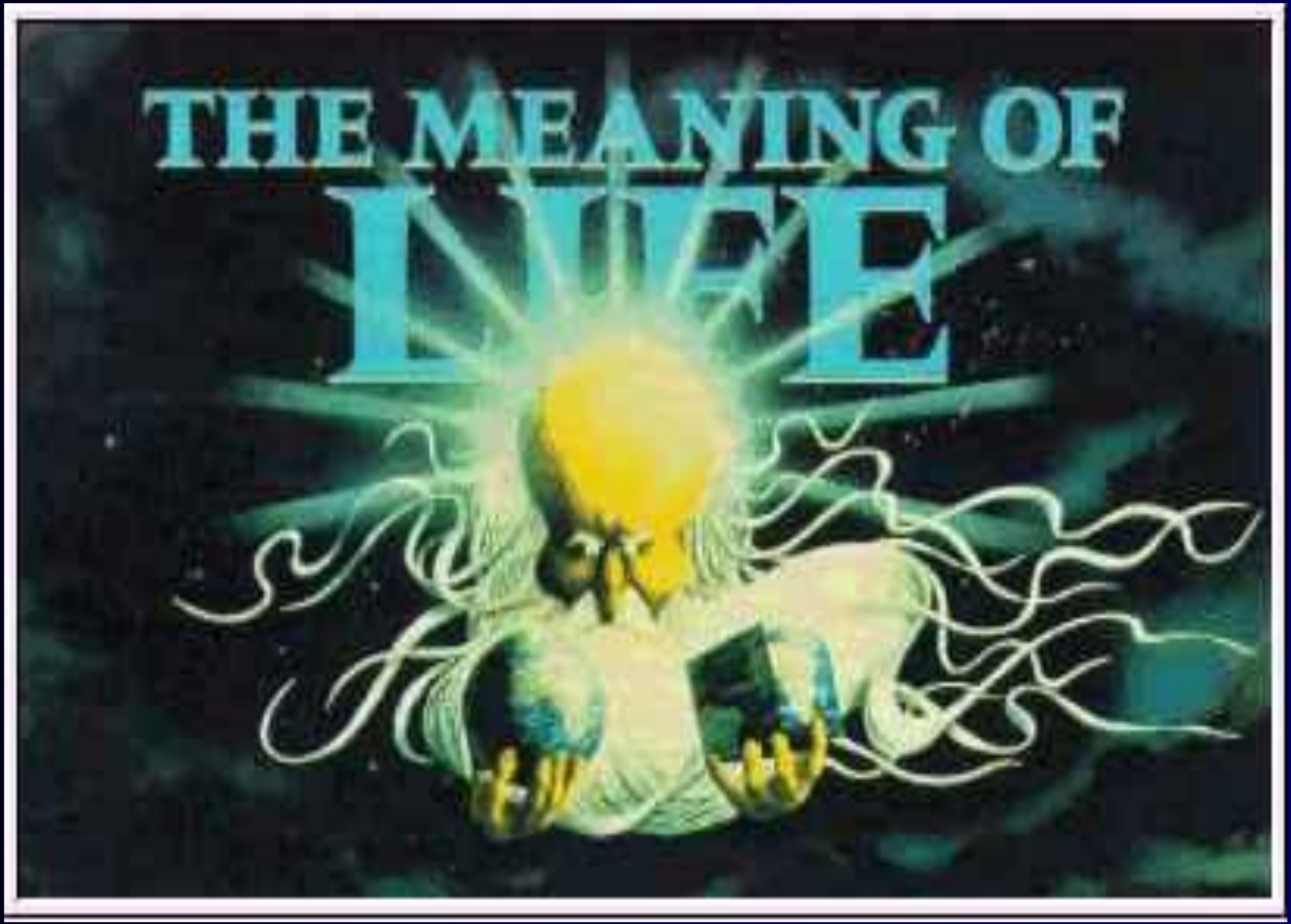
– The measured spectrum E < 10^{15} eV gives spectral index -2.7

However: CR propagation (diffusion) is energy dependent $\propto E^{0.6}$

⟹ source spectrum $\propto E^{-2.1}$: ***excellent agreement!***

PROBLEM: Injection of particles into acceleration mechanism

Maxwell tail not sufficient ⟹ „suprathemal“ particles



- remains unsolved! -

LECTURE 2
Dynamical ISM Processes

II.1 Gas Dynamics & Applications

- ISM is a compressible magnetized plasma
- $\lambda_{mfp} << L$
- Pressure disturbances due to energy + momentum injection: SNe, SWs, SBs, HII regions, jets
- Speed of sound: $c_s = \sqrt{\frac{k_B T}{\mu m}}, \quad 0.3 \leq c_s \leq 120 \text{ km/s}$

  ⟹ ISM motions are supersonic: $M = \frac{u}{c_s} >> 1$
- Shocks (collisionless) propagate through ISM ($\lambda_{mfp} >> \Delta$)

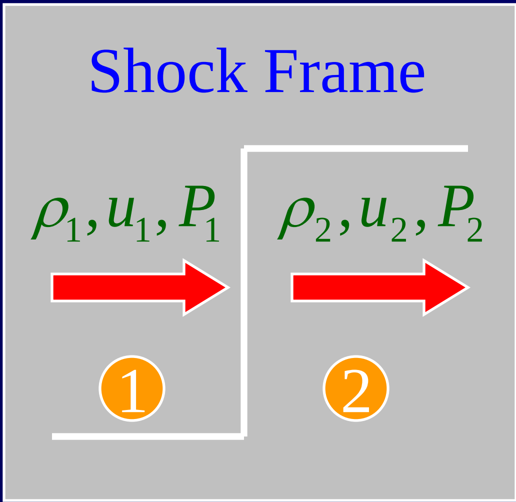
II.2Shocks

- Assumption: Perfect gas, B=0
- Shock thickness $\Delta \le \lambda_{mfp}$ ⟶ $\rho, \; u, P$ time independent across shock discontinuity: steady shock
- Conservation laws: *Rankine-Hugoniot* conditions

$$\rho_1 u_1 = \rho_2 u_2 \quad \text{(mass)}$$

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2 \quad \text{(momentum)}$$

$$\frac{1}{2}\rho_1 u_1^2 + \frac{\gamma}{\gamma - 1}\frac{P_1}{\rho_1} = \frac{1}{2}\rho_2 u_2^2 + \frac{\gamma}{\gamma - 1}\frac{P_2}{\rho_2} \quad \text{(energy)}$$

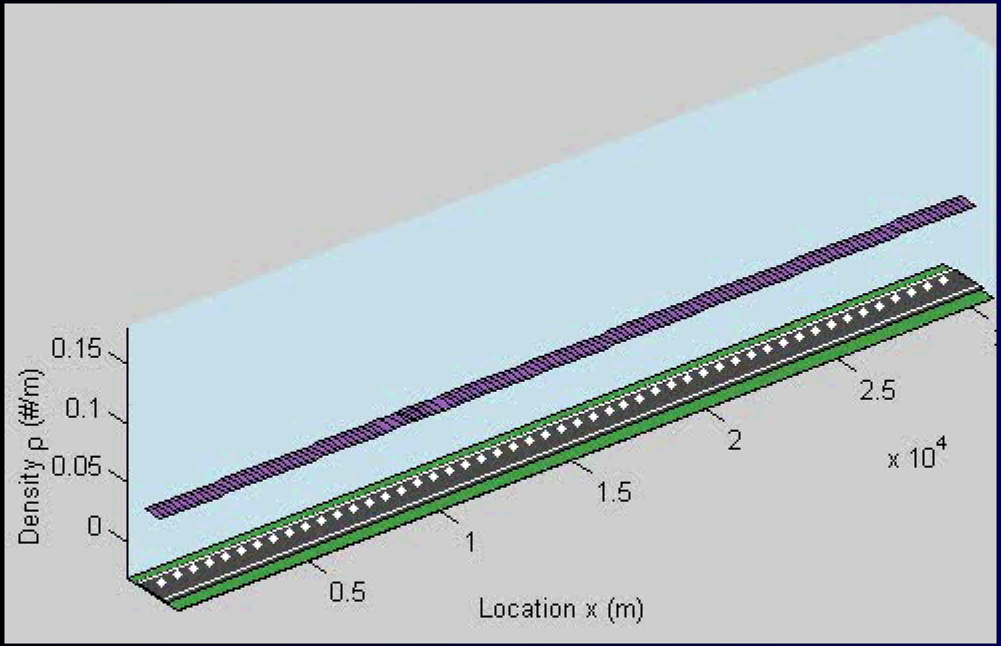


Analoga

1. Traffic Jam:

„speed of sound“ c_s= vehicle distance/reaction time=d/τ

- If car density is high and/or people are „sleeping“: c_s decreases
- If $v_{car} \geq c_s$ then a shock wave propagates backwards due to „supersonic“ driving
- Culprits for jams are people who drive too fast or too slow because they are creating constantly flow disturbances



- For each traffic density there is a maximum current density j_{max} and hence an optimum car speed to make $dj_{max}/d\rho = 0$!

2. *Hydraulic Jump:*

„speed of sound“ $c_s = \sqrt{gh}$ („shallow water“ theory)

Kitchen sink experiment

- total pressure: $P_{tot} = P_{ram} + P_{hyd}$

$$P_{tot} = \rho v^2 + \rho gh$$

- At bottom: $v > c_s$
- Therefore: abrupt jump
- Behind jump:

  $h_2 > h_1$

  $V_2 < V_1$

- $V_1 > c_1$ „supersonic“

  $V_2 < c_2$ „subsonic“

II.3 HII Regions

- Rosette Nebula (NGC2237):
  - Exciting star cluster NGC 2244, formed ~ 4 Myr ago
  - Hole in the centre: Stellar Winds creating expanding bubble

Trifid Nebula:
Stellar photons heat molecular cloud gas
Gasdynamical expansion (e.g. jets)

Facts

- Lyc photon output S_* of O- and B stars ionizes ambient medium to T ~ 8000 K
  - In ionization equilibrium: $\dot{N}_I = \dot{N}_{rec}$
  - Energy input Q per photon: $\dot{N}_I Q = \dot{N}_{rec} Q$

=

  - Energy loss per recombination: $\left(\frac{3}{2} k_B T_e\right) \dot{N}_{rec}$

Equil. Temperature:

$$\Rightarrow \dot{N}_{rec} Q = \left(\frac{3}{2} k_B T_e\right) \dot{N}_{rec}$$

$$\boxed{T_e = \frac{2}{3} \frac{Q}{k_B}}$$

- Q depends on stellar radiation field and frequency dependence of ioniz. cross section

- Assumption: *stellar rad. field is blackbody*

average kinetic energy per photo-electron:

$$\langle Q\rangle = \int_{\nu_L}^{\infty} (h\nu - E_H)\, S_{*\nu}\, d\nu \Big/ \int_{\nu_L}^{\infty} S_{*\nu}\, d\nu\,,\;\; E_H = h\nu_L$$

For a blackbody at temperature T_*:

$$h\nu\, S_{*\nu} \propto B_\nu(T_*) = \frac{2h\nu}{c^2}\left[\exp\left(h\nu/k_B T_*\right) - 1\right]^{-1}$$

For $h\nu/k_B T_* >> 1$

$$\boxed{\langle Q\rangle \approx k_B T_e \quad \Rightarrow T_e \approx \frac{2}{3} T_*}$$

$T_* = 47000$ K (for O5 star) $\Rightarrow$ $T_e \sim 31300$ K

Bad agreement with observation!

Reason: *forbidden line cooling* of heavy elements, like [OII], [OIII], [NII] is missing!

- For H the total recombination coefficient to all excited states is: $\beta^{(2)}(T_e) = 2\times 10^{-10} T_e^{-3/4}\ \mathrm{cm}^3\ \mathrm{s}^{-1}$
- Thus $\dot{N}_{rec} Q = n_e n_H \beta^{(2)} k_B T_*$
- If [OII] is the dominant ionic state: $n_{OII} \approx 6\times 10^{-4} n_e$
- Collisional excitation rate (all ions are approx. in the ground state): $N_{ij} = n_e n_I C_{ij}(T_e)$

$$C_{ij}(T_e) = \left(A_{ij} / T_e^{1/2}\right) \exp[-E_{ij}/k_B T_e]$$

- Radiative energy loss by [OII] for ${}^2D_{5/2}$ and ${}^2D_{3/2}$ levels

$$L_{OII} \approx 1.1\times10^{-32}\, y_{OII}\left(n^2/T_e^{1/2}\right)\exp[-3.89\times10^4\,K/T_e]\,\mathrm{erg\,cm^{-3}s^{-1}}$$

taking $y_{OII} \approx 1$ (i.e. all O is in O$^+$)

and demanding $L_{OII} = G_{heat} = \dot{N}_I Q = \dot{N}_{rec} Q$

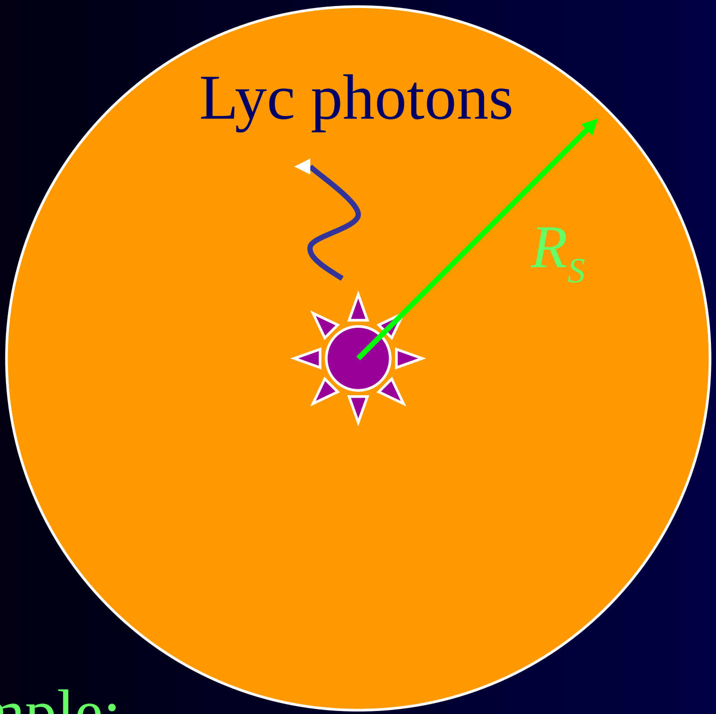
$$\boxed{T_e^{1/4}\exp[-3.89\times10^4\,K/T_e] = 2.5\times10^{-6}\,T_*}$$

For T_*=40000 K, we obtain: T_e~8500 K, in excellent agreement with observations!

- HII regions are thermostats!
- major cooling by forbidden lines

Dynamics of HII Regions

Case A: Static HII Region



- Spherical symmetric Model:
  - Ionization within radius r:

$$S_* = 4\pi\, r^2 J$$

  - Recombination within r:

$$\frac{4}{3}\pi\, R_s^3\, \beta^{(2)} n_H n_e$$

  - Ion. fract. $x \approx 1 \Rightarrow n_e \approx n_H = n$
  - Balance of ion. + recomb.

$$R_S = \left(\frac{3\, S_*}{4\pi\beta^{(2)} n^2} \right)^{1/3}$$

… „Stroemgren“ radius

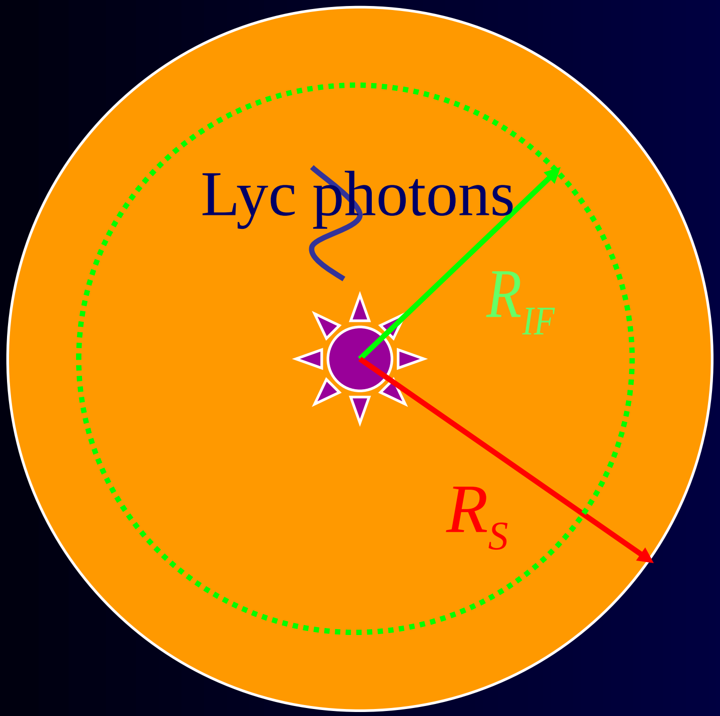
Example:

$\beta^{(2)} = 2\times 10^{-13}\,\mathrm{cm^3 s^{-1}}$ ($T \approx 8000$ K)

$n_H = 10^2\ \mathrm{cm^{-3}}$, $S_* = 10^{49}\ \mathrm{s^{-1}}$ (O6.5 type)

$\Rightarrow R_S \approx 3\,\mathrm{pc}$

Case B: Evolving HII Region



- Photon flux at IF:

$$J = \frac{S_*}{4\pi R^2} - \frac{1}{3}\beta^{(2)} n_0^2 R$$

(conservation of photons)

- Thickness of IF ~ photon mfp: planar geometry; no rec. in IF
- Ambient medium at rest
- IF velocity: $n_0 \frac{dR}{dt} = J$
- Define dimensionless quant.

$$\eta = \frac{R}{R_S},\ \tau = \frac{t}{\tau_{rec}},\ V_R = \frac{R_S}{\tau_{rec}},\quad \tau_{rec} = \left(n_0 \beta^{(2)}\right)^{-1}$$

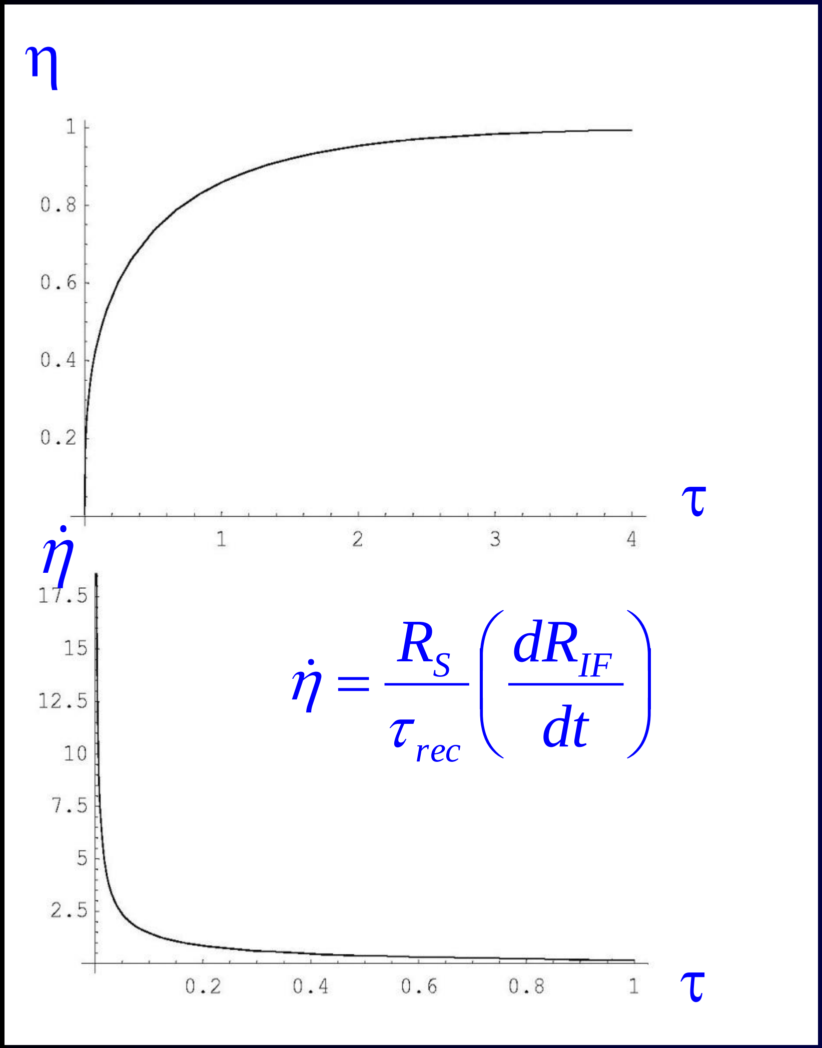
Equation of motion:

$$\dot{\eta} = \left(1 - \eta^3\right) / 3\eta^2$$

Solution:

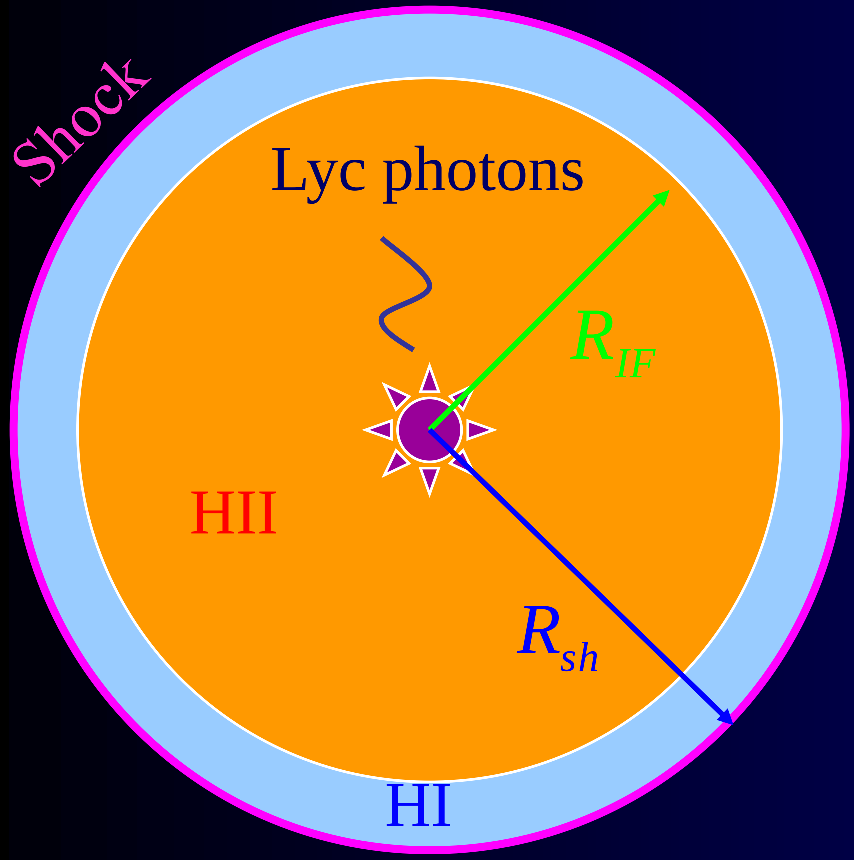
$$\boxed{\begin{aligned} &\eta = C\left(1 - e^{-\tau}\right)^{1/3} \\ &\mathrm{BC}: \tau \to 0, \eta \to 0 \end{aligned}}$$

Evolution of Stroemgren sphere



- R_S is reached only within 1% after $\tau \geq 4\tau_{rec}$
- IF velocity >> c_{II} until R~ R_S
  therefore $n_{II} \approx n_0$
  then gasdynamical expansion
- IF velocity slows down rapidly
- However: $\frac{dR_{IF}}{dt} << c_{II}$
  NOT possible

Case C: Gasdynamical Expansion of HII Region



- When $\dot{R}_{IF} \le c_{II}$ sound waves can reach IF
- $P_{II} >> P_I$: HII gas acts as piston
  ➡ shock driven into HI gas
- Gas pressed into thin shell:
  $R_{IF} \approx R_{sh} := R$
- Pressure uniform in shell & HII region: $\tau_{sc} << \tau_{dyn}$
- Ionization balance in HII reg.

BUT: HII grows in size + mass

$$\frac{dM_{II}}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi R_S^3 n_{II} \overline{m}\right) = -\frac{\overline{m}S_*}{\beta^{(2)}}\frac{1}{n_{II}^2}\frac{dn_{II}}{dt} > 0$$

Simple Model:

- $P_{sh} \approx P_{II} = 2n_{II}k_B T_{II} = n_{II}\overline{m}c_{II}^2$
- For strong shock: $P_{sh} = \dfrac{2}{\gamma+1}\rho_0 V_{sh}^2 = \rho_0 V_{sh}^2 \; (\text{for } \gamma = 1)$
- Ion. Balance: $S_* = \dfrac{4}{3}\pi\, n_{II}^2 \beta^{(2)} R^3 = \dfrac{4}{3}\pi\, n_0^2 \beta^{(2)} R_S^3$

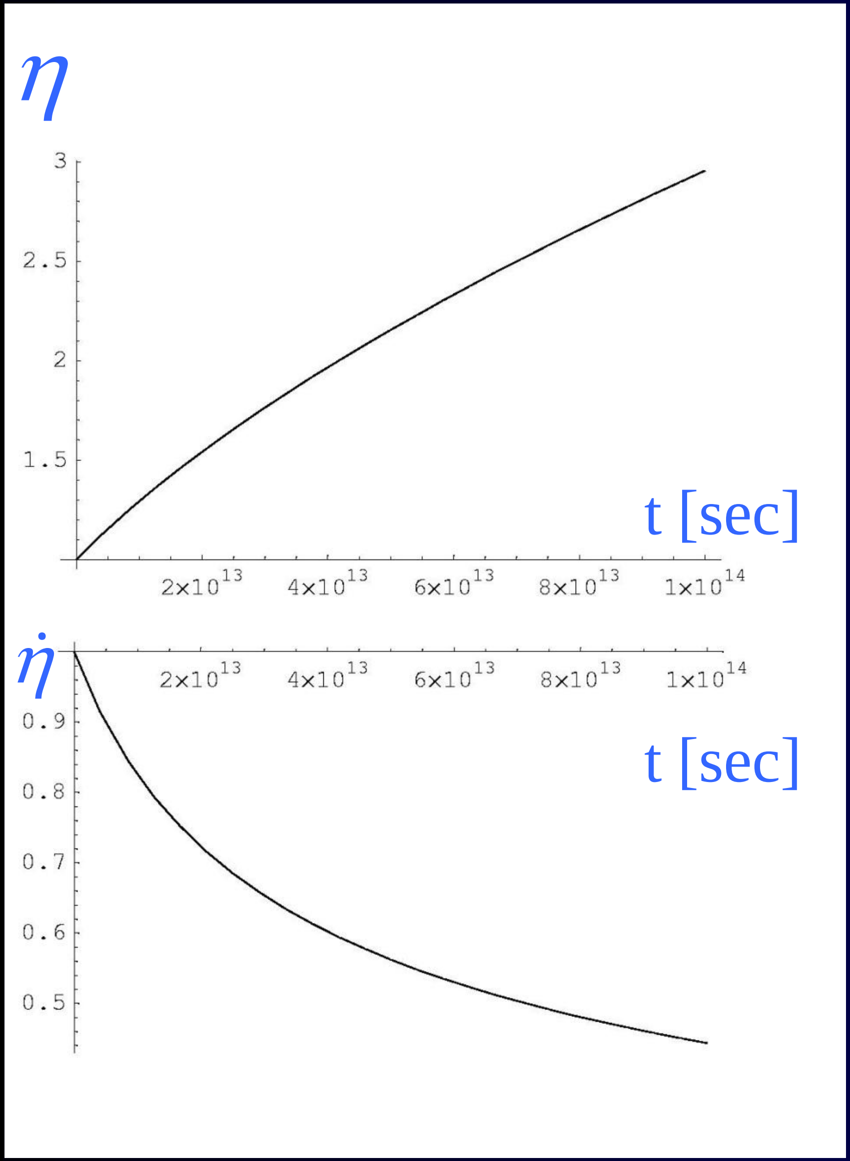
  $\Rightarrow \dfrac{n_{II}}{n_0} = \left(\dfrac{R_S}{R}\right)^{3/2}$
- Ambient medium at rest: $V_{sh} = \dot{R}$
- Thus equation of motion: $\dot{R}^2 = \left(\dfrac{n_{II}}{n_0}\right) c_{II}^2 = \left(\dfrac{R_S}{R}\right)^{3/2} c_{II}^2$
- Dimensionless quantities:

  $\eta = \dfrac{R}{R_S}, \quad N = c_{II}t / R_S, \; \dot{\eta} = \dot{R}/c_{II} \Rightarrow \dot{\eta}\eta^{3/4} = 1$

  $\mathrm{BC}: \eta \to 1, N \to 0$
- Solution:

$$\eta = \left(1 + \frac{7}{4}N\right)^{4/7}, \quad \dot{\eta} = \left(1 + \frac{7}{4}N\right)^{-3/7}$$

Gasdynamical Expansion:



- At $N = 0,\ \dot{R} = c_{II}$
- Note: $t_{\exp} >> \tau_{rec}$
- Is pressure equilibrium reached: $P_{II} \approx P_I$?

$$P_{II} \approx P_I \Rightarrow 2n_f k_B T_{II} = n_I k_B T_I$$

- Ionization balance must hold: $S_* = \frac{4}{3}\pi\, n_f^2 \beta^{(2)} R_f^3$
- Result: $n_f = (T_I / 2T_{II}) n_I$

$$\Rightarrow R_f = (2T_{II} / T_I)^{2/3} R_S$$

- Final mass: $\frac{M_f}{M_S} = \frac{n_f R_f^3}{n_0 R_S^3} = \frac{2T_{II}}{T_I}$

- Example:

$$T_I = 100\,\mathrm{K},\ T_{II} = 8000\,\mathrm{K},\ n_0 = 100\,\mathrm{cm}^{-3}$$

$$\Rightarrow n_f / n_0 = 5\times10^{-3},\ R_f / R_S \approx 34,\ M_f / M_S \approx 100$$

- Initial mass in Stroemgren sphere is a SMALL fraction of final mass

$$\text{For } \eta = 34 \Rightarrow t_f \sim 273\, R_S / c_{II} \approx 1.7\times10^{13}\ \mathrm{n}_0^{-2/3}\mathrm{yr}$$

$$\approx 7.8\times10^{7}\ \mathrm{yr}$$

- Equilibrium never reached, because star leaves main sequence before, unless density is high!

$$n_0 \geq 3\times10^{3}\,\mathrm{cm}^{-3}$$

II.4 Stellar Winds

- Massice star BD+602522 blows bubble into ambient medium
- Ionizing photons produce bright nebula NGC7635
- Diameter is about 2 pc
- Part of bubble network S162 due to more OB stars

Some Facts

6 Taresch et al. : Quantitative Analysis of the FUV, UV and optical spectrum of the O3 star HD 93129A

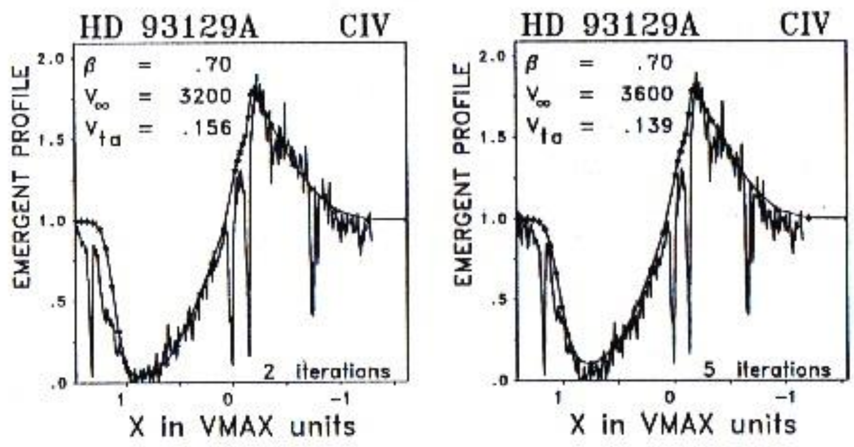


Fig. 5. Line fits to the P-Cygni profile of C IV with V_∞=3200 km s^{-1} and V_∞=3600 km s^{-1}. X is the Doppler shift relative to the laboratory wavelength of the blue doublet component in units of the terminal velocity V_∞. V_{ta} is the microturbulence velocity in units of V_∞.

$V_W \sim 2000 - 3000$ km/s

$\dot{M}_W \sim 10^{-6}$ M$_\odot$/yr

Total outward force:

$$F_W = \frac{\sigma L}{4\pi r^2}$$

In reality:

$$\sigma = \sigma(\lambda)$$

- O- and B-stars:
  - –Burn Hot
  - –Live Fast
  - –Die young

- Strong UV radiation field
- Momentum transfer to gas
- Resonance lines show P Cygni profiles (e.g. CIV, OVI)

Line Driven Stellar Wind:

- Assume a stationary wind flow (ignore P_{th})

$$mv\frac{dv}{dr} = -\frac{GM_*m}{r^2} + \frac{\sigma L}{4\pi\, r^2}$$

radiation pressure

$$v\frac{dv}{dr} = -\frac{GM_*}{r^2}(\Gamma - 1),$$

$$\Gamma = \frac{L_*\sigma}{4\pi GM_*mc} = \frac{L_*}{L_c}$$

$$L_c = \frac{4\pi GM_*mc}{\sigma} \quad \text{(Eddington luminosity)}$$

- Stars with $L_*>L_c$ are radiatively unstable

- Integration:

$$\int_{v_0}^{v} v\,dv = \int_{R_*}^{r} \frac{GM_*}{r^2}(\Gamma - 1)\,dr \Leftrightarrow \frac{1}{2}\left(v^2 - v_0^2\right) = GM_*(\Gamma - 1)\left(\frac{1}{R_*} - \frac{1}{r}\right)$$

- If $v(R_*)=v_0=0$:

$$v(r) = v_\infty\left(1 - \frac{R_*}{r}\right)^{1/2}$$

$$v_\infty = \left[\frac{2GM_*}{R_*}(\Gamma - 1)\right]^{1/2} = v_{esc}\sqrt{\frac{L_*}{L_c} - 1}$$

This is CAK velocity profile ($L_* > L_c$, i.e.$\Gamma>1$ needed)

- If $L >> L_c$ $\quad v_\infty = \sqrt{\frac{\sigma L_*}{2\pi R_* mc}}$

Example:

- For an O5 star: $L_* = 7.9\ 10^5\ L_\odot$, $R_* = 12\ R_\odot$

$$v_\infty = 10\ \text{km/s}\ \left(\frac{\sigma}{\sigma_T}\right)^{1/2}\left(\frac{m_p}{m}\right)^{1/2}\left(\frac{L_*}{L_0}\right)^{1/2}\left(\frac{R_0}{R_*}\right)^{1/2}$$

$$\Rightarrow v_\infty \approx 2566\ \text{km/s}$$

- Mass loss rate (assume one interaction per photon):

$$\frac{L_*}{c} = \dot{M}_W v_\infty$$ Momentum rate

Thus we get: $\dot{M}_W \approx 6.3\times10^{-6}\ M_\odot/\text{yr}$

- Shortcomings: cross sect. λ dep., multiple scattering
- Note: $L_W = \frac{1}{2}\dot{M}_W V_W^2 \approx 1.3\times10^{37}\ \text{erg/s} << L_* = 3\times10^{39}\ \text{erg/s}$

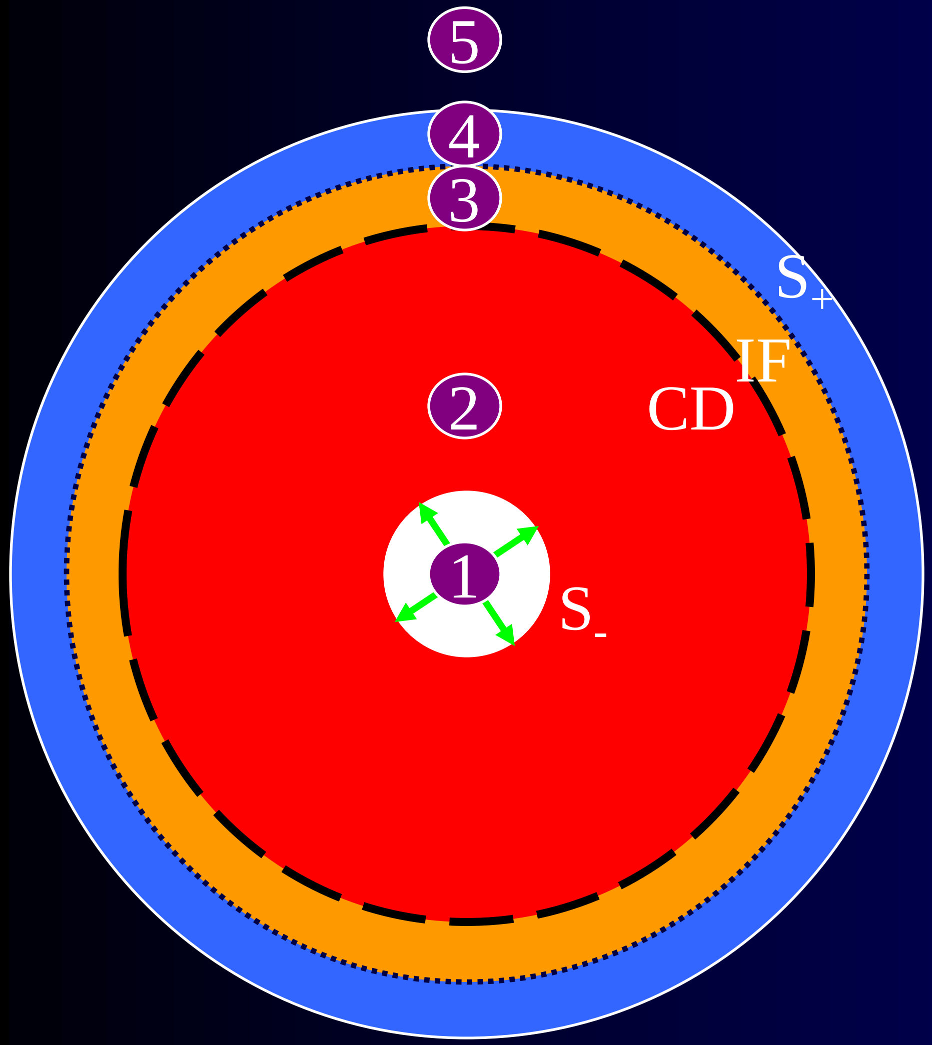
➡ Most of energy lost as radiation!
Less 1 % converted into mechanical luminosity

Effects of stellar winds (SWs) on ISM:

- Observationally V_W is better determined than $\dot{M}_W$
- We use here for estimates: $\dot{M}_W = 10^{-6}\ \mathrm{M_\odot/yr}$

  $V_W = 2000\ \mathrm{km/s}, \quad L_W = \frac{1}{2}\dot{M}_W V_W^2 = 10^{36}\ \mathrm{erg/s}$
- Note: kinetic wind energy >> thermal energy
- Star also has Lyc output: $S_* = 10^{49}\ \mathrm{s^{-1}}$
- Hypersonic wind flows into HII region: $\mathrm{M} = \frac{V_W}{c_{II}} \approx \frac{2000\ \mathrm{km/s}}{10\ \mathrm{km/s}} = 200$
- SW acts as a piston $\Rightarrow$ shock wave formed (once wind feels counter pressure) facing towards star
- Hot bubble pushed ISM $\Rightarrow$ outwards facing shock
- Contact surface separating shocked wind and ISM

Flow Pattern:



- 1 Free expanding wind shocked at S_-
- 2 Energy driven shocked wind bubble bounded by contact discontinuity CD No mass flux across C
- 3 Shell of compressed HII region, bounded by IF
- 4 Shell of shocked HI region (ambient gas) bounded by S_+
- 5 ambient HI gas

Qualitative Discussion:

- Region 1 : free expanding wind has mainly ram pressure, $P \approx \rho_W V_W^2, \ \rho_W = \dfrac{\dot{M}_W}{4\pi r^2 V_W}$

- Region 2 : shocked wind; the post-shock temperature is given by $P_b = \dfrac{3}{4}\rho_b V_W^2 = \dfrac{3}{16}\rho_W V_W^2 = 2 n_b k_B T_b$

$$\Rightarrow T_b = \frac{3}{32}\frac{\overline{m}}{k_B} V_W^2 \approx 4\times10^7 \text{ K}$$

  - Note: S_- moves slowly with respect to wind
  - Since c_b ~ 600 km/s and CD slows down $(c_b >> \dot{R}_b)$, pressure is uniform and energy is mostly thermal
  - n_b low, therefore $\tau_{cool} >> \tau_{dyn} = R_b / \dot{R}_b$ region is adiabatic
  - Density jump by factor 4 ⟹ region extended

- Region 3 : expansion of high pressure region drives outer shock S_+; compresses HII gas into thin shell
  - Density high (at least 4 times n_0)
  - Post-shock temperature $T_{II} << T_b$ since $\dot{R}_b << V_W$

    } cooling high
  - HII region *trapped* in dense outer shell, once

$$\dot{N}_{rec} = 4\pi\ \beta^{(2)} R^2 \delta R\, n_{sh}^2 \geq S_*$$

  - Since wind sweeps up ambient medium into shell:

$$M_{sh} = \frac{4}{3}\pi\rho_0 R_{sh}^3 = 4\pi\ R^2 \delta R\, n_{sh}\overline{m}$$

  - Pressure uniform since $\tau_{sc} = \delta R / c_{sh} << \tau_{dyn}$
- Region 4 : between IF and S_+, shock isothermal $T_I << T_{II}$
- Region 5 : ambient gas at rest at T_I

Simple Model for stellar wind expansion:

- Extension of Regions 3 + 4 is $\delta R \ll R_b$

  $$\Rightarrow R_{sh} = R_b + \delta R \approx R_b$$

- Extension of Region 2 >> Region 1

  therefore it is assumed that 2 occupies all space

- Equations:

$$\mathrm{M_{sh}} = \int^{r} \rho(r')\, d^3 r' \text{ (mass conservation)}$$

$$\mathrm{E_{th}} = \frac{1}{\gamma - 1} \int^{r} p(r')\, d^3 r' \text{ (thermal energy)}$$

$$\frac{d}{dt}(\mathrm{M_{sh}}\, \dot{\mathrm{R}}_{\mathrm{b}}) = 4\pi\, \mathrm{R_b^2}\, \mathrm{P_b} \text{ (momentum conservation)}$$

$$\frac{d\, \mathrm{E_{th}}}{dt} = \mathrm{L_W} - 4\pi\, \mathrm{R_b^2}\, \dot{\mathrm{R}}_{\mathrm{b}} \mathrm{P_b} \text{ (energy conservation)}$$

Similarity Solutions:

- No specific length and time scales involved, i.e. r and t do not enter the equations separately

  ⇒ PDE ⟶ ODE, with variable $\boxed{R_b = At^{\alpha}}$
- Thermal energy given by $E_{th} = \frac{4}{3}\pi R_b^3 \frac{3}{2} P_b = 2\pi P_b R_b^3$
- Combining the conservation equations yields

$$R_b^4 \dddot{R}_b + 12 R_b^3 \dot{R}_b \ddot{R}_b + 15 R_b^2 \dot{R}_b^3 = \frac{3}{2\pi}\frac{L_W}{\rho_0}$$

- Substituting the similarity variable into equation

$$A^5\left[\alpha(\alpha-1)(\alpha-2)+12\alpha^2(\alpha-1)+15\alpha^3\right] t^{5\alpha-3} = \frac{3L_W}{2\pi\rho_0}$$

- RHS is *time-independent* → $5\alpha - 3 = 0 \Rightarrow \alpha = \frac{3}{5}$

$$A = \left(\frac{125}{154\pi}\right)^{1/5} \left(\frac{L_W}{\rho_0}\right)^{1/5}$$

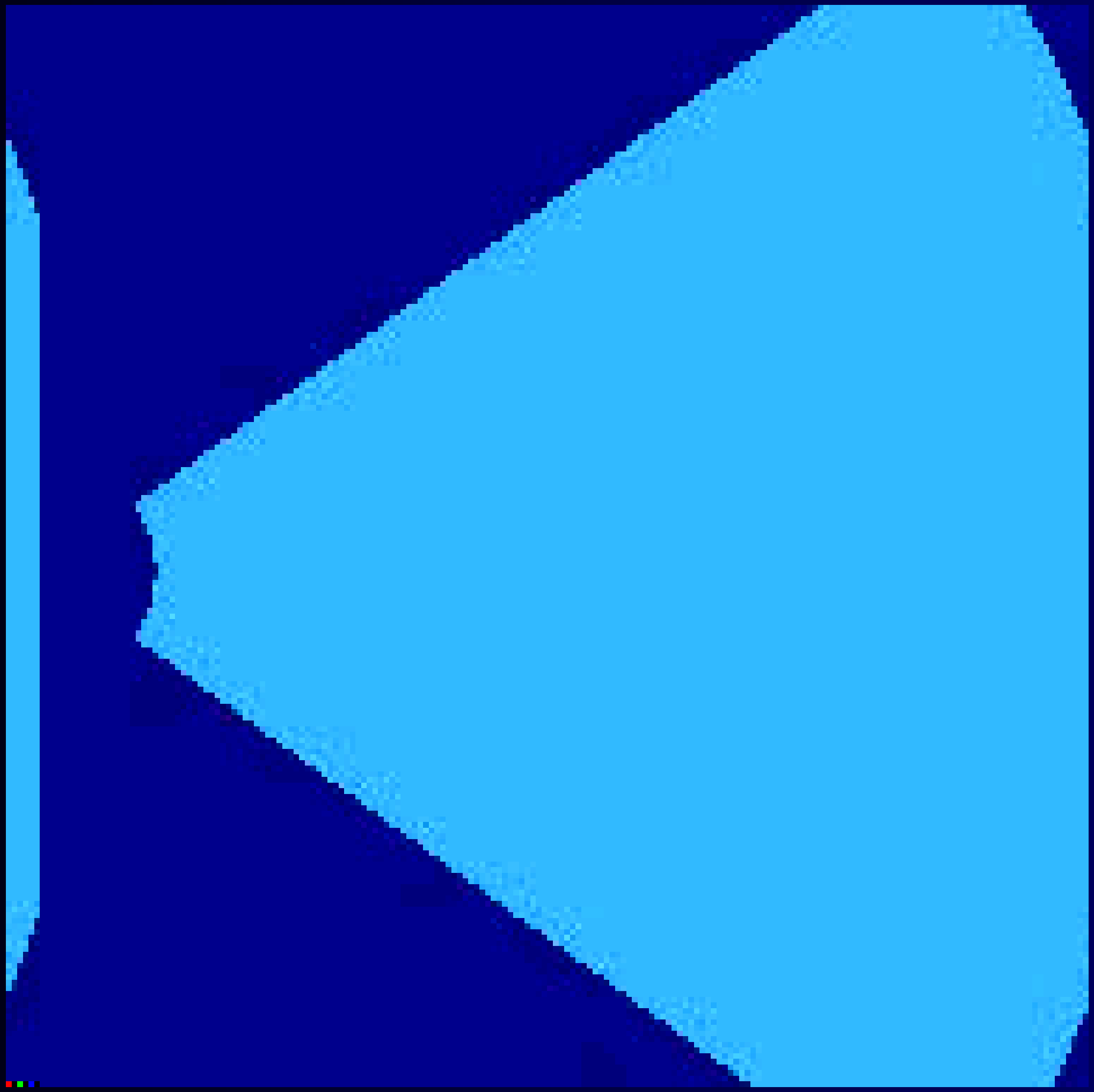
- Thus the solution reads:

$$R_b = A\,t^{3/5} = \left(\frac{125}{154\pi}\right)^{1/5} \left(\frac{L_W}{\rho_0}\right)^{1/5} t^{3/5}$$

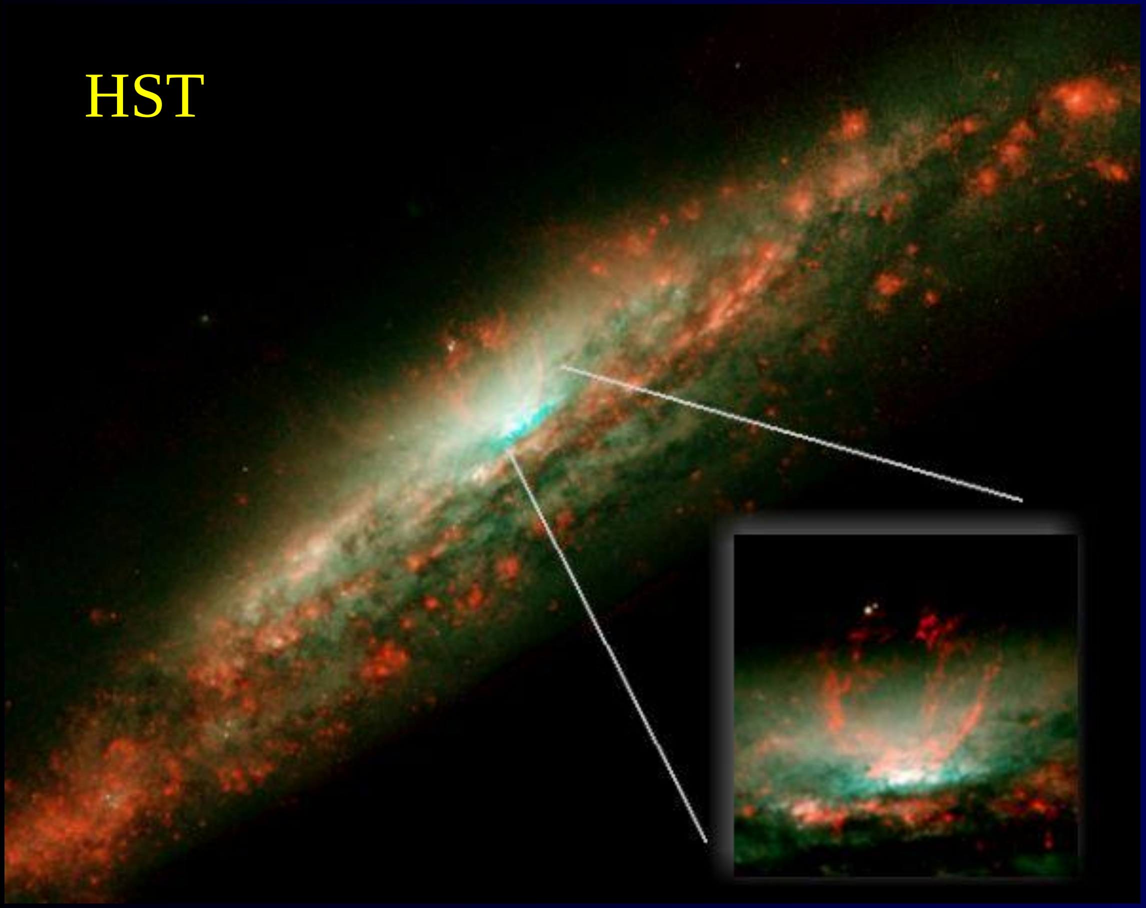
$$\dot{R}_b = V_{sh} = \frac{3}{5}\frac{R_b}{t} = \frac{3}{5} A\,t^{-2/5}$$

- Note: we have assumed $P_I \ll P_{sh} = \rho_0 V_{sh}^2$

Numerical Simulation



II.5 Superbubbles



- NGC 3079: edge-on spiral, D~17 Mpc
- Starburst galaxy with active nucleus
- Huge nuclear bubble generated by massive stars in concert rising to z~700 pc above disk
- Note: substantial fraction of energy blown into *halo!*

More superbubbles:

- Ring nebula Henize 70 (N70) in the LMC
- Superbubble with ~100 pc in diameter, excited by SWs and SNe of many massive stars
- Image by 8.2m VLT (+FORS)

Some Facts

- Massive OB stars are born in associations ($N_* \sim 10^2$-10^3)
- If this happens approx. coeval, SWs and SN explosions are *correlated in space and time!*
- About >50% of Galactic SNe occur in clusters
- MS time is short: $\tau_{\mathrm{MS}} = 3\times10^7 \left(m/10\mathrm{M}_0\right)^{-1.6}\,\mathrm{yr}$

⇒ stars occupy small volume during SB formation

⇒ SB evolution can be described by energy injection from centre of association

- Initially energy input from SWs + photon output rate of O stars dominates $L_W = \frac{1}{2}\dot{M}_W V_W^2 \approx 6\times10^{35}\,\mathrm{erg/s}$ for O7 star

Simple expansion model:

- In early wind phase SB expansion is given by:

$$R_{SB} = 269\,\mathrm{pc}\left({L_{38}}/{n_0} \right)^{1/5} t_7^{3/5}$$

cf. Cygnus supershell: R_S~225 pc
(Cash et al. 1980)

$$L_{38} = {L_W}/{10^{38}\,\mathrm{erg/s}},\quad t_7 = {t}/{10^7\,\mathrm{yr}}$$

McCray & Kafatos, 1987

- After $\tau_{MS} \approx 5\times10^6$ yr last O star leaves main sequence and energy input is dominated by succesive SN explosions
- Thus for $5\times10^6\ \mathrm{yr} \le t \le 5\times10^7\ \mathrm{yr}$ until last SN occurs

(M ~ 7 $M_\odot$) subsequent ejecta input at energy E~10^{51} E_{51} erg ***mimic*** a **stellar wind!**

- If the number of OB stars is N_* energy input is given by

$$L_{SB} = \frac{N_* 10^{51} E_{51}}{\tau_{MS}\left(M_{\min}\right)}$$

- Here we have assumed:
  - A common bubble is formed
  - Bubble is energy driven (like SW case)
  - Energy input by SNe is constant with time (therefore taking MS life time of least massive SN)
  - Ambient density is constant
- The expansion is given by:

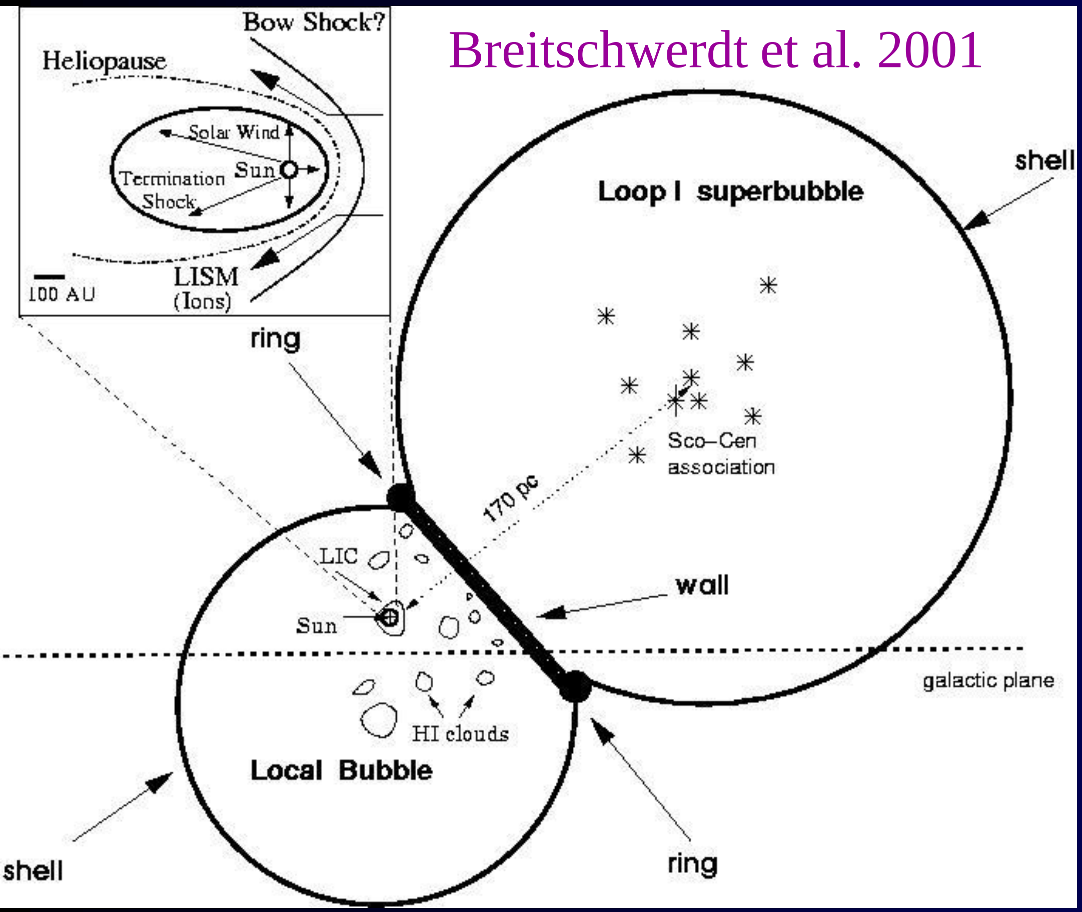
$$R_{SB} = 97\ \mathrm{pc} \left(N_* E_{51} \big/ n_0 \right)^{1/5} t_7^{3/5}$$

$$V_{SB} = 5.7\ \mathrm{km/s} \left(N_* E_{51} \big/ n_0 \right)^{1/5} t_7^{-2/5}$$

- Most of SB size is due to SN explosions!
- SB velocity larger than stellar drift

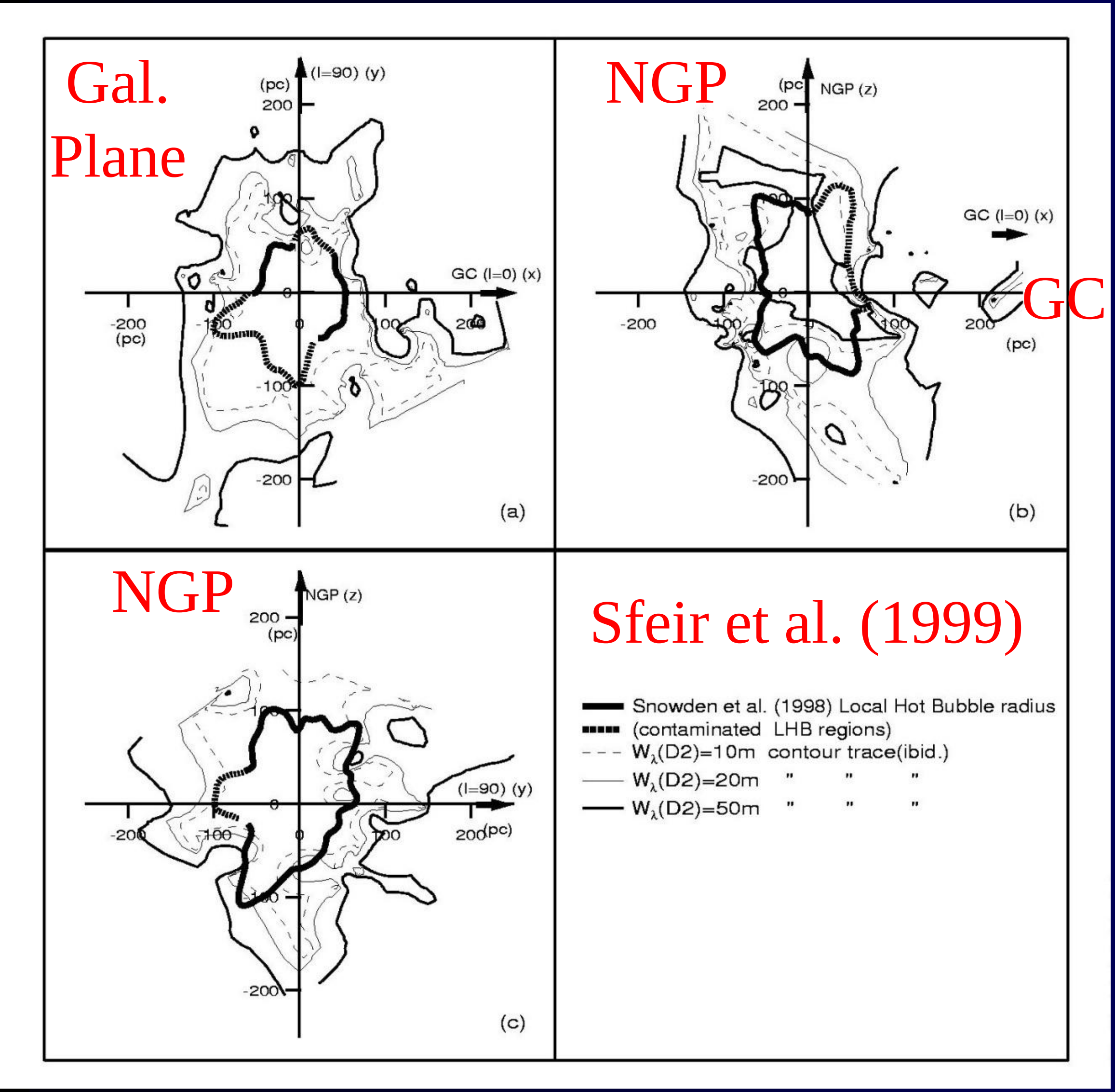
→ explosions occur always *INSIDE* bubble

Example: Local (Super-)Bubble



- Solar system shielded from ISM by heliosphere
- Nearest ISM is diffuse warm HI cloud (LIC)
- LIC embedded in low density soft X-ray emitting region: Local Bubble (R_{LB} ~ 100 pc)
- Origin of LB: multi-SN?

NaI absorption line studies



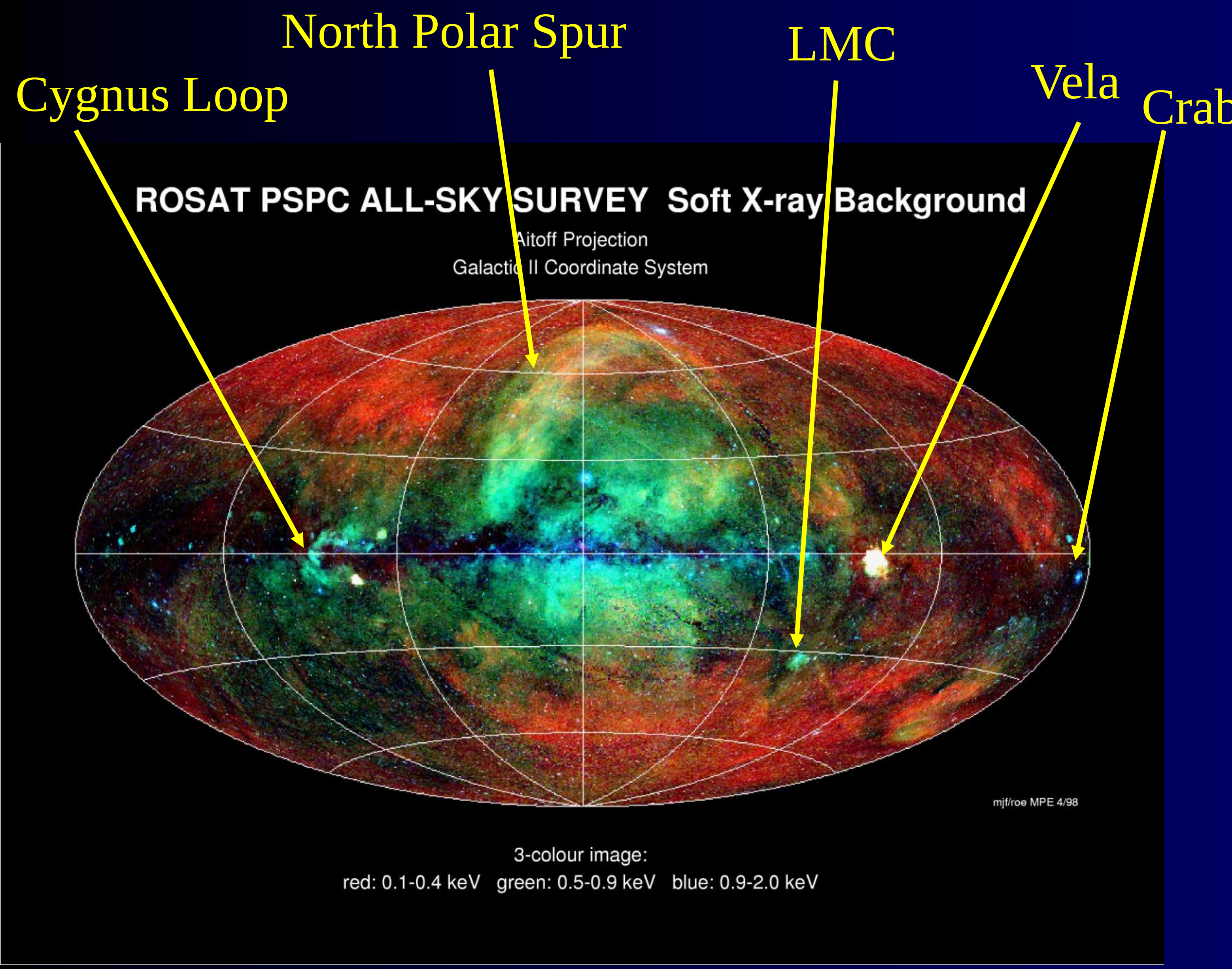
- 3 sections through Local Bubble
- LB inclined ~ 20° wrt NGP

→ LB ⊥ Gould's Belt

- LB open towards NGP?

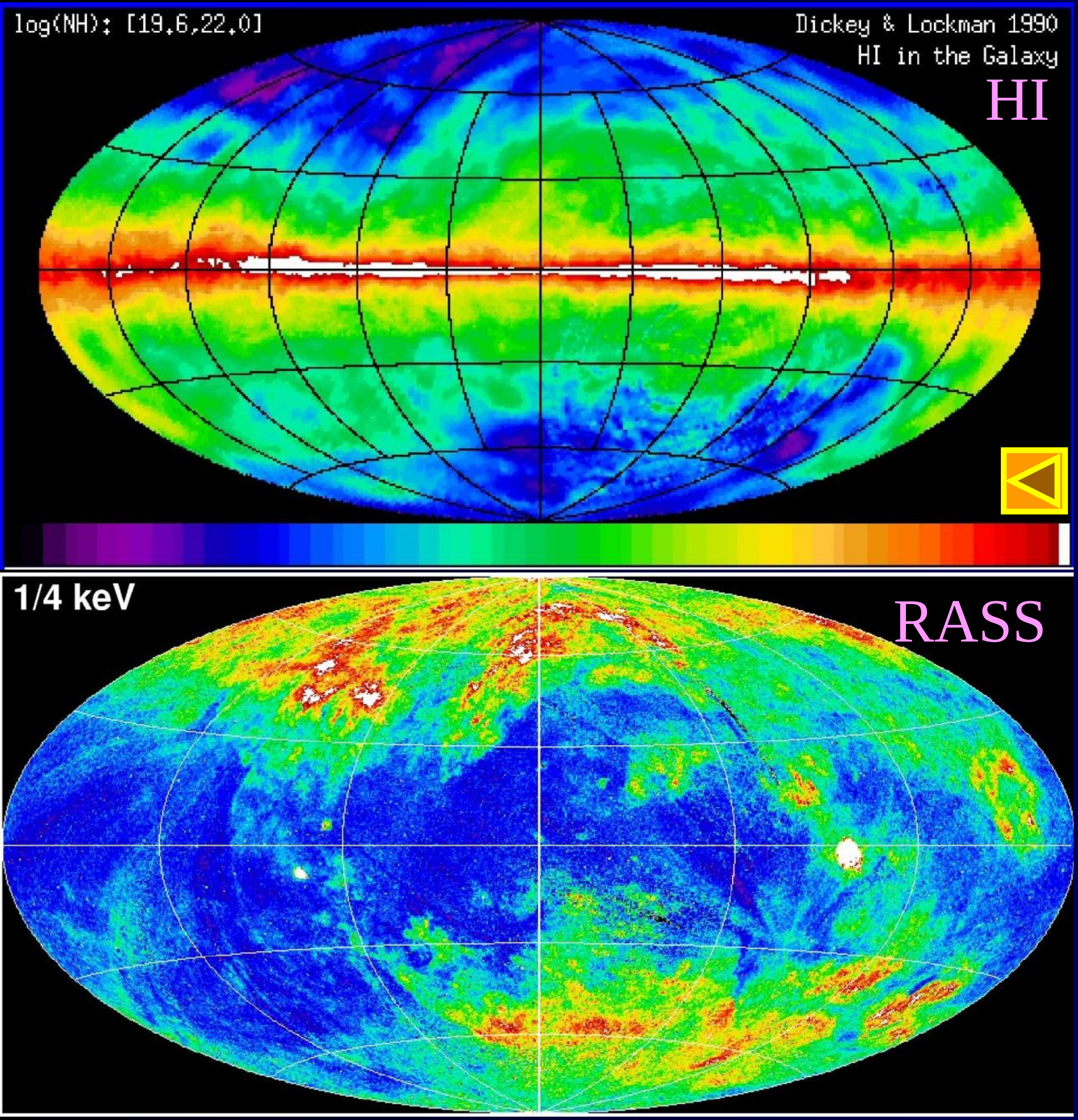
→ Local Chimney

RASS



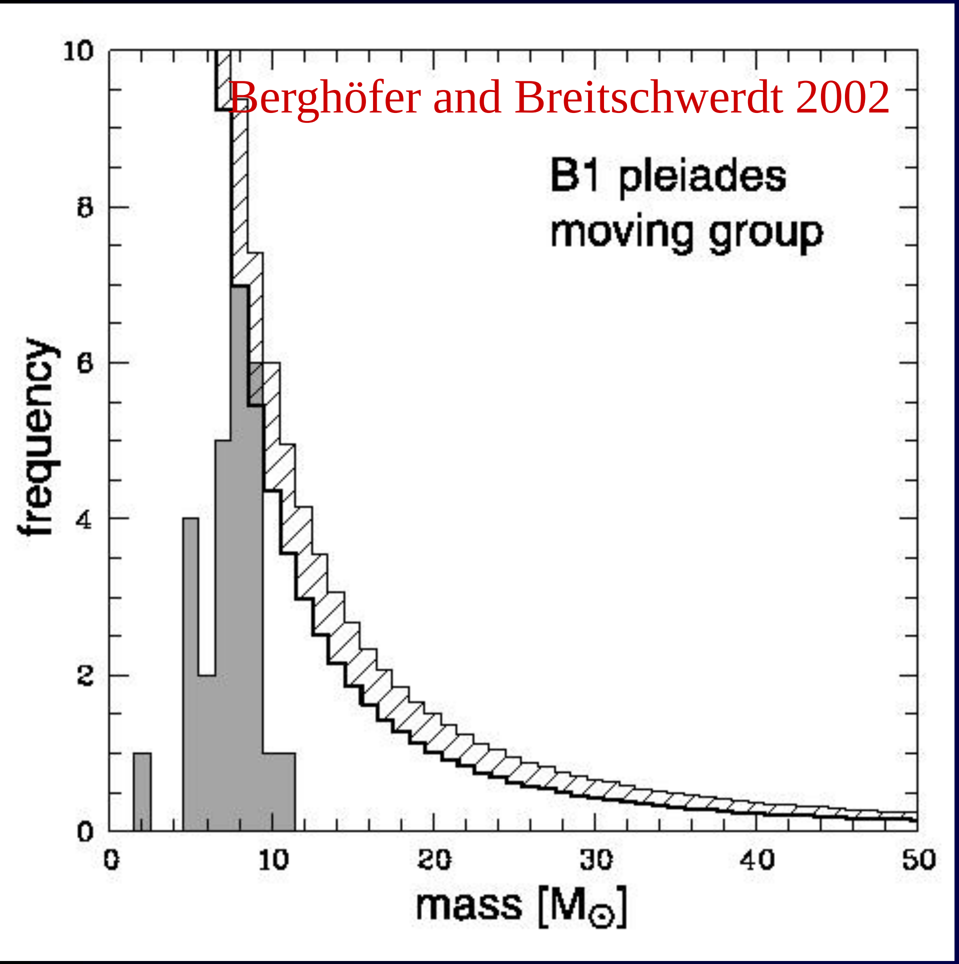
North Polar Spur
LMC
Cygnus Loop
Vela
Crab
ROSAT PSPC ALL-SKY SURVEY Soft X-ray Background
Aitoff Projection
Galactic II Coordinate System
mjf/roe MPE 4/98
3-colour image:
red: 0.1-0.4 keV green: 0.5-0.9 keV blue: 0.9-2.0 keV

Anticorrelation: SXRB – N(HI)



- Anticorrel. on large angular scales for soft emission
- Increase of SXR flux: disk/pole ~3
- Absorption effect: ~50% local em.

Local Stellar Population

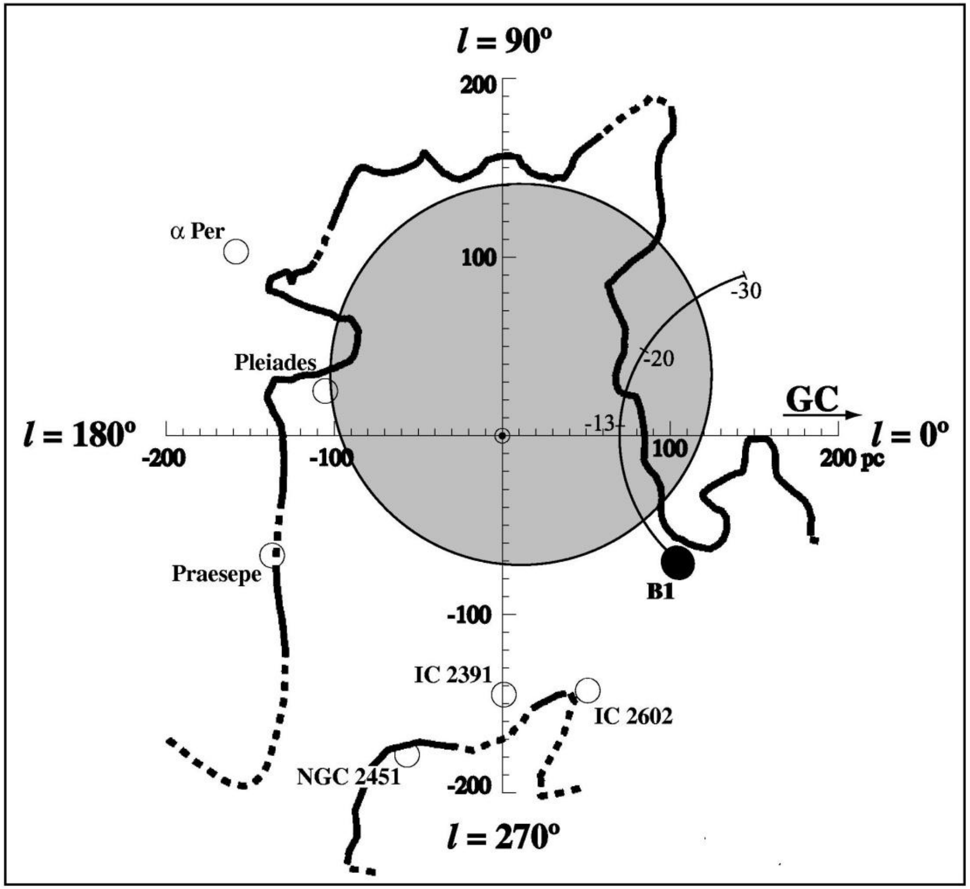


- Local moving groups (e.g. Pleiades, subgr. B1)
- 1924 B-F-MS stars (kin.): *Hipparcos* + photometric ages (Asiain et al. 1999)
- Youngest SG B1: 27 B, $\tau \approx 20 \pm 10$ Myr, $D_0 \approx 120$ pc
- Use evol. track (Schaller 1992): det. stellar masses
- IMF (Massey et al 1995):

$$\Rightarrow N(m) = N_0 \left(\frac{m}{m_0} \right)^{\Gamma - 1}, \Gamma = -1.1$$

Young Stellar Content and Motion

Berghöfer and Breitschwerdt 2002



- Adjusting IMF (B1):

$$N(m = 8M_0) = 7$$

$$\Rightarrow N(m) = 551.6\left(\frac{m}{M_0}\right)^{\Gamma-1}$$

$$N(m_{max}) \leq 1 \Rightarrow m_{max} \cong 20\, M_0$$

$$\Rightarrow N_{SN} = \int_{m_{min}}^{m_{max}} N(m)\, dm \approx 21$$

- 2 B stars still active
- SNe explode in LB

Formation and Evolution of the Local Bubble

- Energy input by sequential SNe

$$\tau_{\mathrm{MS}} = 3\times10^{7}\left(\frac{m}{10\,\mathrm{M}_0}\right)^{-\alpha}\mathrm{yr},\ \alpha = 1.6 \Rightarrow \mathrm{m} = \mathrm{m}(\tau)$$

$$\mathrm{L}_{\mathrm{SB}} = \mathrm{E}_{\mathrm{SN}}\frac{d\mathrm{N}_{\mathrm{SN}}}{dt} = \mathrm{L}_0 \mathrm{t}^{\delta},\ \delta = -(\Gamma/\alpha+1) \approx -0.3$$

- Assumption: coeval star formation

Star deficiency: $m \geq 10\,\mathrm{M}_0 \Rightarrow \tau_{\mathrm{cl}} \leq 2.5\times10^{7}\,\mathrm{yr}$

First explosion: 15 Myr ago $(\mathrm{m}_{\mathrm{max}} = 20\,\mathrm{M}_0)$

Superbubble Evolution

Analytic Model:

$$\mathrm{M_{sh}} = \int^{r} \rho(r')\, d^3r' \ \text{(mass conservation)}$$

$$\mathrm{E_{th}} = \frac{1}{\gamma - 1}\int^{r} p(r')\, d^3r' \ \text{(thermal energy)}$$

$$\frac{d}{dt}(\mathrm{M_{sh}}\, \dot{\mathrm{R}}_{\mathrm{b}}) = 4\pi\, \mathrm{R_b^2}\, \mathrm{P_b} \ \text{(momentum conservation)}$$

$$\frac{d\,\mathrm{E_{th}}}{dt} = \mathrm{L_{SB}}(t) - 4\pi\, \mathrm{R_b^2}\, \dot{\mathrm{R}}_{\mathrm{b}} \mathrm{P_b} \ \text{(energy conservation)}$$

Similarity solution:

$$R_b = At^{\mu}, \quad \mu = \frac{\delta+3}{5-\beta}, \quad \rho = \tilde{\rho}\left(\frac{r}{R_0}\right)^{-\beta} = K_0 r^{-\beta}$$

$$\mathrm{IF}: \ \beta = 0, \ \alpha = 1.6, \ \Gamma = -1.1 \Rightarrow \mu \approx 0.54$$

- Note that similarity exponent μ is between SNR (Sedov phase: μ=0.4) AND SW/SB (μ=0.6)
- The mass of the shell is given by:

$$M_{sh} = \int_0^{R_b} 4\pi r^2 K_0 r^{-\beta} dr = \frac{4\pi}{3-\beta} K_0 R_b^{3-\beta}$$

- After some tedious calculations we find:

$$A = \left[\frac{(5-\beta)^3(3-\beta)}{2\pi(\alpha+3)(7\alpha-\beta-\alpha\beta+11)(4\alpha-2\beta-\alpha\beta+7)}\frac{L_0}{K_0}\right]^{\frac{1}{5-\beta}}$$

Local Bubble (Analytic results)

- Local Bubble at present:

$$n_0 = 30\,\mathrm{cm}^{-3},\ \tau_{exp} = 13\,\mathrm{Myr} \Rightarrow R_b \approx 146\,\mathrm{pc},\ V_{sh} \approx 5.9\,\mathrm{km/s}$$

- Present LB mass: $M_{LB} \geq 600\,M_0,\ M_{ej} \leq 200\,M_0$

Mass loading! (decreases radius)

- Average SN rate in LB: $f_{SN} \cong 1/(6.5\times10^5\ \mathrm{yr})$
- Cooling time: $\tau_c \approx 13\,\mathrm{Myr}\ (n_b \approx 5\times10^{-3}\,\mathrm{cm}^{-3},\ T_b \approx 10^6\,\mathrm{K})$

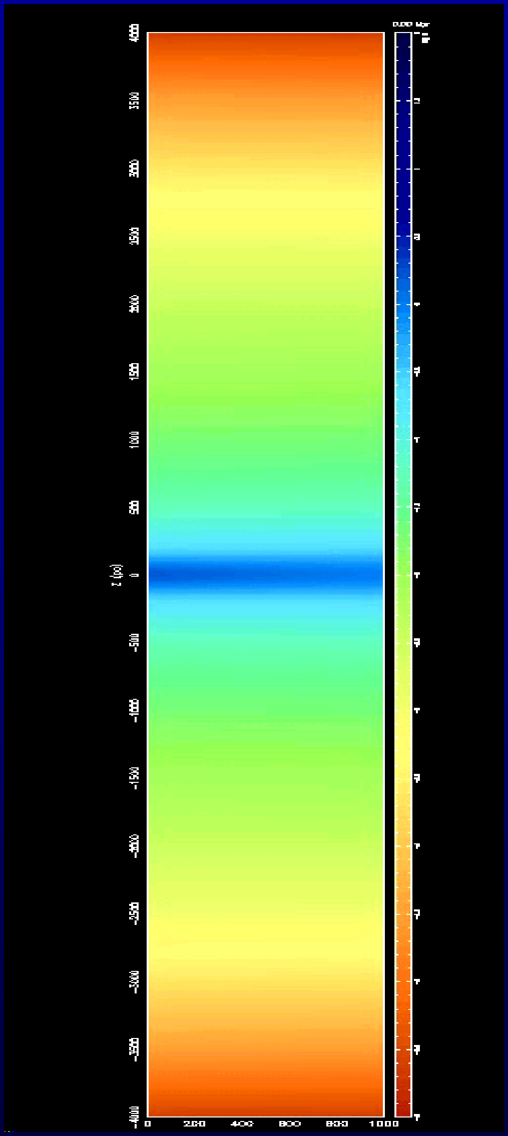
X-ray emission due to last SN(e)!

- Energy input by recent SNe: $\dot{E}_{SN} = E_{SN}\dfrac{dN}{dt}$

$$\dot{E}_{SN} \approx 4.3\times10^{36}(1+t_7)^{0.31} \approx 5.2\times10^{36}\,\mathrm{erg/s}$$

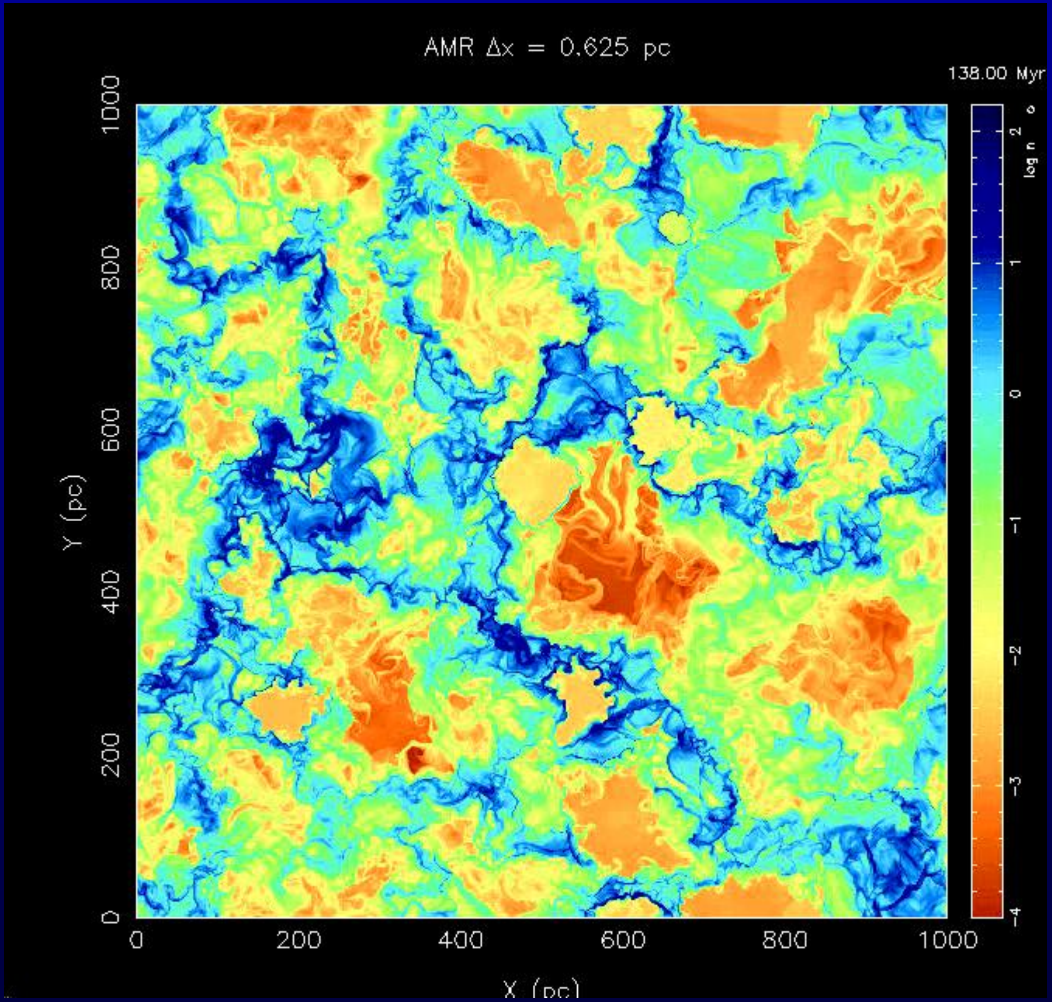
Disturbed background ambient medium

z

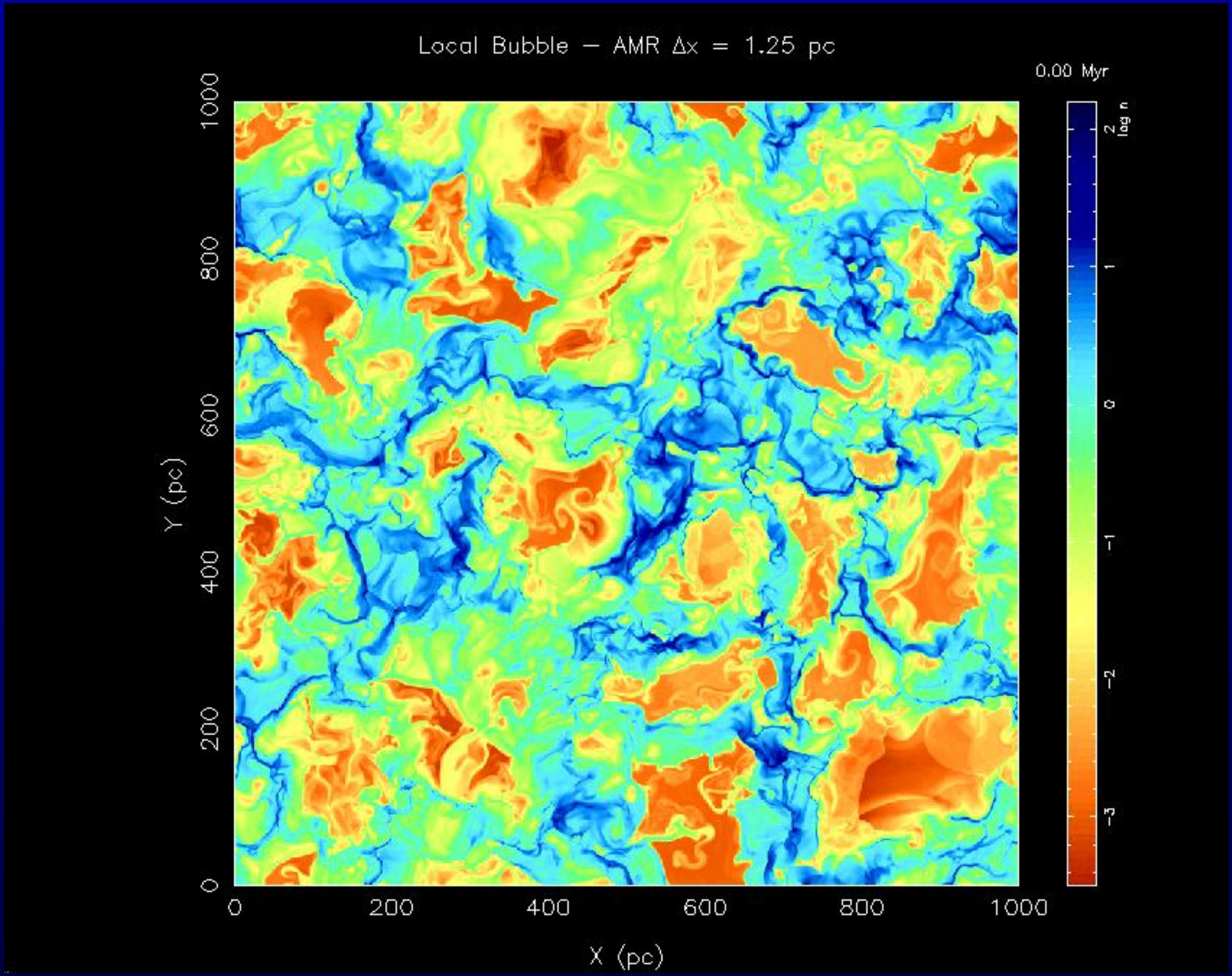


x

y



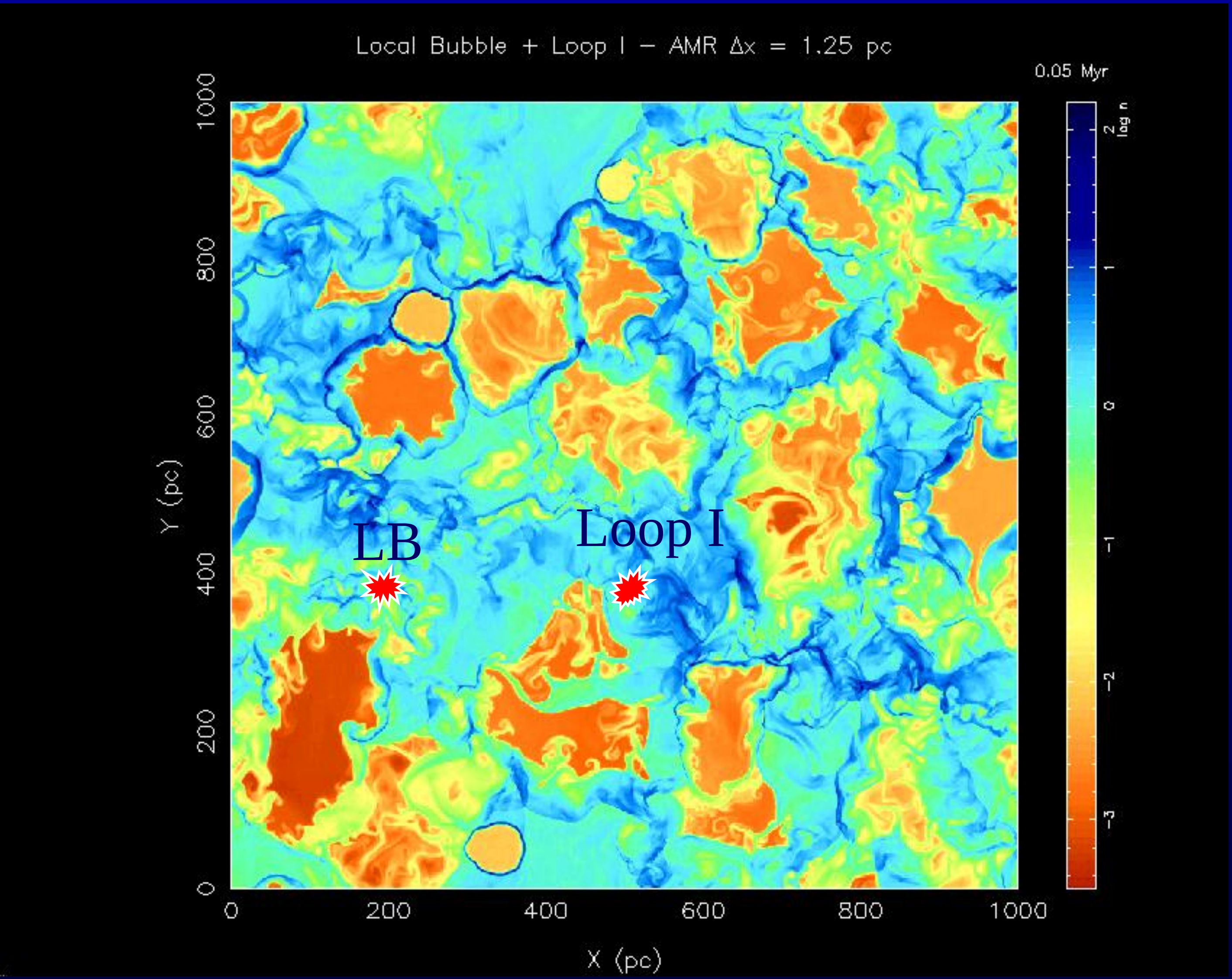
Local Bubble Evolution: Realistic Ambient Medium



- Density
- Cut through Galactic plane
- LB originates at (x,y) = (200 pc, 400 pc)
- SNe Ia,b,c & II at Galactic rate
- 60% of SNe in OB associations
- 40% are random
- Grid resolution:

  1.25 and 0.625 pc

Numerical Modeling (II)



- Density
- Cut through galactic plane
- LB originates at (x,y) = (200 pc, 400 pc)
- Loop I at (x,y) = (500 pc, 400 pc)

Results

Bubbles collided ~ 3 Myr ago

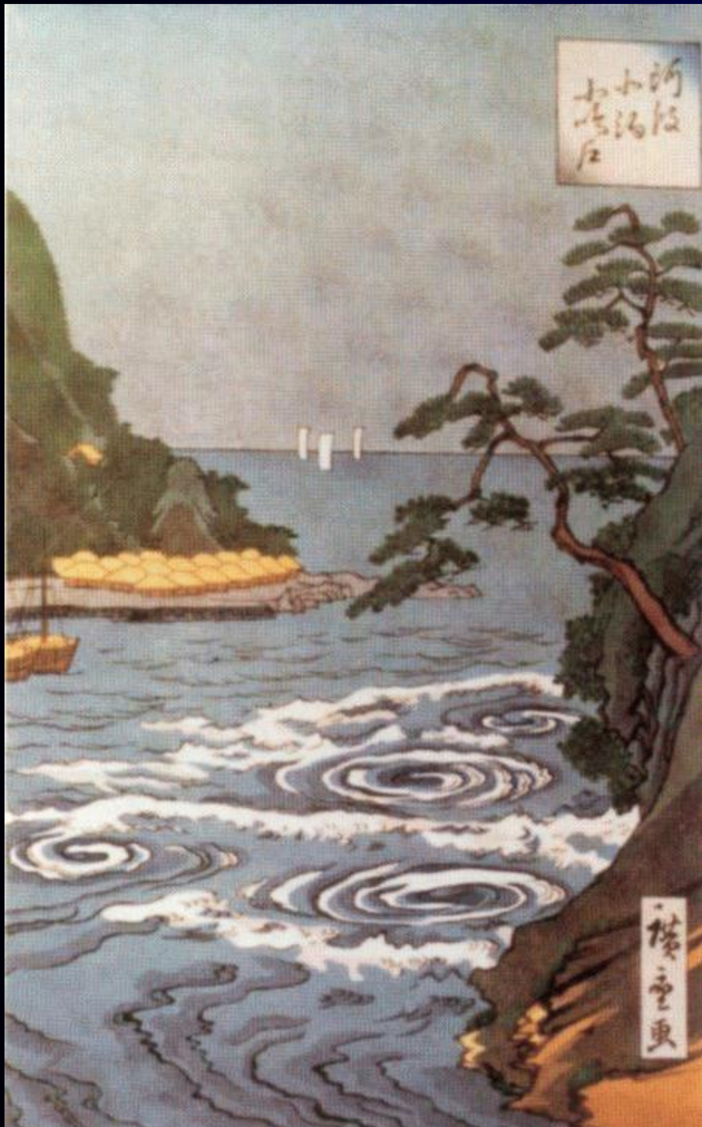
Interaction shell fragments in ~3Myrs

Bubbles dissolve in ~ 10 Myrs

II.6 Instabilities

- Equilibrium configurations (especially in plasma physics) are often subject instabilities
- Boundary surfaces, separating two fluids are susceptible to become unstable (e.g. contact discontinuity, stratified fluid)
- Simple test: linear perturbation analysis
  - Assume a static background medium at rest
  - Subject the system to finite amplitude disturbances
  - Test for exponential growth
  - However: no guarantee, system can be 1st order stable and 2nd or higher order unstable

Kelvin-Helmholtz instability



- Why does the surface on a lake have ripples?

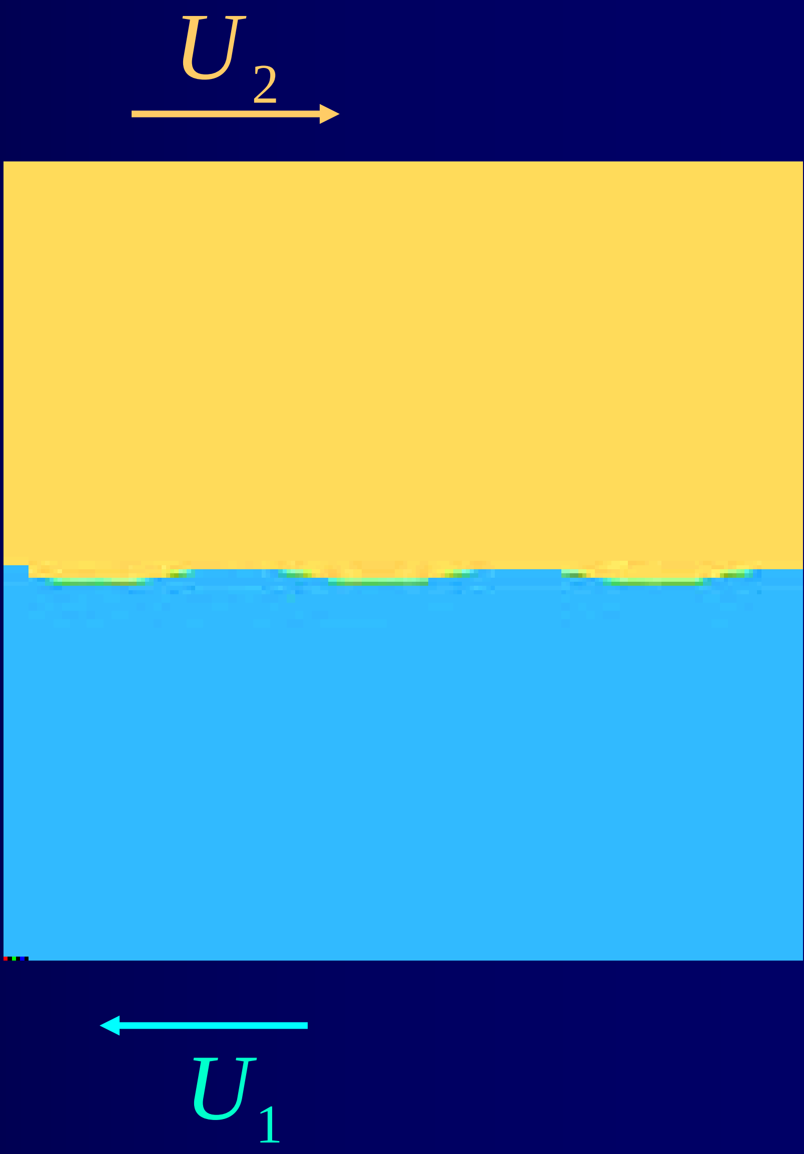
Examples:

Cloud – wind interface:

- Shear flow generates ripples and kinks in the cloud surface due to Kelvin-Helmholtz instability
- „Cat‘s eye“ pattern typical

2D Simulation of Shear Flow

Vincent van Gogh knew it!



Normal modes analysis:

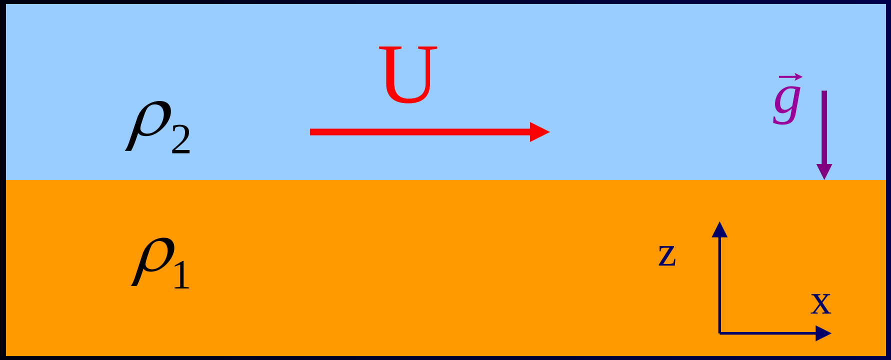
- Assume incompressible inviscid fluid at rest ($\vec{\nabla}\vec{v} = 0$)
- Two stratified fluids of different densities move relative to each other in horizontal direction x at velocity U
- Let disturbed density at (x,y,z) be $\rho + \delta\rho$

  corresponding change in pressure is δP

  the velocity components of perturbed state are:

  $U + u,\ v$ and w in x-, y- and z-direction

  thus the perturbed equations read:



Stratified fluid

$\vec{g}$ can stands for any acceleratiion!

$$\rho\frac{\partial u}{\partial t}+\rho U\frac{\partial u}{\partial x}+\rho w\frac{dU}{dz}=-\frac{\partial}{\partial x}\delta P$$

$$\rho\frac{\partial v}{\partial t}+\rho U\frac{\partial v}{\partial x}=-\frac{\partial}{\partial y}\delta P$$

$$\rho\frac{\partial w}{\partial t}+\rho U\frac{\partial w}{\partial x}=-\frac{\partial}{\partial z}\delta P-g\delta\rho$$

$$\rho\frac{\partial \delta\rho}{\partial t}+U\frac{\partial \delta\rho}{\partial x}=-w\frac{d\rho}{dz}$$

$$\frac{\partial \delta z_s}{\partial t}+U_s\frac{\partial \delta z_s}{\partial x}=w(z_s)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

where $U_s=U(z_s)$

z_s is the surface at which ρ changes discontinously

Disturbances vary as

$$\exp i\left[k_x x+k_y y+\sigma t\right]$$

Instability if $\mathrm{Im}(\sigma)<0$

- Inserting the normal modes yields a DR

$$\frac{d}{dz}\left\{\rho[\sigma+k_xU]\frac{dw}{dz}-\rho k_x\left(\frac{dU}{dz}\right)w\right\}-k^2\rho[\sigma+k_xU]w=\frac{gk^2w}{[\sigma+k_xU]}\frac{d\rho}{dz}$$

- At the interface U is discontinous, but the perturbation velocity *w* must be unique
- Integrate over a small „box“ $(z_s-\varepsilon, z_s+\varepsilon)$, with $\varepsilon\to 0$

$$\Delta_s\left\{\rho[\sigma+k_xU]\frac{dw}{dz}-\rho k_x\left(\frac{dU}{dz}\right)w\right\}=gk^2\Delta_s(\rho)\left(\frac{w}{\sigma+k_xU}\right)_s$$

BC

where $\Delta_s(f)=f_{z=z_s+0}-f_{z=z_s-0}$

- Since we have a discontinuity in U and ρ the DR becomes

$$\left(\frac{d^2}{dz^2}-k^2\right)w=0 \qquad \text{since } \frac{dU}{dz}=\frac{d\rho}{dz}=0$$

- Since $\dfrac{w}{\sigma + k_x U}$ must be continous at z_s and w must not increase exponentially on either side, we must have

$$w_1 = A(\sigma + k_x U_1)\exp[+kz],\ (z < 0)$$
$$w_2 = A(\sigma + k_x U_2)\exp[-kz],\ (z > 0)$$

- Applying the BC to solutions:

$$\rho_2(\sigma + k_x U_2)^2 + \rho_1(\sigma + k_x U_1)^2 = gk(\rho_1 - \rho_2)$$

- Expanding and rearranging yields the growth rate

$$\boxed{\sigma = -k_x \frac{\rho_1 U_1 + \rho_2 U_2}{\rho_1 + \rho_2} \pm \left[gk \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} - k_x^2 \frac{\rho_1 \rho_2 (U_1 - U_2)^2}{(\rho_1 + \rho_2)^2} \right]^{1/2}}$$

where $\vec{k}\vec{U} = kU\cos\vartheta$ and $k_x = k\cos\vartheta$

- Two solutions are possible
  - If $k_x = 0$

    the growth rate is simply $\sigma = \pm\sqrt{gk\dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}}$ **R-T instability**

<u>Perturbations transverse to streaming are unaffected by it</u>

  - For every other directions of wave vector instability occurs if:

    $$k > \frac{g\left(\rho_1^2 - \rho_2^2\right)}{\rho_1\rho_2\left(U_1 - U_2\right)^2\cos^2\vartheta}$$

    Kelvin-Helmholtz instability

  - Even for stable stratification $\rho_1 > \rho_2$ (against R-T)

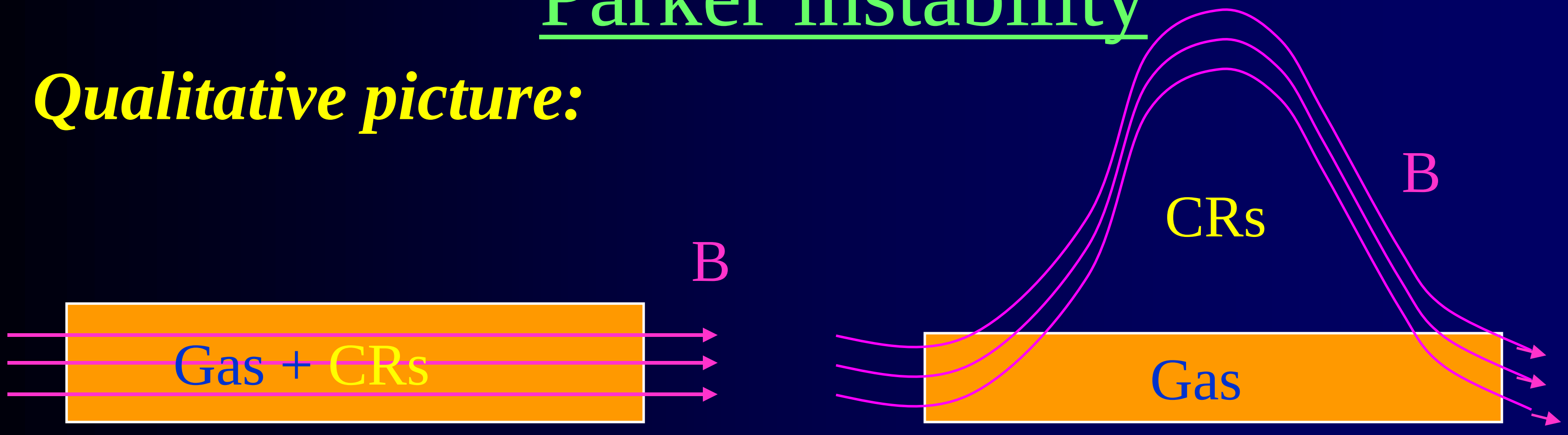
    There is ALWAYS instability no matter how SMALL

    $U_1 - U_2$ is!

  - For large velocity difference large wavelength instability

Parker instability

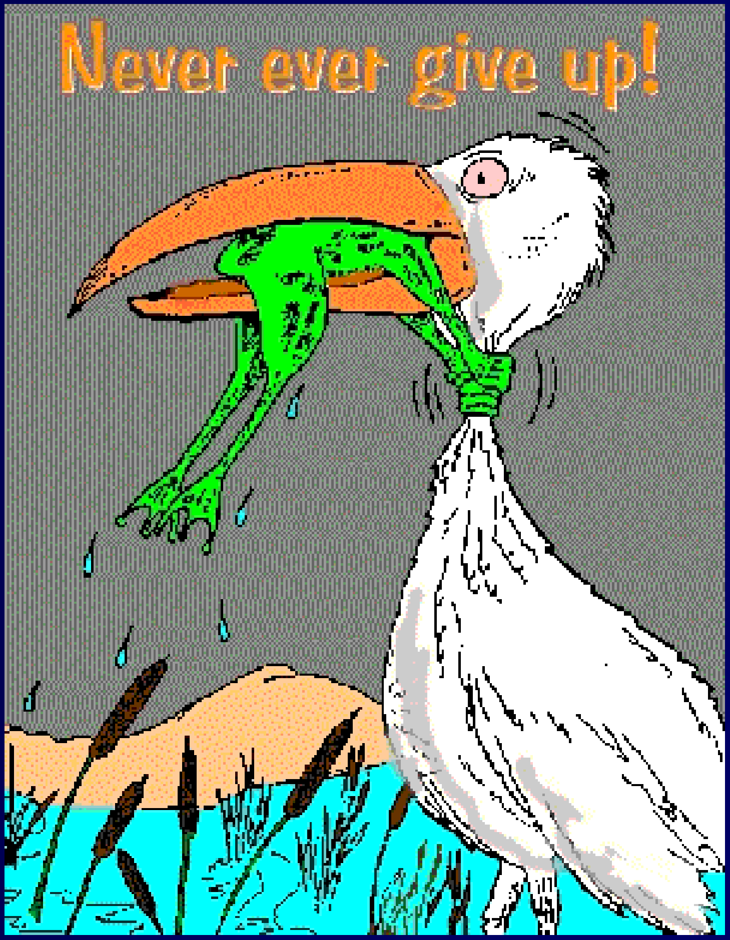
- ***Qualitative picture:***



- *CRs are coupled to magnetic field which is frozen into gas*
- *Magnetic field is held down to disk by gas!*
- CRs exert *buoyancy forces* on field -> *gas slides down along lines to minimize potential energy -> increases buoyancy -> generates magnetic („Parker“) loops!*
- *Note:* CRs and magnetic field have tendency to expand if unrestrained

Do not despair if you did not understand everything right away!

BUT



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Bubbles everywhere!!!

- The End -